Proton-neutron symmetry in valence mirror nuclei

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Nuclear states of yrast and yrare type have been investigated in pairs of valence mirror nuclei: ^{110}Sn-^{142}Nd, ^{114}Sn-^{146}Gd, ^{43}Ca-^{51}V, ^{45}Ca-^{53}Mn, as well as in some even calcium isotopes with their corresponding $N=28$ valence mirror isotones. The observed similarities can be related to an approximate proton-neutron symmetry in valence mirror nuclear states.

I. INTRODUCTION

The impressive number of approximate symmetries that have been discovered recently in the frame of the interacting boson model[1-5] indicates that the search for approximate symmetries in nuclear spectra is a fruitful enterprise. The study of those approximate symmetries will surely deepen our understanding of nuclear structures, especially when we have a wealth of data while we do not have easily calculable theories. The similarities of the spectra of mirror nuclei as well as the existence of isobaric analog states in heavy nuclei show a proton-neutron symmetry which results from the charge independence of nuclear force. In a recent paper[6] we pointed out that the similarities of the spectra can also be found in pairs of valence mirror nuclei. Figure 1 shows schematically a pair of mirror nuclei with a magic core and a pair of valence mirror nuclei. As shown in Fig. 1(a) mirror nuclei are just a pair of nuclei (Z_1,N_1) and (Z_2,N_2) with the number of the protons and neutrons exchanged $(Z_1=N_2, Z_2=N_1)$. This kind of nuclei show strong similarities in their excited states, which has been thoroughly studied in the past and indicates the isospin symmetry.[7] Valence mirror nuclei [Fig. 1(b)] have different magic cores, but they still have the same numbers of valence protons Z_v and valence neutrons N_v in the major shell, respectively (e.g., $N_1=82$, $Z_1=50+k$ and $N_2=50+k$, $Z_2=50$). Such pairs of valence mirror nuclei have also been referred to as quasi mirror nuclei.[8,9] We expect that the different nuclei with equal number of valence protons and valence neutrons in the same major shell have a set of mirrored valence nucleon excited states. Due to the fact that the single magic nuclei are not expected to have deformation, they can properly be described within the framework of spherical shell model. As we have stated in Ref. 6, the shell model would predict a close analogy of the spectra in valence mirror nuclei if the following conditions are fulfilled.

(i) The two nuclei have identical single particle energies for the valence protons and neutrons, respectively. (Note that this implies that the effects of the Coulomb force on relative single particle energies in the valence shell are small.)

(ii) The residual force for the valence protons and valence neutrons is the same.

(iii) Only states with an unbroken magic core are considered.

The last condition implies that only states with either pure proton or pure neutron shell model configurations are considered. If all these conditions are fulfilled, we expect an identity of the spectra corresponding to the same mirror shell model configurations.

Before we turn to the comparison between the valence mirror nuclei, we want to introduce the *yrast filter* concept corresponding to our limitation of an unbroken magic core. We remind the reader that there are neutron or proton particle-hole states which break the magic core already at low excitation energy. These states are not the above-mentioned mirror valence nuclear states. They normally have lower spins and do not belong to the yrast

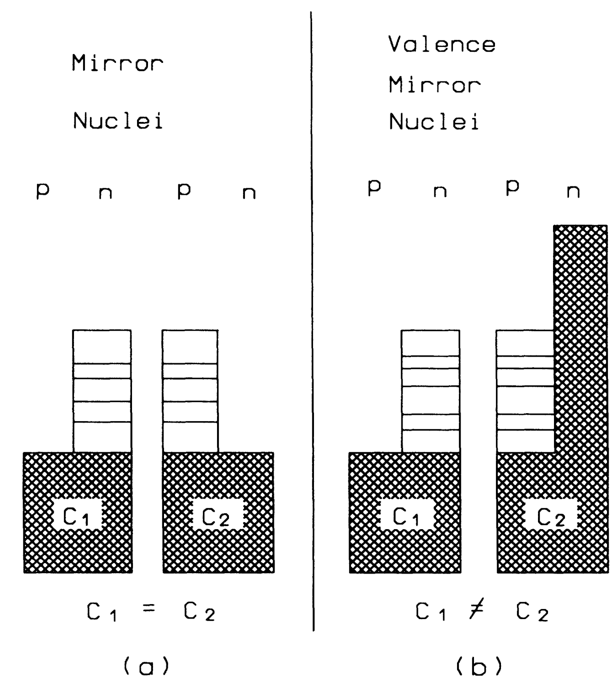

FIG. 1. Schematical representation of mirror states in (a) magic mirror nuclei and (b) valence mirror nuclei. The number of core nucleons is labeled by $C(C=Z_{\mathrm{Core}}+N_{\mathrm{Core}})$. Valence mirror nuclei differ from magic mirror nuclei by having different magic cores (C_1 and C_2).

and yrare band. If we consider only the states of the yrast and yrare bands (i.e., with yrast filter), the condition of unbroken magic core will normally hold up to a certain excitation energy. Let us also emphasize that the members of the yrast band and to some degree the members of the yrare band are exactly the states which are strongly populated in the γ decay following fusion reactions and thus can be easily studied by in-beam γ-spectroscopy experiments.

II. EXPERIMENTAL EVIDENCE AND DISCUSSION

Figure 2 shows the comparison of the low-lying excited yrast states of positive parity of even Ca isotopes on one hand and of their corresponding valence mirror nuclei on the other hand. The data are taken from Refs. 10–16, 24, and 25. A very good agreement of the excitation energies of the levels with given spin and parity can be observed, especially for the pairs ^{42}Ca-^{50}Ti and ^{46}Ca-^{54}Fe. Figure 3 shows a similar comparison of low-lying negative parity states of the odd Ca isotopes ^{43}Ca, ^{45}Ca and their $N=28$ valence mirror isotones ^{51}V, ^{53}Mn. The data are taken from Refs. 11 and 17–25. The similarity of the spectra is also well pronounced with the exception of the $\frac{13}{2}^-$ state of ^{53}Mn. The valence mirror nuclei with proton number Z or neutron number N, when $20 < Z$ (or N) < 28, have only the $1f_{7/2}$ subshell to be filled by the valence nucleons. The condition (i) is then fulfilled. The low-lying valence nucleon excited states can be related to the proton or neutron excitations with $\pi(1f_{7/2})^{Z_v}$ or $\nu(1f_{7/2})^{N_v}$ shell model configurations.[26,27] On the assumption that conditions (ii) and (iii) are approximately held, we would have the similar spectra. The exception is the $\frac{13}{2}^-$ state of ^{53}Mn which clearly cannot belong to the $(1f_{7/2})^{Z_v}$ configuration due to the Pauli exclusion principle. No such candidates in other three nuclei were found. At high energies extended single-particle spaces to f-p shell become more important. We notice that the excitation of valence nucleons to the next major shell ($2p_{1/2}$, $2p_{3/2}$, $1f_{5/2}$, $1g_{9/2}$ orbitals) requires about the same energy as that needed to break the Z (or N)$=20$ magic core. States at such energies will, therefore, be complex and hence we do not expect a similarity of the spectra at high energies. Earlier calculations[28,29] already indicated that core excitation effect could be significant above about 2.5 MeV for these nuclei. For the state $\frac{13}{2}^-$ of ^{53}Mn, Lister *et al.*[19] suggested that it may come from the particle hole excitations that break the neutron $N=28$ core.

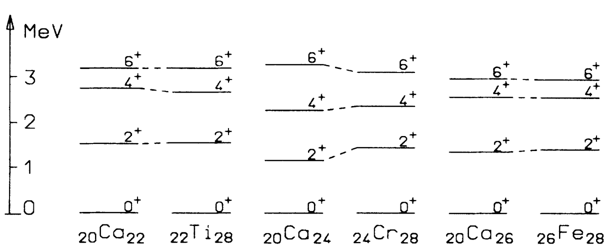

FIG. 2. Comparison of the low-lying positive yrast states of the even Ca isotopes and their corresponding $N=28$ valence mirror isotones. The data are taken from Refs. 10–16, 24, and 25.

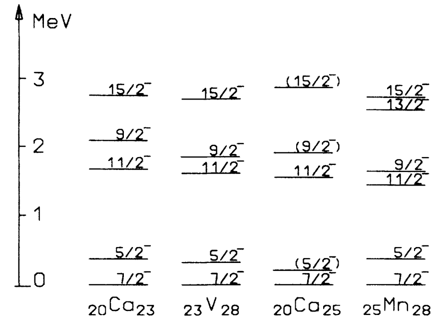

FIG. 3. Excited negative parity states chosen with yrast filter in the valence mirror nuclei ^{43}Ca-^{51}V and ^{45}Ca-^{53}Mn. They can be related to proton $\pi(1f_{7/2})^{Z_v}$ or neutron $\nu(1f_{7/2})^{N_v}$ shell model configurations except for the $\frac{13}{2}^-$ state in ^{53}Mn. The data are taken from Refs. 11 and 17–25.

From recent intensive high spin studies of semimagic $N=82$ isotones and $Z=50$ Sn isotopes by heavy ion fusion reactions we are able to compare pairs of valence mirror nuclei in this region up to a certain high spin. In Fig. 4 we show the yrast filter states for the pair of valence mirror nuclei ^{110}Sn (Refs. 30–32) and ^{142}Nd (Refs. 33 and 34). The 3^- state of ^{110}Sn was not observed in the γ-spectroscopic experiments; it was taken from Ref. 35. We can see in particular that the relative energies of the yrast states with spin values 4 to 11 agree rather well. A reasonable agreement is found also for the energies of the observed yrare states. A similar comparison with the yrast filter for the pair of valence mirror nuclei ^{146}Gd (Refs. 36–38) and ^{114}Sn (Refs. 32 and 39) up to an excitation energy of about 5.5 MeV is given in Fig. 5. The left-hand side of the ^{114}Sn spectrum displays also the bandlike structure, which was identified as collective excitations with a broken proton magic core.[40] These collective states do not have partner states in the ^{146}Gd nucleus. Nevertheless the main part of the spectrum of ^{114}Sn shows a surprising similarity to the spectrum of its valence mirror nucleus ^{146}Gd. In particular, the relative energies of the state with $I^\pi = 5^-, \ldots, 10^+$ agree rather well. We note that some levels are missing or without spin/parity assignments in one nucleus of the pair. This concerns in particular the 5_1^- level in the pair of ^{110}Sn-^{142}Nd, the 8_2^- level and the levels above 10_1^+ in the pair of ^{114}Sn-^{146}Gd, which are not observed or where no spin and/or parity assignments could be made due to differences in the sensitivity of the various experiments. A significant deviation is found in the energies of the low-lying first 2^+ and first 3^- levels. The 2_1^+ and 3_1^- states are more or less collective, and therefore, they imply core excitation admixtures. For instance, the first 3^- state is known nearly in all even-mass $N=82$ isotones, and it contains essential admixtures of neutron particle-hole excitations,[41] which is beyond our limitation of an unbroken magic core. This indicates also the approximation and limitation of our yrast filter.

For the $Z=50$ Sn isotopes and $N=82$ isotones, they have a significant difference of magic core, i.e., $C_1=100$ and $C_2=132$ ($C=Z_{\mathrm{Core}}+N_{\mathrm{Core}}$, see Fig. 1). This difference would bring about the different character in the core excitations. Numerous shell model calculations with complex nucleon-nucleon interactions, and extended single particle spaces including particle hole excitations that break the magic cores, indicate this point.[28,42–45] If we think that the core excitations are responsible for the deviations of the 2_1^+ state and the 3_1^- state in the comparison of even-even valence mirror nuclei, it should also affect the states which result from the coupling of an odd nucleon with the 2_1^+ or 3_1^- states in the corresponding odd-even valence mirror nuclei. This coupling effects make the spectra more complicated and would strongly influence the similarities of the excited spectra in the neighboring odd-even valence mirror nuclei. In fact, we have seen the discrepancies from the comparison of odd-even valence mirror pair ^{111}Sn (Ref. 8) and ^{143}Pm (Ref. 46) in the low excited states. Due to the lack of experimental data on high spin states, the expected similarity of the spectra above spin value $\frac{19}{2}$ cannot yet be verified. On the other hand, we may lose the similarities at high energies due to the particle-hole excitation admixtures. More experimental works on high spin states for the odd-even valence mirror nuclei are therefore appreciated.

We recall that the valence nucleons lie in the fifth major shell in the frame of the simple shell model. The proton single particle energies of the $N=82$ single neutron magic nuclei have been discussed in Ref. 47 and 48. The neutron single particle energies in the Sn isotopes seem to be less well defined. Nevertheless, we notice that the $s_{1/2}$ and $d_{3/2}$ orbitals lie higher than $g_{7/2}$ and $d_{5/2}$ orbitals. If the Fermi level lies between the $d_{5/2}$ and $g_{7/2}$ orbits, the yrast filter will strike out the levels which are formed from the $s_{1/2}$ and/or $d_{3/2}$ combining configurations. The yrast filter states shown in Figs. 4 and 5 are, therefore, mostly of seniority $v \leq 4$ quasiparticle configurations in the $g_{7/2}$, $d_{5/2}$, and $h_{11/2}$ subshells. [In this point of view, the yrast filter tends to select the unique parity configurations (here it refers to $h_{11/2}$). Hence, it can be widely used with the other main shells of the unique parity orbital $g_{9/2}$ and $i_{13/2}$, i.e., N (or Z)=29–50 and N (or Z)=82–126 shells, etc.). Since the Coulomb force tends

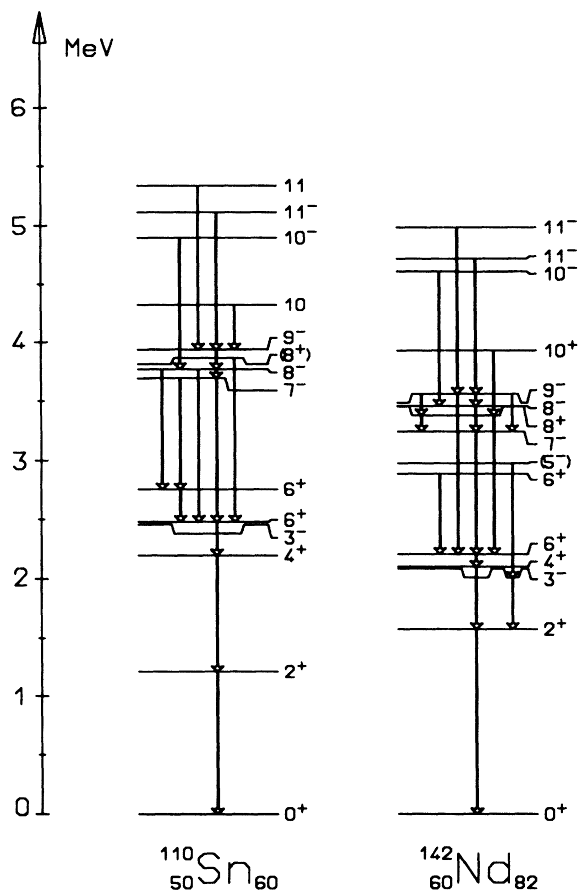

FIG. 4. Comparison of yrast filter states for the pair of valence mirror nuclei ^{110}Sn and ^{142}Nd. The data are taken from γ-spectroscopic experiments with heavy-ion-induced fusion reactions for ^{110}Sn (Refs. 30–32) and ^{142}Nd (Refs. 33 and 34), except for the 3^- state in ^{110}Sn, which was taken from Ref. 35.

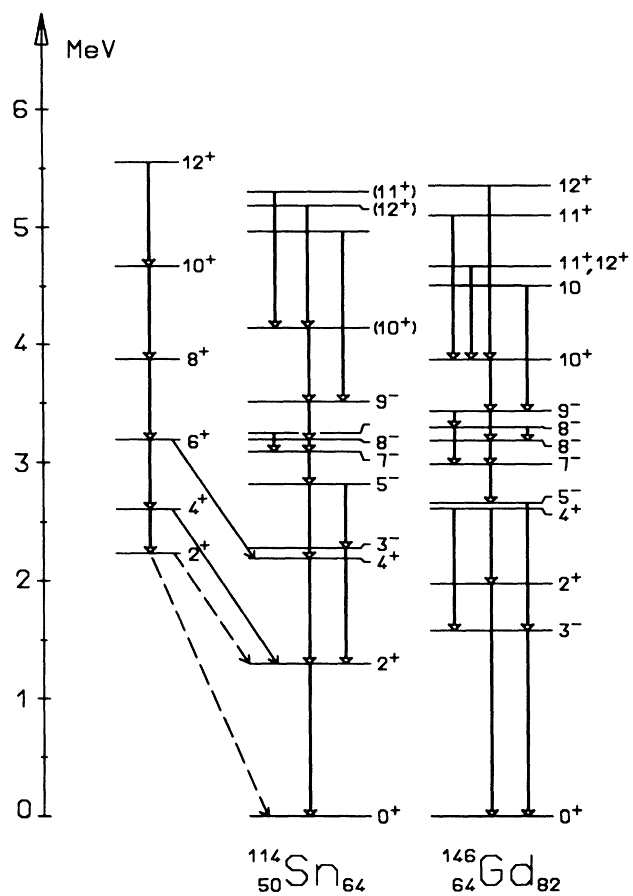

FIG. 5. Comparison of the spectra obtained with yrast filter for the pair of valence mirror nuclei ^{114}Sn and ^{146}Gd. The left-hand side of the spectrum of ^{114}Sn also displays the bandlike excited states which were considered as collective excitations with a broken $Z=50$ core (Ref. 40). The data are taken from γ-spectroscopic experiments with heavy-ion-induced fusion reactions for ^{114}Sn (Refs. 32 and 39) and ^{146}Gd (Refs. 36–38).

to shift all the proton single-particle energies together, it is reasonable to assume that their relative energies are not much changed by the Coulomb force. Then, the similarity of the valence mirror states will depend on the combined effects of the residual interaction and of the underlying single-particle energies (which are, themselves dependent on residual interaction between the closed shells and the valence nucleons). Due to the ambiguities of the known single-particle energies that were directly obtained from the experimental levels or have been used for fitting the experimental data, we cannot definitely decide at the present moment whether the single particle energies and the residual interactions are really identical for these valence mirror nuclei. If we compare the proton single particle energies in ^{143}Pm (Refs. 47 and 48) with the neutron single-particle energies in ^{131}Sn (Ref. 49), we see that they are rather similar. For lighter Sn isotopes, however, Prade *et al.*[8] got significantly different single-particle energies from that obtained by Bonsignori *et al.*,[50] although both groups described the same nucleus (see also Table 1 in Ref. 6). The uncertainties may be an indication that single-particle energies used in theoretical calculations cannot be uniquely determined by demanding a best fit to the experimental data. Moreover, different single-particle energies can be used if the residual interaction is also changed. This poses a more complicated question concerning how the residual interactions in the valence shell are treated.

III. SUMMARY AND CONCLUSIONS

We have compared states in valence mirror nuclei with the yrast filter for the pairs ^{110}Sn-^{142}Nd, ^{114}Sn-^{146}Gd, ^{43}Ca-^{51}V, ^{45}Ca-^{53}Mn as well as some even calcium isotopes with their corresponding $N=28$ valence mirror isotones, respectively. We found evidence of an approximate proton-neutron symmetry for the valence mirror nuclei in the calcium region. With the yrast filter and gamma transition cascade information a subset of the valence-nuclear states from the spectra of the semimagic nuclei can be obtained. The similarity in the valence-nuclear states with medium-high spins for the pairs ^{110}Sn-^{142}Nd, ^{114}Sn-^{146}Gd connects either pure neutron or pure proton shell model configurations. It is difficult to draw definite conclusions on equal single-particle energies and identical residual force in these valence mirror nuclei because of the ambiguities surrounding these quantities, however. While direct information on single-particle energies can be obtained only from nuclei with one particle (hole) plus a doubly magic core, single-particle energies in non doubly magic nuclei can be extracted from the experiment only if we define at the same time the residual interaction or if we have extensive information on spectroscopic factors from transfer reactions. A further study of the similarity in pairs of valence mirror nuclei should lead to a wider knowledge of the A dependence of both residual interactions and the single-particle energies, and it will help us decide whether the proposed proton-neutron symmetry holds also in the heavier nuclei.

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