Collective enhancement of nuclear level density in the interacting boson model

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The interacting boson model (IBM) is applied at finite temperature for evaluation of a collective enhancement factor in nuclear level densities in the form of a canonical partition function. The most crucial variable turns out to be the effective boson number, whose temperature dependence is studied in the frame of a Nilsson-Bardeen-Cooper-Schrieffer model. Numerical results are obtained in the three IBM limiting symmetries.

I. INTRODUCTION

Excitations at finite temperature play an important role in understanding the basic properties of nuclei: regions of low temperature and spin have been explored for a long time by means of neutron- or light-ion-induced reactions. More recently, heavy-ion reactions have opened the way to the study of equilibrated nuclei at high temperature and spin. In general, nuclear motion at finite temperature consists of contributions from different degrees of freedom, both of single-particle and collective character. While the former feature has been extensively investigated and many even sophisticated models developed to deal with it, relatively less attention has been paid to the latter.

As far as the low-temperature and spin region is concerned, many semiempirical formulas have been deduced for the single-particle component of nuclear level densities; on the other hand, a few simple expressions have been worked out for the collective component, mainly describing schematic cases of rotational or harmonic vibrational motions.

In the last fifteen years the interacting boson model (IBM) (Ref. 1) has met with remarkable success in reproducing collective features of the low-energy spectrum and transition rates of even-even nuclei in a large region of the Periodic Table, including many transitional isotope chains.

The purpose of this paper is to investigate the finite-temperature behavior of the IBM spectra in its dynamic symmetry limits and utilize it for evaluation of the collective contribution to the density of nuclear states at finite temperature. The paper is organized as follows: Sec. II contains the definition of the collective enhancement factor in nuclear level densities, Sec. III contains an analysis of the finite-temperature behavior of the effective boson number; in Sec. IV we give the expression of the IBM partition function and finally, in Sec. V, we show the results of calculations for three nuclei, one for each of the dynamic symmetry limits of IBM together with the conclusions and perspective for future works.

II. NUCLEAR STATE DENSITY

It is intended here to investigate the energy region where noncollective, or quasiparticle, nuclear excitations already form a continuum, but the coupling to collective degrees of freedom is so weak that every quasiparticle excitation can be thought of as the bandhead of a discrete collective spectrum similar to the one built on the ground state.[2] It thus becomes possible to factorize the total density of nuclear states in the form of a quasiparticle state density times a collective factor, which can be approximated by the canonical partition function for a collective Hamiltonian.

With the assumption of decoupling of intrinsic (noncollective) and collective degrees of freedom, the excitation energy of the nucleus, U, can be written as a sum of intrinsic and collective excitation energies, $U=E_i+E_c$, where $E_c \ll U$. The total state density is thus factorized as follows:

$$\rho(U)=\int \rho_i(E_i)\sum_c \delta(U-E_i-E_c)dE_i \ , \tag{1}$$

where the intrinsic state density $\rho_i(E_i)$ is so high that E_i can be taken as a continuous variable and

$$\sum_c \delta(U-E_i-E_c)$$

is the collective state density at energy $E_c=U-E_i$, the sum being over set c of quantum numbers labeling the collective states.

Formula (1) can be rewritten in the equivalent form

$$\rho(U)=\sum_c \rho_i(U-E_c) \ . \tag{2}$$

Since E_c is small with respect to U, the right-hand side of Eq. (2) can be expanded in a Taylor series to the first order in E_c:

$$\rho(U)\simeq \sum_c \left[\rho_i(U)-E_c\frac{\partial\rho_i(U)}{\partial U}\right]$$
$$=\sum_c \left[\rho_i(U)-\frac{E_c}{T}\rho_i(U)\right] \ , \tag{3}$$

where use has been made of the definition of temperature T in energy units, corresponding to an excitation energy U.[2]

Formula (3) is the expansion to the first order in E_c/T of the more general expression

$$\rho(U) \simeq \rho_i(U) \sum_c \exp(-\beta E_c) \,, \quad (4)$$

where $\beta = 1/T$ and

$$Z_c(\beta) = \sum_c \exp(-\beta E_c) \quad (5)$$

is the definition of the canonical partition function for the collective degrees of freedom of the system.

III. EFFECTIVE BOSON NUMBER AT FINITE TEMPERATURE

Evaluation of the collective factor in formula (4) generally requires diagonalization of a collective Hamiltonian $\hat{H}_c$ whose parameters depend on the nuclear temperature. The IBM Hamiltonian concerning this work simulates the excitation of particle-particle, or hole-hole pairs in the valence shell of the nucleus through s- and d-boson interactions. Therefore, the possible temperature dependence of the IBM parameters should be seen, from a microscopic viewpoint, as a consequence of the T dependence of the effective nucleon-nucleon interactions. For a not excessively high temperature ($T \lesssim 5$ MeV), the effective interaction has been assumed constant,[3] thus leading to constant IBM parameters which can be adjusted at zero temperature so as to reproduce the experimental discrete levels and electromagnetic transitions at low excitation energy.

On the other hand, the disappearance of shell effects with increasing temperature[3] washes out collective features as well. Such an effect can be reproduced within the IBM framework by giving the effective boson number N_B a proper temperature dependence. To this end we resort to a microscopic interpretation of bosons such as Cooper pairs in doubly degenerate Nilsson levels[4,5] and deal with the residual pairing interaction at zero, or finite temperature in the Bardeen-Cooper-Schrieffer (BCS) approximation,[6] by assuming that the energies ε_k of the Nilsson levels do not change with temperature in the range of interest ($T \lesssim 5$ MeV).[3] This microscopic interpretation deals with proton and neutron degrees of freedom separately, according to version 2 of the IBM.[1] Let $a_k^\dagger$ be the creator of a nucleon in the kth Nilsson level, characterized by a positive projection of the angular momentum, and $a_{\bar{k}}^\dagger$ the corresponding operator for the time reversed state at the same energy ε_k, but with negative angular momentum projection; we introduce the usual Bogoliubov-Valatin (BV) transformations from particles to quasiparticles (qp's):[6]

$$\eta_k^\dagger = U_k a_k^\dagger - V_k a_{\bar{k}} \,, \quad (6a)$$

$$\eta_{\bar{k}}^\dagger = U_k a_{\bar{k}}^\dagger + V_k a_k \quad (6b)$$

where U_k and V_k are real numbers whose squares are interpreted as complementary occupation probabilities, so that $U_k^2 + V_k^2 = 1$.

Solution of finite-temperature BCS equations provides us with Fermi energies $\lambda(T)$, pairing gaps $\Delta(T)$, qp energies $E_k = [(\varepsilon_k - \lambda)^2 + \Delta^2]^{1/2}$, and qp occupation probabilities:

$$\langle \eta_k^\dagger \eta_k \rangle = \langle \eta_{\bar{k}}^\dagger \eta_{\bar{k}} \rangle = \frac{1}{1+\exp(\beta E_k)} \equiv f_k \,, \quad (7)$$

where the angular brackets denote the statistical average over a grand canonical ensemble of noninteracting qp's.[6]

The average numbers of particle and hole pairs in a given set of Nilsson levels, conveniently assumed to be centered around the Fermi energy, are, respectively,

$$N_{pp} = \sum_{k=k_1,k>0}^{k_2} \langle a_k^\dagger a_{\bar{k}}^\dagger a_{\bar{k}} a_k \rangle = \sum_{k=k_1,k>0}^{k_2} [V_k^2(1-2f_k)+f_k^2] \,, \quad (8a)$$

$$N_{hh} = \sum_{k=k_1,k>0}^{k_2} \langle a_{\bar{k}} a_k a_k^\dagger a_{\bar{k}}^\dagger \rangle = \sum_{k=k_1,k>0}^{k_2} [U_k^2(1-2f_k)+f_k^2] \,, \quad (8b)$$

where k_1 and k_2 indicate the lowest and highest Nilsson level, respectively, and $k>0$ denotes the summation over positive spin projection only.

We note, however, that

$$f_k^2 = f_k f_{\bar{k}} = \langle \eta_k^\dagger \eta_k \rangle \langle \eta_{\bar{k}}^\dagger \eta_{\bar{k}} \rangle$$

expresses the probability of two uncorrelated quasiparticles in the kth level and we subtract,

$$\sum_{k=k_1,k>0}^{k_2} f_k^2$$

from the rhs's of formulas (8a) and (8b) in order to estimate the number of collective pairs corresponding to our bosons.

The effective boson number, for protons and neutrons separately, is thus given by

$$N_B = \min \left[\sum_{k=k_1,k>0}^{k_2} V_k^2(1-2f_k),\ \sum_{k=k_1,k>0}^{k_2} U_k^2(1-2f_k) \right] . \quad (9)$$

N_B depends in general on the width of the energy interval where the Nilsson levels are counted. We assume that the interval is symmetric with respect to the Fermi energy and take the value of the half width $\sqrt{3}\Delta(0)$ fixed at any temperature. This roughly corresponds to the zero-temperature half-width half-maximum interval of the distribution of the elements of the pairing tensor:[6]

$$t_{k\bar{k}} = \langle a_k^\dagger a_{\bar{k}}^\dagger \rangle = \langle a_{\bar{k}} a_k \rangle = U_k V_k (1-2f_k) \quad (10)$$

and measures the extent to which the pairing interaction is effective around the Fermi energy.

In the zero-temperature limit where the qp occupation numbers f_k vanish, the above prescription for the effective boson number in spherical nuclei is tantamount to counting the number of particle pairs above the closest shell closure below the Fermi energy and the number of hole pairs below the next shell closure and choosing the smaller of the two numbers. In the case of deformed nuclei in the middle of a major shell, this prescription gives a boson number smaller than the usual spherical shell estimate, due to the influence of deformed closures.[7] In all cases the boson number given by formula (9) decreases with increasing temperature and vanishes at infinity, since $\lim_{T\to\infty} f_k = \frac{1}{2}$: This corresponds to the vanishing of the collective features in equilibrated nuclei at high temperatures.

IV. THE BOSON PARTITION FUNCTION

With the two main assumptions previously described, namely the T dependence of the boson number N_B and the T independence of the other Hamiltonian parameters, diagonalization of the IBM Hamiltonian at any temperature provides us with the collective energies E_c to be inserted into the definition (5) of the partition function. Here we are dealing only with the three limiting symmetries of the IBM Hamiltonian, thus reduced to a linear combination of Casimir operators for subgroups of U(6), the group spanned by the six collective degrees of freedom corresponding to one s and five d bosons. In all cases the U(6) decomposition chain must contain O(3) so that total angular momentum J is a good quantum number. The collective energies are obtained analytically and the corresponding states belong to irreducible representations (irrep's) of the subgroups appearing in the decomposition chains.

We merely state these well-known results, with the notation of Ref. 1, by labeling each group with the quantum numbers defining its irrep:

Chain I: $U(6) \supset U(5) \supset O(5) \supset O(3)$

$$N_B \quad n_d \quad v, n_\Delta \quad J$$

According to formula (5), the partition function is

$$Z_{\mathrm{I}}(\beta)=\sum_{n_d}\sum_{v,n_\Delta}\sum_J (2J+1)\exp\{-\beta[E_0+\varepsilon_0 n_d+\alpha_0 n_d(n_d+4)+2\beta_0 v(v+3)+2\gamma_0 J(J+1)]\} \ . \tag{11}$$

As is known, the energy spectrum of chain I corresponds to that of an anharmonic vibrator.

Chain II: $U(6) \supset SU(3) \supset O(3)$

$$N_B \quad (\lambda,\mu),\tilde{\chi} \ J \ .$$

with

$$Z_{\mathrm{II}}(\beta)=\sum_{\lambda,\mu,\tilde{\chi}}\sum_J (2J+1)\exp\{-\beta[E_0+\tfrac{2}{3}\delta_0(\lambda^2+\mu^2+\lambda\mu+3\lambda+3\mu)+2\gamma_0 J(J+1)]\} \ . \tag{12}$$

The spectrum of chain II is that of a rotor with axial symmetry, whose ground-state band belongs to the $(2N_B,0)$ irrep of SU(3), while the β and γ bands are contained in the $(2N_B-4,2)$ irrep with $\tilde{\chi}=0$ and $\tilde{\chi}=2$, respectively.

Chain III: $U(6) \supset O(6) \supset O(5) \supset O(3)$

$$N_B \quad \sigma \quad \tau,\tilde{v}_\Delta \quad J$$

and

$$Z_{\mathrm{III}}(\beta)=\sum_{\sigma}\sum_{\tau,\tilde{v}_\Delta}\sum_J (2J+1)\exp\{-\beta[E_0+2\beta_0\tau(\tau+3)+2\eta_0\sigma(\sigma+4)+\gamma_0 J(J+1)]\} \ . \tag{13}$$

Chain III yields the spectrum of a nucleus whose ground state is unstable with respect to γ vibrations.

The summations over the quantum numbers of the three chains include all the irrep's contained in the totally symmetric $[N_B]$ irrep's of U(6); in particular, the angular momentum J runs from 0 to $J_{max}=2N_B$ and the factor $(2J+1)$ expresses the degeneracy of a state having spin J. It is worth pointing out that the degeneracy factor has to be included in all cases, independently of the geometrical interpretation of the dynamic symmetry, since J is always a good quantum number for IBM.

The IBM parameters E_0, ε_0, α_0, β_0, γ_0, δ_0, and η_0 are adjusted on the experimental low-lying levels of positive parity. While the remaining parameters do not change with temperature, the value of E_0 is adjusted at any temperature in such a way that the collective spectrum always begins with a 0^+ state.

Since the summations over the quantum numbers are not mutually independent, our $Z_{coll}(\beta)$ cannot be written in the form of a product of vibrational and rotational partition functions $Z_{vib}Z_{rot}$, as is usually done in the traditional approaches to the calculation of collective enhancement factor:[2] rotations and vibrations are coupled in the irrep's of the reduction chains.

V. RESULTS, COMMENTS, AND PERSPECTIVES

The temperature dependence of the boson number and the partition function has been worked out in three cases representative of the limiting symmetries: ^{110}Cd for chain I, ^{238}U for chain II, and ^{196}Pt for chain III. In order to evaluate the boson number according to formula (9), NBCS calculations for protons and neutrons have been performed at zero and finite temperature[8] by using the Nilsson model parametrization of Ref. 9 and the systematics of pairing interactions of Ref. 10. Solving the finite-temperature NBCS equations allows us to determine the temperature T corresponding to a given excitation energy U shown in Fig. 1 for the three nuclei under investigation.

We have assumed the validity of the ground-state Nilsson parameters and single-particle energies at finite temperature as well; such an assumption would certainly break down for $T \gtrsim 5$ MeV. On the other hand, the nonconservation of the nucleon number in the NBCS formalism reflects itself in a phase transition at low temperature, $T_c \simeq 0.567\Delta(0)$, resulting in a change of slope for the curves in Fig. 1. The number of collective pairs for protons and neutrons are separately calculated according to formula (9) and summed together, thus giving the effective boson numbers plotted versus the temperature in Fig. 2. Here again, the nonconservation of the nucleon number in NBCS calculations has a consequence, the fluctuation in the boson number, which is maximum at zero temperature, as discussed in Ref. 7 and vanishes above the critical temperature T_c.

Since we are mainly interested in the T dependence of N_B, we have not corrected the ground-state boson number for the above mentioned effect. It is worth stressing that the values $N_B(0)=6$ for ^{196}Pt and $N_B(0)=7$ for ^{110}Cd obtained in this work agree with the usual estimate based on the spherical shell closure, while $N_B(0)=12$ for ^{238}U is smaller than the spherical estimate $N_{\rm spher}(0)=15$, owing to the influence of deformed shell closures at

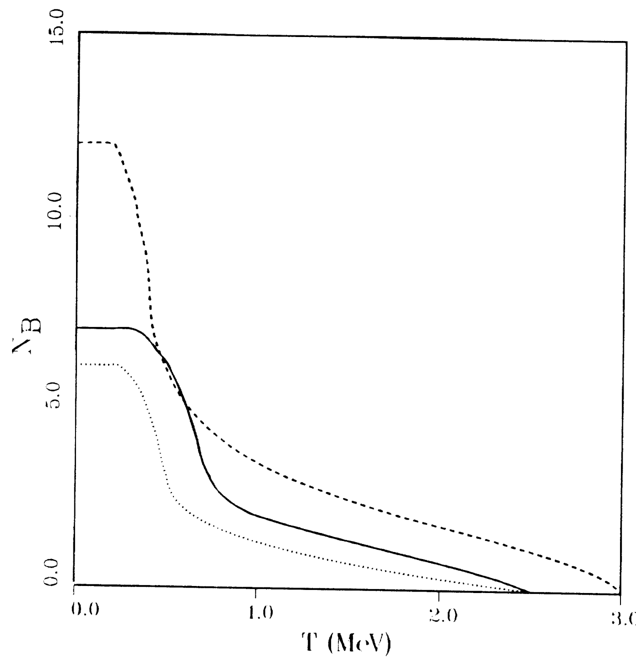

FIG. 2. Boson number versus temperature: solid line—^{110}Cd; dashed line—^{196}Pt; dotted line—^{238}U.

$Z=100$, $N=152$ for the Nilsson levels of Ref. 9 adopted in this work. An interesting feature of Fig. 2 is the vanishing of the boson number, and hence of collective effects at $T \simeq 2.5$ MeV for ^{110}Cd and ^{196}Pt and $T \simeq 3$ MeV for ^{238}U, the latter being in agreement with the results of the self-consistent Hartree-Fock calculations in Ref. 3 for the strongly deformed nucleus ^{168}Yb.

Finally, the boson partition functions in the three symmetries are plotted versus temperature in Fig. 3. The IBM parameters appearing in formulas (11)–(13) have

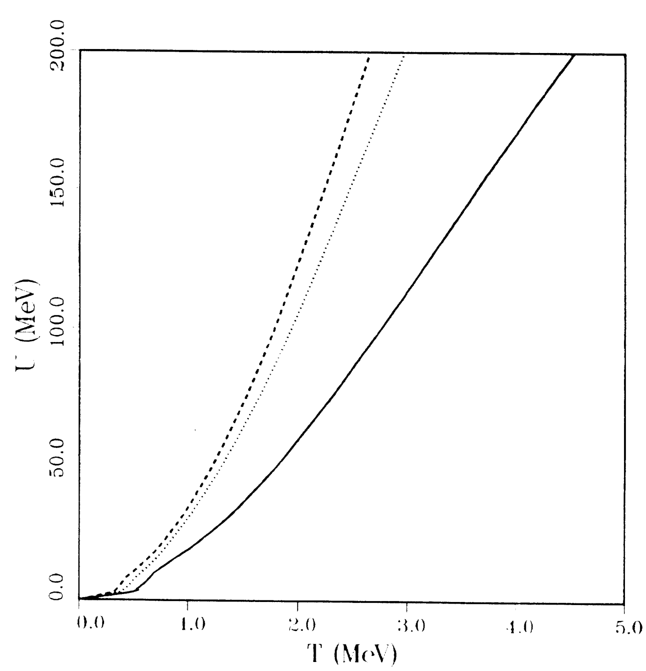

FIG. 1. Excitation energy versus temperature: solid line—^{110}Cd; dashed line—^{196}Pt; dotted line—^{238}U.

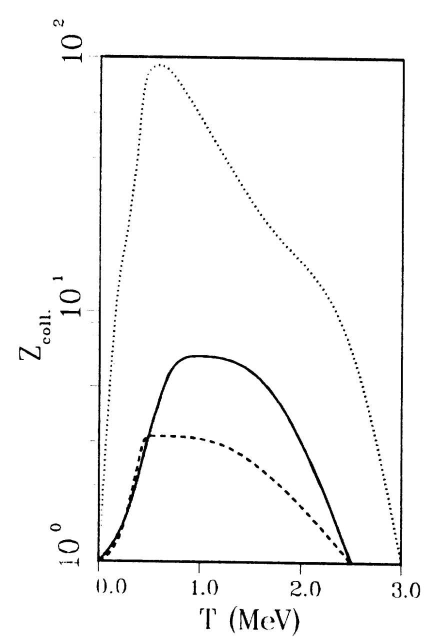

FIG. 3. Partition function versus temperature: solid line—^{110}Cd; dashed line—^{196}Pt; dotted line—^{238}U.

TABLE I. IBM parameters at zero temperature.

Nucleus	N_B	E_0 (MeV)	ε_0 (MeV)	α_0 (MeV)	β_0 (MeV)	γ_0 (MeV)	δ_0 (MeV)	η_0 (MeV)
^{110}Cd	7	0.0	0.1533	0.0972	−0.0013	0.0024	0.0	0.0
^{196}Pt	6	1.5129	0.0	0.0	0.0098	0.0876	0.0	−0.0126
^{238}U	12	4.3538	0.0	0.0	0.0	0.0037	−0.0101	0.0

been adjusted on the experimental low-energy spectrum[11] of each nucleus and are listed in Table I. The calculations have been repeated at different temperatures with constant IBM parameters ε_0, α_0, β_0, γ_0, δ_0, and η_0 but with different boson numbers, $N_B(T)$ and all the states of the subgroup irrep's contained in the totally symmetric $[N_B(T)]$ irrep's of U(6) have been generated in each reduction chain. As expected from the low-energy spectra, the nuclei of class I and III have similar collective factors, while the nuclei of class II show a more pronounced collective enhancement, in qualitative agreement with the traditional approaches.[2]

The main purpose of this work is to show the gross features of the IBM at finite temperature, without attempting a detailed comparison with other collective model predictions, intended for a future work; we only mention that vibrational factors of the same order of magnitude as our $Z_{\rm I}$ have been obtained in the $A=60$ mass region by the particle-hole random phase approximation (ph RPA) at $T \lesssim 2$ MeV,[12] while rotational factors based on Elliott's SU(3) model[13] in the $A=160$ mass region compare relatively well with our $Z_{\rm II}$ at low excitation energy ($U \lesssim 60$ MeV).

Starting from the canonical partition function it is possible to obtain any thermodynamic variable suitable for the investigation of phase transitions at finite temperature, for instance specific heat,

$$C=\beta^2 \frac{\partial^2 \ln Z}{\partial \beta^2} \tag{14}$$

as is done in a recent paper[14] on ^{20}Ne, on the basis of realistic fermion interactions in the s-d shell. The approach to the canonical partition function is similar to the present one and makes it possible to predict transitions from deformed to spherical shape at temperatures ranging from 1.7 to 2.5 MeV, depending on the effective interaction used in the calculation.

One great advantage of the simple phenomenological approach proposed in this work over more complex microscopic models is that the IBM is not limited to a particular symmetry, whether vibrational or rotational, but, in principle, allows a finite-temperature description of transitional nuclei in a large region of the periodic table, exhibiting collective features intermediate between the limiting symmetries discussed here. The only further requirement for the treatment of transitional nuclei will be the numerical diagonalization of the IBM Hamiltonian instead of the analytical solutions used in this work.

Again, simultaneous evaluation of the IBM collective enhancement factor and of the NBCS intrinsic state density will provide us with a total state density $\rho(U)$, according to formula (4), or a spin-dependent level density $\rho(U,J)$, to be compared with experimental data. This study is in progress and will be the subject of further publications.

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