PHYSICAL REVIEW C VOLUME 42, NUMBER 3 SEPTEMBER 1990

ARTICLES

Radiative neutron capture on ^{3}He

J. Carlson
Theory Division, Los Alamos National Laboratory, Los Alamos, New Mexico 87545

D. O. Riska
Department of Physics, University of Helsinki, 00170 Helsinki, Finland

R. Schiavilla
Continuous Electron Beam Accelerator Facility, Newport News, Virginia 23606

R. B. Wiringa
Physics Division, Argonne National Laboratory, Argonne, Illinois 60439
(Received 2 April 1990)

The thermal neutron cross section for the reaction $n+{}^3\mathrm{He}\rightarrow{}^4\mathrm{He}+\gamma$ is calculated with the Monte Carlo method from realistic variational wave functions obtained for the Argonne v_{14} two-nucleon and Urbana VII three-nucleon interaction models. The calculated cross section is found to be almost entirely due to exchange currents. With the exchange current operator constructed so as to satisfy the continuity equation with the Argonne v_{14} interaction, the calculated cross section is 71 μb, which is close to the empirical value 54$\pm$6 μb. The less certainly known contributions from model-dependent exchange current mechanisms increase the predicted value to about 112 μb. The calculated cross section depends strongly on the $n+{}^3\mathrm{He}$ scattering length, varying between 140 and 71 μb as the scattering length is changed from 3.25 to 3.75 fm.

I. INTRODUCTION

The radiative thermal neutron capture reactions $n+{}^2\mathrm{H}\rightarrow{}^3\mathrm{H}+\gamma$ and $n+{}^3\mathrm{He}\rightarrow{}^4\mathrm{He}+\gamma$ are interesting in that exchange current mechanisms contribute a major part of their total cross sections.[1-3] The reason for this is that the single-nucleon current operator cannot connect the main S-state components of the ground state wave functions of the initial and final nuclei at low energies.[3-5] Because of this "pseudo-orthogonality" only the small components of the wave functions contribute in the impulse approximation. As on the other hand the exchange current operators can connect the S-state components, their matrix elements are exceptionally large in comparison with those of the single-nucleon current operator in these reactions.

The exchange current contributions to the reaction $n+{}^3\mathrm{He}\rightarrow{}^4\mathrm{He}+\gamma$ have been estimated by Towner and Khanna[3] and by Tegnér and Bargholtz[6] with simple wave function models. Towner and Khanna find the exchange current contribution to the cross section to be considerably larger than that of the single-nucleon current, whereas Tegnér and Bargholtz find the exchange current contribution to be the smaller one by far. Since both pairs of authors use similar models for the exchange current operator (the meson exchange current operator of Chemtob and Rho,[7] with some short-range modifications), the discrepancy between the results must derive from the schematic wave function models used. In an attempt to reduce this uncertainty we have calculated the cross section for the reaction $n+{}^3\mathrm{He}\rightarrow{}^4\mathrm{He}+\gamma$ with realistic wave functions that correspond to the Argonne v_{14} two-nucleon[8] and Urbana VII three-nucleon potential models,[9] taking into account the interaction effects in the initial state as well. The quality of the variational wave functions has previously been assessed by quantitatively successful predictions of the binding energies and asymptotic D/S state ratios in the $d+p$ and $d+d$ breakup channels of ^{3}He and ^{4}He,[9] as well as the electromagnetic form factors of the helium isotopes.[10-12] Further tests of their accuracy have been carried out by direct comparison with results obtained with exact Faddeev[13] and Green's function Monte Carlo[14] wave functions for the two-body correlation functions,[15] magnetic form factors,[11] and longitudinal energy weighted sum rules.[16]

The result of the present essentially complete calculation of the cross section in the impulse approximation is a very small 6 μb. This value falls within the range 2–14 μb predicted by Towner and Khanna, but is much smaller than the result of Tegnér and Bargholtz. The smallness of the impulse approximation cross section indicates that most of the empirical cross section, which according to the most recent measurement is 54$\pm$6 μb,[17] is due to exchange currents. The new empirical cross section value is in good agreement with two earlier measurements,[18,19] although not with the smaller value reported in Ref. 20.

The two-body exchange current operators consist of model-independent and model-dependent parts. The former ones are constrained by the NN interaction through the continuity equation.[10,11,21] The latter ones, which are of smaller numerical significance, are purely transverse and thus not directly linked to the NN interaction model. Furthermore, they depend on a set of only approximately known effective coupling constants and cutoff parameters. The exchange current contribution will thus consist of a term that is determined by the NN interaction and a smaller very model-dependent term. Indeed, we have recently shown that an excellent description of the magnetic[10,11] and charge[12] form factors of the three- and four-nucleon systems, as well as of the deuteron electrodisintegration at threshold at backward angles,[11] is obtained if the Argonne two-nucleon interaction model[8] is used to construct both the electromagnetic current operator and the wave functions. In the present work, which should be viewed as a continuation of the above investigation, the model-independent term together with the single-nucleon current leads to a cross section value of 71 μb. When the contribution from the model-dependent exchange currents are added the calculated total cross section increases to 112 μb, which is considerably larger than the empirical value. This overprediction is probably partly a consequence of the "hard" isospin-dependent pseudoscalar and vector tensor components[10] of the Argonne v_{14} interaction, which are used in the construction of the exchange current operator, and partly due to an overestimate of the model-dependent exchange current contributions. An additional factor of uncertainty is the sensitivity of the calculated cross section to the $n+{}^3$He scattering length.

This paper is organized into five sections. In Sec. II we present the basic formalism and the cross section calculation in the impulse approximation. The model-independent (or potential-constrained) and model-dependent exchange current contributions are discussed in Secs. III and IV, respectively. Finally, Sec. V contains a summary along with some concluding remarks.

II. THE CROSS SECTION IN THE IMPULSE APPROXIMATION

The spin-averaged differential cross section for the reaction $n+{}^3\text{He}\rightarrow{}^4\text{He}+\gamma$ can be written in the form

$$\frac{d\sigma}{d\Omega}=\frac{\alpha}{8\pi}\frac{mk}{q}\sum_M |\langle {}^4\text{He}|\mathbf{j}_T^\dagger(\mathbf{k})|n+{}^3\text{He};1M\rangle|^2 . \tag{2.1}$$

Here α is the fine structure constant, m the neutron mass, and q and k the momenta of the incident neutron and final photon in the lab frame, respectively. The matrix elements of the transverse components of the electromagnetic current operator $\mathbf{j}_T(\mathbf{k})$ involve the initial $n+{}^3$He spin-triplet and the final ^{4}He spin-singlet states. In the magnetic dipole approximation the total cross section is obtained by simply multiplying the expression (2.1) by 4π.

In the impulse approximation the current operator is

$$\mathbf{j}_T(\mathbf{k})=\sum_{i=1,A} e^{i\mathbf{k}\cdot\mathbf{r}_i}\left\{\frac{1}{2}(1+\tau_{i,z})\frac{\mathbf{p}_{i,T}}{m} -i\frac{\mathbf{k}\times\boldsymbol{\sigma}_i}{2m}[\tfrac{1}{2}(1+\tau_{i,z})\mu_p +\tfrac{1}{2}(1-\tau_{i,z})\mu_n]\right\} , \tag{2.2}$$

where μ_p and μ_n are the proton and neutron magnetic moments, respectively. The initial state is an antisymmetrized $n+{}^3$He spin-triplet scattering wave function, which in the asymptotic region has the form

$$\begin{aligned}\psi_{1M}(n+{}^3\text{He})=\frac{1}{\sqrt{4}}[&|\phi_n(1)\psi_{^3\text{He}}(234)\rangle_{1M}\\ &-|\phi_n(2)\psi_{^3\text{He}}(341)\rangle_{1M}\\ &+|\phi_n(3)\psi_{^3\text{He}}(412)\rangle_{1M}\\ &-|\phi_n(4)\psi_{^3\text{He}}(123)\rangle_{1M}] .\end{aligned} \tag{2.3}$$

The single-particle neutron state ϕ_n is characterized at low energies by a scattering length a_n. This scattering length can be determined microscopically from the interaction model using the variational method described in Ref. 22. The full wave function has the form

$$\psi_{1M}(1234)=S\left[\prod_{i<j}F_{ij}\right]\phi ,$$

where S is a symmetrization operator, F_{ij} pair correlation operators, and ϕ is given by

$$\phi=A\left[\prod_{j\neq 1}f^c(\bar{r}_{1j})\right]^{-1}\phi_1(\mathbf{r}_1-(\mathbf{r}_2+\mathbf{r}_3+\mathbf{r}_4)/3) . \tag{2.4}$$

The spin-isospin dependent pair correlations go to zero and $\bar{r}_{ij}\rightarrow r_{ij}$ at large distances, ensuring that the correct asymptotic form for the scattering state is obtained. Variational methods are used to determine the pair correlation functions in F_{ij} as well as the radial form of the one-body state ϕ_1. For the Argonne v_{14} (Ref. 8) plus Urbana VII interaction model,[9] this procedure yields a scattering length of 3.5 fm, with a statistical error of approximately 0.25 fm. This result is in good agreement with the empirical value of 3.50±0.25 fm,[23] but slightly larger than that obtained in an R-matrix analysis,[24] 3.25±0.10 fm. The sensitivity of the results for the radiative capture cross section to the triplet scattering length is discussed at the end of Sec. IV. The asymptotic form given in Eq. (2.4) ignores possible couplings to the $p+{}^3$H scattering state, as well as couplings to D-state components in the relative $n+{}^3$He wave function at large distances. The pair correlation operators, of course, introduce such terms into the wave function within the interaction region. R-matrix analyses of the four-body problem indicate that these couplings are very weak.[24] We have also computed the coupling to the $p+{}^3$H state microscopically, and found it to be small.[22]

The ^{4}He bound state wave function has also been determined variationally with the Argonne v_{14} plus Urbana VII interaction model. These wave functions contain

TABLE I. The matrix elements of the single-nucleon (IA) operator and those associated with the pseudoscalar (PS) and vector (V) parts of the isospin-dependent tensor component, the spin-orbit [SO; Eqs. (3.1)–(3.3)], L^2 (LL) and quadratic spin-orbit (SO2) components of the Argonne v_{14} interaction, and the model-dependent $\rho\pi\gamma$, Δ_{33}, and $\omega\pi\gamma$ mechanisms [Eqs. (4.1)–(4.5)]. The V contribution also includes a small correction due to the central isospin-dependent component of the Argonne v_{14}. The numerical difference between Re$\langle j_x \rangle$ and Im$\langle j_y \rangle$ is due to the statistical errors in the Monte Carlo integration; we estimate these errors to be 3–4 % in the matrix element or 5–8 % in the cross section.

Contribution	Re$\langle j_x \rangle$ (fm$^{3/2}$)	Im$\langle j_y \rangle$ (fm$^{3/2}$)	σ (μb)
IA	−0.001 61	−0.001 67	6
IA+PS	0.004 47	0.004 42	41
IA+ $\cdots$ +V	0.005 85	0.005 79	70
IA+ $\cdots$ +SO	0.005 72	0.005 64	67
IA+ $\cdots$ +LL	0.005 57	0.005 50	63
IA+ $\cdots$ +SO2	0.005 90	0.005 83	71
IA+ $\cdots$ +$\rho\pi\gamma$	0.005 76	0.005 70	68
IA+ $\cdots$ +$\Delta(\pi+\rho)$	0.006 82	0.006 74	95
IA+ $\cdots$ +$\omega\pi\gamma$	0.007 39	0.007 32	112

fairly large D-state admixtures: approximately 9.2% for ^{3}He and 17.5% for ^{4}He. The results for the capture matrix elements do not appear to depend critically upon these values, although artificially eliminating the D state can have a large effect.

The numerical evaluation of the matrix elements of the transverse components of the electromagnetic current operator in Eq. (2.1) is carried out by the Monte Carlo methods developed in Ref. 10. Although the magnitudes of the $j_x(k\hat{z})$ and $j_y(k\hat{z})$ matrix elements are in fact equal, we have evaluated them separately in order to have an automatic test of the program. The matrix elements of $j_x(k\hat{z})$ and $j_y(k\hat{z})$ are given in Table I, along with their associated statistical errors. These statistical errors are 3–4 % in the matrix element, or 5–8 % in the cross section. The impulse approximation has somewhat larger errors, but is a small part of the total result.

For the total cross section in the impulse approximation (IA) we obtain the value 6 μb. This value falls between the two values 2.1 and 14.6 μb obtained by Towner and Khanna[3] with simple phenomenological exponential and oscillator wave function models. Since the cross section in the impulse approximation is entirely due to the small components in the wave function, the values predicted with phenomenological wave functions must, however, be viewed as very uncertain, as reflected in the large difference between the two values obtained by Towner and Khanna.[3]

The predicted cross section value in the impulse approximation also depends sensitively on the scattering length of the $n+{}^3$He system. A change of the scattering length from 3.5 to 3.25 fm would lead to an enhancement of the impluse approximation value by about 40%. Our conclusions is in any case that the contribution of the single-nucleon current to the cross section is very small in comparison to that of the two-body exchange currents.

III. MODEL-INDEPENDENT EXCHANGE CURRENT CONTRIBUTIONS

The exchange current density operator, which contributes the main part of the total cross section for the thermal neutron capture reaction $n+{}^3\text{He}\rightarrow{}^4\text{He}+\gamma$, can be separated into "model-independent" and "model-dependent" parts. The former is required by the nucleon-nucleon interaction for current conservation, and contains no parameters beyond those contained in the interaction. We consider this exchange current term here and the model-dependent one in Sec. IV below.

We construct the exchange current operator that is associated with the Argonne v_{14} interaction model[8] in a way which ensures current conservation and consistency with the wave function models.[10,11] The resulting "model-independent" exchange current operator can in turn be split into two terms: one associated with the isospin-dependent central and tensor interactions, and another associated with the velocity-dependent components of the interaction.

The most important exchange current operator is the isovector one associated with the isospin-dependent spin-spin and tensor interactions. We use here the method developed in Ref. 21 to construct this exchange current operator from the corresponding potential components. The method involves separation of these potential components into a sum of two terms, which may be associated with exchange of isospin-one pseudoscalar bosons (PS or "generalized pion exchange") and exchange of isospin-one spin-one bosons (V or "generalized vector meson exchange"). When these separated potential components are then used in place of the simple Yukawa potentials in the expressions for the pion and ρ-meson exchange current operators, one obtains an exchange current operator that is consistent with boson exchange models and which satisfies the continuity equation with the given realistic potential model (in this case the Argonne v_{14} interaction).

The resulting exchange current operator then appears as the sum of a "generalized pion" and a "generalized ρ-meson" exchange current operator. The former represents a generalization of the conventional "seagull" (or "pair") and "mesonic" pion exchange current operators (Fig. 1),[7] but which satisfies the continuity equation with a realistic potential, rather than with the simple one-pion-exchange potential. The latter may be viewed

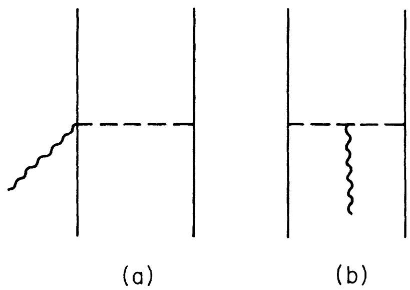

FIG. 1. Feynman diagrams that describe the "seagull" (a) and "mesonic" (b) exchange current operators that are associated with the pion and ρ-meson exchange interactions.

as a generalization of the corresponding ρ-meson exchange current operator.[21] The explicit expressions for these exchange current operators, which have the isospin structure $(\tau_1\times\tau_2)_z$, are given in Ref. 10 for the case of the Argonne v_{14} potential. The resulting exchange current operator is found to give satisfactory predictions for the backward electrodisintegration cross section of deuteron near threshold[11] and for the magnetic form factors of the trinucleons.[10,11] For completeness, we point out that the generalized vector meson exchange current operator also contains a term associated with the central interaction, but which is numerically of only minor significance.

The construction of the exchange current operators that are associated with the spin-orbit, quadratic spin-orbit, and L^2 components of the interaction is less straightforward.[25,26] The simplest approach would be to construct these exchange current operators by direct minimal substitution in the corresponding potential components. While this method ensures current conservation, the form of the resulting exchange current operators does not agree with that of the corresponding operators obtained with boson exchange models.[25] We shall therefore construct the exchange current operator that is associated with the spin-orbit interaction by generalizing the relevant single scalar and vector meson exchange current operators to a form which meets the requirement of current conservation with the central and spin-orbit components of the Argonne v_{14} potential.

For this purpose we assume that the isospin-independent central and spin-orbit interactions of the Argonne potential can be written as sums of a term associated with exchange of spin-zero bosons (generalized scalar meson exchange) and a term that is associated with exchange of spin-one bosons (generalized ω-meson exchange). By then using the known expressions for the single scalar and vector meson exchange current operators,[27] and separating the scalar and vector components of the interactions, it is possible to construct generalized scalar and vector meson exchange current operators which satisfy the continuity equation with the phenomenological potential. The resulting generalized ω-exchange current operator then has the form in momentum space

$$\mathbf{j}(\mathbf{k}_1,\mathbf{k}_2)=-\frac{i}{16m^2}(1+\tau_{1,z})[v^C(k_2)+2m^2v^{SO}(k_2)]\times[(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\times\mathbf{k}_2-i(\mathbf{p}_2+\mathbf{p}_2')]+(1\rightleftharpoons2)\,. \tag{3.1}$$

The momenta of the initial and final nucleon pairs are $\mathbf{p}_1,\mathbf{p}_2$ and $\mathbf{p}_1',\mathbf{p}_2'$, while the fractions of the photon momentum $\mathbf{k}$ that are delivered to nucleons one and two are denoted $\mathbf{k}_1$ and $\mathbf{k}_2$, respectively. The functions $v^C(p)$ and $v^{SO}(p)$ are the Fourier transforms of the isospin-independent central and spin-orbit components [note that the spin-orbit interaction in momentum space has the form $\frac{1}{2}i(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot\mathbf{p}'\times\mathbf{p}v^{SO}(p)$].

The corresponding generalized scalar exchange current operator has the expression

$$\mathbf{j}(\mathbf{k}_1,\mathbf{k}_2)=-i\frac{3}{16m^2}(1+\tau_{1,z})[v^C(k_2)-\tfrac{2}{3}m^2v^{SO}(k_2)]\times[\boldsymbol{\sigma}_1\times\mathbf{k}-i(\mathbf{p}_1+\mathbf{p}_1')]+(1\rightleftharpoons2)\,. \tag{3.2}$$

In the case of single isospin-zero scalar and vector meson exchange interactions the exchange current expressions (3.1) and (3.2) reduce to the σ- and ω-exchange current operators given by Blunden,[27] with the exception of an insignificant nonlocal operator that is proportional to the (small) ω-nucleon tensor coupling constant.

In the case of the isospin-dependent interaction one cannot separate the central and spin-orbit interactions into terms associated with scalar- and vector-like exchanges in a unique way, because of the large isovector tensor coupling to the nucleon. We therefore in this case make the simple assumption that only the exchange current associated with vector-like exchanges (generalized ρ-meson exchange) is important. The form of this exchange current operator can then be obtained from the expression (3.1) by the substitution

$$(1+\tau_{1,z})\rightarrow(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2+\tau_{2,z})\,. \tag{3.3}$$

Note that the isovector term that involves the isospin operator $(\tau_1\times\tau_2)_z$, which is associated with the central vector-exchange interaction, has been included with the similar isovector exchange current operators that are associated with the isospin-dependent tensor interaction above.

The exchange current operators (3.1) and (3.2) [and that obtained by the substitution (3.3)] satisfy the continuity equation with the corresponding central and spin-orbit interactions, when the single-nucleon charge operator that appears in the continuity equation includes the relativistic corrections proportional to m^{-2}. The importance of taking into account this relativistic correction in the continuity equation has been stressed by Blunden[27] and by Ichii *et al.*[28] It should be mentioned here that the present current operator associated with the spin-orbit components of the v_{14} is constructed in a different way from that discussed in Ref. 10. However, in contrast to what was found in Ref. 10, the prediction for the isoscalar combination of the magnetic moments of the trinucleons obtained with the currents (3.1)–(3.3) is in excellent agreement with the experimental value.[11]

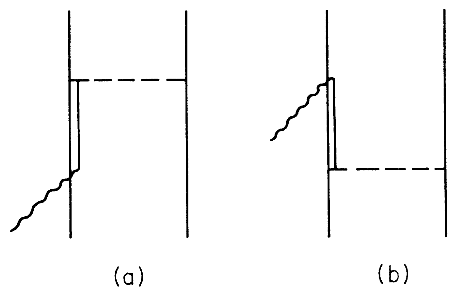

FIG. 2. The Δ_{33}-excitation exchange current diagrams.

There is no simple way to construct the exchange current operators that are associated with the quadratic spin-orbit and L^2 components in the interaction so that they would have a form that is consistent with that obtained from meson exchange mechanisms. Since on the other hand the numerical significance of these exchange current operators is small we shall here be content to construct these by direct minimal substitution in the corresponding interactions. The resulting expressions for these exchange current operators are then those obtained in Ref. 10.

The predicted total cross section values are also listed in Table I. When only those exchange current corrections that are associated with the isospin-dependent tensor and spin-spin interactions are taken into account the predicted cross section value is 70 μb, which is only slightly above the empirical value 54±6 μb. It should be noted that the contribution due to the isospin-dependent central interaction as well as that associated with the momentum-dependent interactions is very small. The 70 μb prediction for the model-independent isovector PS and V current contributions is much larger than the value quoted by Towner and Khanna[3] from the corresponding pion and ρ-meson exchange current operators. The reason for the small value of Towner and Khanna seems to be a large cancellation between the SS- and SD-state matrix elements of these exchange current operators. However, it should be pointed out that the above authors have not included the contribution from the diagonal DD-matrix elements.

IV. MODEL-DEPENDENT EXCHANGE CURRENT CONTRIBUTIONS

In addition to the model-independent exchange current operators that are required by the nucleon-nucleon interaction one has to consider model-dependent exchange current mechanisms, which are described by purely transverse current density operators. The best established such mechanisms are the pion and ρ-meson exchange current operators that involve excitation of virtual intermediate Δ_{33} resonances (Fig. 2). Although there is a strong cancellation between these pion and ρ-meson exchange contributions[10,29] they do contribute a non-negligible correction to the total cross section.

The pion exchange current operator associated with excitation of virtual intermediate Δ_{33} resonances (in the sharp resonance approximation) is

$$\mathbf{j}_{\pi\Delta}(\mathbf{k}_1,\mathbf{k}_2)=i\frac{2\sqrt{2}f_\pi^2}{15mm_\pi^2(m_\Delta-m)}\mu_{\gamma N\Delta}\left[4\tau_{1,z}\frac{\boldsymbol{\sigma}_1\cdot\mathbf{k}_1\mathbf{k}_1}{m_\pi^2+k_1^2}+4\tau_{2,z}\frac{\boldsymbol{\sigma}_2\cdot\mathbf{k}_2\mathbf{k}_2}{m_\pi^2+k_2^2}\right.$$
$$\left.-(\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2)_z\left[\frac{(\boldsymbol{\sigma}_1\times\mathbf{k}_2)(\boldsymbol{\sigma}_2\cdot\mathbf{k}_2)}{m_\pi^2+k_2^2}-\frac{(\boldsymbol{\sigma}_2\times\mathbf{k}_1)(\boldsymbol{\sigma}_1\cdot\mathbf{k}_1)}{m_\pi^2+k_1^2}\right]\right]\times\mathbf{k}\ . \tag{4.1}$$

Here m_π and m_Δ are the pion and Δ_{33} masses, respectively. In this expression the $\pi N\Delta$ coupling strength has been expressed in terms of the πNN coupling constant ($f_\pi\simeq 1$) by means of the static quark model.[30] For the $\gamma N\Delta$ transition moment $\mu_{\gamma N\Delta}$ we use the empirical value 3,[31] which is about 30% smaller than the value obtained by expressing it in terms of the isovector nucleon magnetic moment by using the static quark model. In Eq. (4.1) the momentum variables $\mathbf{k}_1$ and $\mathbf{k}_2$ are the fractions of the total momentum $\mathbf{k}=\mathbf{k}_1+\mathbf{k}_2$ that are delivered to nucleons one and two, respectively.

The corresponding ρ-meson exchange current operator, which involves excitation of intermediate Δ_{33} resonances, has a similar form[32]

$$\mathbf{j}_{\rho\Delta}(\mathbf{k}_1,\mathbf{k}_2)=-i\frac{g_\rho^2(1+\kappa)^2}{15\sqrt{2}m^3(m_\Delta-m)}\mu_{\gamma N\Delta}\left[4\tau_{1,z}\frac{(\boldsymbol{\sigma}_1\times\mathbf{k}_1)\times\mathbf{k}_1}{m_\rho^2+k_1^2}+4\tau_{2,z}\frac{(\boldsymbol{\sigma}_2\times\mathbf{k}_2)\times\mathbf{k}_2}{m_\rho^2+k_2^2}\right.$$
$$\left.-(\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2)_z\left[\frac{\boldsymbol{\sigma}_1\times[(\boldsymbol{\sigma}_2\times\mathbf{k}_2)\times\mathbf{k}_2]}{m_\rho^2+k_2^2}-\frac{\boldsymbol{\sigma}_2\times[(\boldsymbol{\sigma}_1\times\mathbf{k}_1)\times\mathbf{k}_1]}{m_\rho^2+k_1^2}\right]\right]\times\mathbf{k}\ . \tag{4.2}$$

In this expression the static quark model has again been used to express the $\rho N\Delta$ transition coupling strength in terms of the corresponding ρNN coupling constant combination $g_\rho(1+\kappa)$. We shall here use the values $g_\rho^2/4\pi\simeq 0.5$ and $\kappa=6.6$.[32]

The effect of the finite extent of the nucleons and mesons is taken into account by introduction of monopole form factors at the πNN and ρNN vertices in the current operators (4.1) and (4.2):[32]

$$f_a(p)=\frac{(\Lambda_a^2-m_a^2)}{(\Lambda_a^2+p^2)},\quad a=\pi,\rho\ . \tag{4.3}$$

The cutoff masses Λ_π and Λ_ρ do of course represent arbitrary parameters. The values $\Lambda_\pi=1.2$ GeV and $\Lambda_\rho=2$ GeV, suggested by studies of the reaction $\pi^+d\rightarrow pp$,[33] are used in the $\Delta(\pi+\rho)$ contribution listed in Table I.

The contribution from the combined pion and ρ-meson exchange currents that are associated with excitation of Δ_{33} resonances is shown in Table I. Inclusion of this contribution increases the calculated cross section from the value 71 μb (impulse approximation + model-independent exchange currents) to 99 μb. There is, however, a remaining theoretical uncertainty in the Δ_{33}-exchange current correction, which is associated with the uncertainty in the $\pi N\Delta$, ρNN, and $\rho N\Delta$ coupling constants. The magnitude of this uncertainty may be of the order 20%. The sensitivity of the matrix element on the values chosen for Λ_π and Λ_ρ is discussed at the end of Sec. IV.

The remaining model-dependent exchange currents that have a long-range component (apart from those involving excitation of pion-nucleon resonances with high energy) are the $\rho\pi\gamma$ and $\omega\pi\gamma$ exchange mechanisms illustrated in Fig. 3. The relative importance of these at low values of momentum transfer is small, and depends rather sensitively on the treatment of the short-range behavior

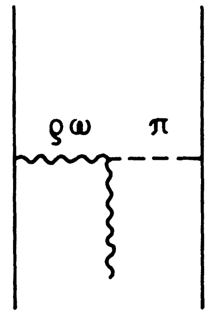

FIG. 3. The $\gamma\pi\rho$ and $\gamma\pi\omega$ exchange current mechanisms.

The expressions for $\rho\pi\gamma$ and $\omega\pi\gamma$ exchange current operators are given by

$$j_{\rho\pi}(\mathbf{k}_1,\mathbf{k}_2)=i\frac{f_\pi g_\rho g_{\rho\pi\gamma}}{m_\pi m_\rho}\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\mathbf{k}_1\times\mathbf{k}_2\left[\frac{\boldsymbol{\sigma}_1\cdot\mathbf{k}_1}{(k_1^2+m_\pi^2)(k_2^2+m_\rho^2)}-\frac{\boldsymbol{\sigma}_2\cdot\mathbf{k}_2}{(k_1^2+m_\rho^2)(k_2^2+m_\pi^2)}\right] \tag{4.4}$$

and

$$\mathbf{j}_{\omega\pi}(\mathbf{k}_1,\mathbf{k}_2)=i\frac{f_\pi g_\omega g_{\omega\pi\gamma}}{m_\omega m_\pi}\mathbf{k}_1\times\mathbf{k}_2\left[\frac{\boldsymbol{\sigma}_1\cdot\mathbf{k}_1}{(k_1^2+m_\pi^2)(k_2^2+m_\omega^2)}\tau_{1,z}-\frac{\boldsymbol{\sigma}_2\cdot\mathbf{k}_2}{(k_1^2+m_\omega^2)(k_2^2+m_\pi^2)}\tau_{2,z}\right]. \tag{4.5}$$

For the $\rho\pi\gamma$, $\omega\pi\gamma$, and ωNN coupling constants we use the values 0.4 and 0.63 [from the measured widths of $\rho\rightarrow\pi+\gamma$ (Ref. 34) and $\omega\rightarrow\pi+\gamma$ (Ref. 7)] and 14.6 (from the Bonn potential[35]), respectively. Because of the recently raised question of the proper sign of the $\rho\pi\gamma$ exchange current operator,[36] we emphasize that the expression (4.4) agrees with the corresponding magnetic moment expression given by Chemtob and Rho.[7] We finally introduce monopole form factors at the pion and vector meson vertices in the $\rho\pi\gamma$ and $\omega\pi\gamma$ exchange current operators (the value $\Lambda_\omega=2$ GeV is used in the $\omega\pi\gamma$ contribution listed in Table I).

The matrix elements of the exchange current operators (4.4) and (4.5) (modified by the monopole form factors) are listed in Table I. The contribution of the $\rho\pi\gamma$ exchange current operator is very small, whereas that of the $\omega\pi\gamma$ mechanism is non-negligible. Adding these corrections to the previously considered ones increases the calculated cross section to 112 μb, which is about twice the empirical value. The most likely reasons for this overprediction will be discussed in the following section.

We have also calculated the cross section as a function of scattering length, and find that it varies from 140 to 71 μb as the scattering length goes from 3.25 to 3.75 fm. This sensitivity is an important source of uncertainty in the present calculation, and a more accurate experimental determination of the scattering length would be valuable. Furthermore, the $\rho\pi\gamma$ and $\omega\pi\gamma$ contributions are very sensitive to the cutoff values in the πNN, ρNN, and ωNN vertices. Changing these from 1.2, 2.0, and 2.0 GeV to 0.6, 1.0, and 1.0 GeV decreases the magnitude of the $\rho\pi\gamma$ and $\omega\pi\gamma$ matrix elements by factors roughly equal to 6 and 3, respectively. However, the $\Delta(\pi+\rho)$ contribution increases by 10%, so that the reduction of the cutoff values to 0.6, 1.0, and 1.0 GeV decreases the contribution of the model-dependent exchange currents by only 0.000 23 fm$^{3/2}$, or roughly 3% of the total matrix element. Finally, we have performed calculations using the Chemtob-Rho expressions for the pion and ρ-meson exchange currents in place of the PS and V exchange currents derived from the Argonne potential, and find a total cross section of 123 μb. Including only the impulse approximation and the pion exchange current gives a much smaller value, 42 μb.

V. DISCUSSION

While the qualitative conclusion of the present calculation of the total cross section for $n+{}^3\mathrm{He}\rightarrow{}^4\mathrm{He}+\gamma$, that the cross section is almost entirely due to exchange current contributions, is fairly certain, the substantial overprediction of the cross section is unsatisfactory. This overprediction can already be inferred from the recent calculation of the exchange current contribution to the isovector combination of the trinucleon magnetic moments in Refs. 10 and 11. The exchange current contribution to this quantity obtained in Refs. 10 and 11, with essentially the same model for the exchange current operator, was too large by about 20%. This translates into a 45% overprediction of any cross section that is dominated by the isovector exchange current contribution, such as the thermal neutron cross section.

The exchange current contribution depends on the nucleon-nucleon interaction both explicitly through the exchange current operator and indirectly through the wave function. The strong pseudoscalar and vector tensor components[10] of the Argonne v_{14} interaction model are likely to be partly responsible for the present overprediction. We have verified that if one leaves out the *D*-state components in the wave function and the tensor correlation in the initial scattering state one would obtain a 40% reduction of the predicted cross section value. This demonstrates the sensitivity to the tensor force. Moreover, as already pointed out at the end of Sec. IV, the calculated cross section is very sensitive to the $n+{}^3\mathrm{He}$ scattering length.

Among the model-dependent exchange current contributions considered above, the least certain is the $\omega\pi\gamma$ exchange current operator (4.5). The magnitude of its contribution to the cross section in Table I should be viewed as an upper limit, as it could easily have been reduced by reducing the mass values in the cutoff factors from the large values $\Lambda_\pi=1.2$ GeV and $\Lambda_\omega=2.0$ GeV. One could in fact view the fact that its contribution increases the calculated cross section from the fairly reasonable value 95 μb to the clearly too large value 112 μb as evidence that it has been overestimated.

Improving on the present overprediction of the exchange current contribution to the cross section for the reaction $n+{}^3\mathrm{He}\rightarrow{}^4\mathrm{He}+\gamma$ will probably require a model for the nucleon-nucleon interaction that has somewhat weaker isospin-dependent pseudoscalar and vector tensor components. At intermediate ranges these components should, however, not be weaker than those of the Argonne v_{14} potential because otherwise the good fit to the empirical threshold electrodisintegration cross section of the deuteron[11] and the magnetic form factors of ^{3}H and ^{3}He (Refs. 10 and 11) cannot be maintained.

ACKNOWLEDGMENTS

The authors would like to thank V.R. Pandharipande and S.C. Pieper for valuable discussions. J.C. and R.S. would also like to thank the Physics Division of Argonne National Laboratory, where much of this work was undertaken. The calculations were made possible by grants of time on the Cray supercomputers at the National Energy Research Supercomputer Center in Livermore. Finally, J.C., R.S., and R.B.W. would like to thank the Nuclear Physics Division of the U.S. Department of Energy for support.

[1]E. Hadjimichael, Phys. Rev. Lett. **31** 183 (1973).
[2]J. Torre and B. Goulard, Phys. Rev. C **28**, 529 (1983).
[3]I. S. Towner and F. C. Khanna, Nucl. Phys. **A356**, 445 (1981).
[4]N. Austern, Phys. Rev. **83**, 672 (1951).
[5]A. C. Phillips, Nucl. Phys. **A184**, 337 (1972).
[6]P. E. Tegnér and C. Bargholtz, Astrophys. J. **272**, 311 (1983).
[7]M. Chemtob and M. Rho, Nucl. Phys. **A163**, 1 (1971).
[8]R. B. Wiringa, R. A. Smith, and T. L. Ainsworth, Phys. Rev. C **29**, 1207 (1984).
[9]R. Schiavilla, V. R. Pandharipande, and R. B. Wiringa, Nucl. Phys. **A449**, 219 (1986).
[10]R. Schiavilla, V. R. Pandharipande, and D. O. Riska, Phys. Rev. C **40**, 2294 (1989).
[11]R. Schiavilla and D. O. Riska, Phys. Lett. B (to be published).
[12]R. Schiavilla, V. R. Pandharipande, and D. O. Riska, Phys. Rev. C **41**, 309 (1990).
[13]C. R. Chen, G. L. Payne, J. L. Friar, and B. F. Gibson, Phys. Rev. C **33**, 1740 (1986).
[14]J. Carlson, Nucl. Phys. **A508**, 141c (1990).
[15]J. Carlson, Phys. Rev. C **38**, 1879 (1989).
[16]R. Schiavilla, A. Fabrocini, and V. R. Pandharipande, Nucl. Phys. **A473**, 290 (1987).
[17]F. L. H. Wolfs, S. J. Freedman, J. E. Nelson, M. S. Dewey, and G. L. Greene, Phys. Rev. Lett. **63**, 2721 (1989).
[18]L. M. Bollinger, R. J. Specht, and G. E. Thomas, Bull. Am. Phys. Soc. **18**, 591 (1973).
[19]M. Suffert and R. Berthollet, Nucl. Phys. **A318**, 54 (1979).
[20]V. P. Alfimenkov *et al.*, Yad. Fiz. **31**, 21 (1980) [Sov. J. Nucl. Phys. **31**, 10 (1980)].
[21]D. O. Riska, Phys. Scr. **31**, 471 (1985).
[22]J. Carlson (unpublished).
[23]H. Kaiser, H. Rauch, W. Bauspiess, and U. Bonse, Phys. Lett. **71B**, 321 (1977).
[24]G. Hale, private communication.
[25]D. O. Riska, Phys. Scr. **31**, 107 (1985).
[26]D. O. Riska and M. Poppius, Phys. Scr. **32**, 581 (1985).
[27]P. G. Blunden, Nucl. Phys. **A464**, 525 (1987).
[28]S. Ichii, W. Bentz, and A. Arima, Nucl. Phys. **A464**, 575 (1987).
[29]D. O. Riska, Phys. Rep. **181**, 207 (1989).
[30]G. E. Brown and W. Weise, Phys. Rep. **22**, 279 (1975).
[31]C. E. Carlson, Phys. Rev. D **34**, 2704 (1986).
[32]D. O.Riska, Prog. Part. Nucl. Phys. **11**, 199 (1984).
[33]J. Chai and D. O. Riska, Nucl. Phys. **A338**, 349 (1980).
[34]M. M. Nagels *et al.*, Nucl. Phys. **B147**, 189 (1979).
[35]K. Holinde and R. Machleidt, Nucl. Phys. **A256**, 479 (1976).
[36]P. Sarriguren, J. Martorell, and D. W. L. Sprung, Phys. Lett. B **228**, 285 (1989).