# Electric quadrupole excitations in relativistic nucleus-nucleus collisions

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Calculations are presented for electric quadrupole excitations in relativistic nucleus-nucleus collisions. The theoretical results are compared to an extensive data set and it is found that electric quadrupole effects provide substantial corrections to cross sections, especially for heavier nuclei.

#### I. INTRODUCTION

The search for a fundamentally new state of matter in the form of a quark-gluon plasma' has stimulated the production of very high-energy nuclear beams. The hope is to observe the quark-gluon plasma in a relativistic nucleus-nucleus collision. At the Berkeley Bevalac a variety of light nuclei such as  ${}^{12}C$ ,  ${}^{16}O$ , and  ${}^{20}Ne$  can be accelerated up to energies of 2.1 GeV/nucleon and heavier nuclei such as  $^{139}$ La and  $^{238}$ U can be accelerated to 1.26 and 0.96 GeV/nucleon, respectively. At the<br>Brookhaven National Laboratory, <sup>16</sup>O and <sup>28</sup>Si beams are available at 14.6 GeV/nucleon and at the CERN Super Proton Synchrotron (SPS) in Geneva, beams of  $^{16}O$  and  $32S$  are both produced at 60 and 200 GeV/nucleon. The relativistic heavy-ion collider (RHIC) is expected to produce two colliding beams at 100 GeV/nucleon to give a total center-of-mass energy of 200 GeV/nucleon, which corresponds to a single beam energy of 21 TeV/nucleon. Grabiak<sup>2</sup> has pointed out that nuclear beams of 3.5 and 8 TeV/nucleon may be possible at the CERN Large Hadron Collider (LHC) or the Superconducting Super Collider (SSC). By way of comparison, the majority of galactic rays have energies<sup>3</sup> of about 1 GeV/nucleon, with a range<sup>3</sup> typically from 10 MeV/nucleon to 1 TeV/ nucleon. However, the JACEE (Japanese-American Cooperative Emulsion Experiment) collaboration<sup>4</sup> has made observations as high as 1000 TeV/nucleon.

Nucleus-nucleus reactions proceed mainly through either the strong or electromagnetic (EM) interactions. Historically, strong interaction processes have been the main object of study,<sup>5</sup> however, with the availability of the above high-energy nuclear beams there has been a resurgence of interest in EM interactions in relativistic nucleus-nucleus collisions.

The primary theoretical tool for studying these relativistic EM processes has been via the Weizsacker-Williams (WW) method<sup>6,7</sup> of virtual quanta. The nucleus-nucleus total EM reaction cross section is

$$
\sigma = \int N_{\rm WW}(E_\gamma) \sigma(E_\gamma) dE_\gamma \tag{1}
$$

where  $E_{\gamma}$  is the virtual photon energy,  $N_{WW}(E_{\gamma})$  is the WW virtual photon spectrum, and  $\sigma(E_{\gamma})$  is the photonuclear reaction cross section. For high accuracy it is important to use experimental photonuclear data for  $\sigma(E_{\gamma})$ . (For an excellent compilation of photoneutron data see Ref. 8.) However, a more exact formulation of  $\sigma$  involves a breakdown into the various EM multipolarities such as electric dipole  $(E1)$ , electric quadrupole  $(E2)$ , magnetic dipole (M1), etc. The most important contributions to  $\sigma$ are from  $E1$  and  $E2$  so that

$$
\sigma = \sigma_{E1} + \sigma_{E2}
$$
  
= 
$$
\int [N_{E1}(E_{\gamma})\sigma_{E1}(E_{\gamma}) + N_{E2}(E_{\gamma})\sigma_{E2}(E_{\gamma})]dE_{\gamma}
$$
, (2)

where  $N_{E_i}(E_\gamma)$  is the virtual photon spectrum of a particular multipolarity due to the projectile nucleus and  $\sigma_{E_i}(E_\nu)$  is the photonuclear reaction cross section of the target nucleus. Bertulani and Baur<sup>6</sup> have derived expressions for  $N_{E_i}(E_\nu)$  and found that the electric dipole spectrum is the same as the WW spectrum, i.e.,  $N_{E1}(E_\gamma) = N_{WW}(E_\gamma)$ . Furthermore, at very high projectile energies all  $N_{E_i}(E_\gamma)$  and  $N_{Mi}(E_\gamma)$  are equal so that Eq. (1) is seen to be a very high-energy approximation to all multipolarities included in Eq. (2). Bertulani and Baur<sup>6</sup> have made a crude estimate of the EM cross section using Eq. (2) but they pulled  $N_{E_1}(E_\gamma)$  and  $N_{E_2}(E_\gamma)$ outside the integral and evaluated them at a single energy and used sum rules to evaluate  $\int \sigma_{E_i}(E_\gamma) dE_\gamma$ . A more accurate calculation can be performed if one uses experimental data for the photonuclear cross section and evaluates the full integral numerically without removing the energy dependence in the photon spectra. Thus I undertook a more exact study<sup>9</sup> leaving Eq. (2) as it stands and using experimental data for the photonuclear cross sections by defining

$$
\sigma_{E1}(E_{\gamma}) \equiv \sigma_{\text{expt}}(E_{\gamma}) - \sigma_{E2}(E_{\gamma}), \qquad (3)
$$

where  $\sigma_{\text{expt}}(E_{\gamma})$  is the *experimentally* measured photonuclear cross section and  $\sigma_{E_2}(E_\gamma)$  is a theoretical calculation based on a Lorentzian shape for the electric giant quadrupole resonance (GQR). Details for this procedure can be found in Ref. 9. As was noted in that reference, the above procedure yields very accurate values for the sum  $\sigma_{E1} + \sigma_{E2}$  (which is to be compared to nucleusnucleus reaction experiments) even though the GQR parameters are uncertain. The basic reason for this, as can be seen from Eq. (3), is that an under (over) estimate in  $\sigma_{E2}(E_{\gamma})$  will give an over (under) estimate in  $\sigma_{E1}(E_{\gamma})$ , so that the combined  $\sigma_{E_1}+\sigma_{E_2}$  in Eq. (2) will not change very much.

In Ref. 9 a detailed study of  $E1$  and  $E2$  was undertak-

en for the reaction  ${}^{89}Y(RHI, X) {}^{88}Y$ , where RHI refers to various relativistic heavy ions and  $X$  is anything. It was found that the E2 effects account for a considerable fraction of the cross section, and that inclusion of  $E2$  [via Eq. (2)] provides improved agreement with experiment over the WW method. Given this situation, it was decided to compare this theoretical approach to as much experimental data as possible. Thus, the present work involves a comparison to neutron emission from  $89Y$ ,  $197Au$ , and  $^{19}$ Co and neutron and proton emission from  $^{12}$ C,  $^{16}$ O, and  $^{18}$ O which includes both electric dipole and quadrupole effects. This complements earlier work<sup>7</sup> which involved an extensive comparison of the WW theory to experiment.

## II. CALCULATIONAL METHOD

The basic calculational method is outlined in Ref. 9 and the discussion will not be repeated here. Also, Ref. 7 includes a very detailed summary of which photonuclear data were used for  $\sigma_{\text{expi}}(E_y)$  in Eq. (3). The same data is used in the present work. All isoscalar GQR parameters were taken from the compilation of Refs. 10 and 21 and are listed in Table I. As mentioned in the Introduction, even though these parameters are somewhat uncertain, the total EM cross section  $\sigma_{E_1}+\sigma_{E_2}$  is expected to be very accurate<sup>9</sup> due to the subtraction procedure of Eq. (3). The most inaccurate results would be expected for the  $^{12}C$ ,  $^{16}O$ , and  $^{18}O$  GQR parameters where the isoscalar GQR is fragmented into several components.<sup>10</sup> Only a single Lorentzian<sup>9</sup> was used in the present work. However,  $\sigma_{E2}$  is found to be quite small for these nuclei (see below) so that my conclusion that the calculated  $\sigma_{E1}+\sigma_{E2}$  is accurate remains valid

For the nuclei <sup>12</sup>C, <sup>16</sup>O, and <sup>18</sup>O, proton (p) emission occurs as well as neutron  $(n)$  emission. Thus, Eq. (3) needs to be modified to incorporate the branching ratio. I assume that the excited nucleus decays only by proton or neutron emission and that the (photon) energy-

TABLE I. Isoscalar giant quadrupole resonance {GQR) parameters taken from the compilation of Refs. 10 and 21. E is the GQR resonance excitation energy,  $\Gamma$  is the full-width at half maximum, and  $f$  is the fractional depletion of the energy weighted sum rule. {The GQR of light nuclei are fragmented into several peaks, so that the parameters below represent an estimated average value. )

<b>Nucleus</b>	E (MeV)	Г (MeV)	
$^{16}$ O	$22.0^{b}$	6.0 <sup>a</sup>	$0.4^{\rm c,d}$
$^{18}$ O	24.0 <sup>e</sup>	6.0 <sup>a</sup>	0.4 <sup>a</sup>
59 <sub>Co</sub>	$16.3^{b}$	$5.6^{b}$	0.61 <sup>b</sup>
89Y	$13.8^{b}$	$3.2^{b}$	0.55 <sup>c</sup>
$197$ Au	10.8 <sup>e</sup>	2.9 <sup>b</sup>	0.95 <sup>c</sup>

'Estimate.

<sup>b</sup>Best value from Table 4 of Ref. 10. 'From Fig. 23 of Ref. 10.

 ${}^d$ From Fig. 17 of Ref. 21.

 ${}^{\text{e}}E$  is calculated from 63  $A$ <sup>-1/3</sup>.

dependent neutron branching ratio is defined as

$$
f_n(E_\gamma) \equiv \frac{\sigma_{\text{expt}}(E_\gamma, n)}{\sigma_{\text{expt}}(E_\gamma, n) + \sigma_{\text{expt}}(E_\gamma, p)} \tag{4}
$$

This is simply a statement that the fraction of neutrons emitted at a given energy is determined by dividing the experimental neutron cross section by the total cross section at the same energy. The total cross section is given as the sum of the neutron and proton cross sections. Thus,

$$
\sigma_{E2}(E_{\gamma}, n) = f_n(E_{\gamma})\sigma_{E2}(E_{\gamma}), \qquad (5)
$$

where  $\sigma_{E2}(E_{\nu})$  is the photonuclear GQR cross section. Thus, for proton and neutron emission Eq. (3) becomes

$$
\sigma_{E1}(E_{\gamma}, n) = \sigma_{\text{expt}}(E_{\gamma}, n) - f_n(E_{\gamma})\sigma_{E2}(E_{\gamma})
$$
 (6a)

and

$$
\sigma_{E1}(E_{\gamma},p) = \sigma_{\text{expt}}(E_{\gamma},p) - [1 - f_n(E_{\gamma})] \sigma_{E2}(E_{\gamma}). \quad (6b)
$$

Equations (4)—(6) were used for nucleon emission from <sup>12</sup>C, <sup>16</sup>O, and <sup>18</sup>O. For <sup>59</sup>Co, the  $(\gamma, p)$  cross section is not available and so a constant value of  $f_n=0.7$  (suggested from Ref. 11) was used. For  $89Y$  and  $197Au$  I used  $f_n = 1.0$ .

## III. RESULTS AND DISCUSSION

The calculated results are listed in Table II, along with the experimental results of various groups.<sup>1</sup>  $\sigma_{E1} + \sigma_{E2}$  is the calculated result to be compared with the data  $\sigma_{\text{expt}}$ . Also listed are the results of WW calculations.<sup>7</sup> In all cases two theoretical cross sections are listed. The first is calculated using an expression for the minimum impact parameter as

$$
b_{\min} = R_{0.1}(T) + R_{0.1}(P) , \qquad (7)
$$

where  $R_{0,1}$  represents the 10% charge-density radius<sup>7</sup> of the target or projectile. The second theoretical cross section listed in parentheses in Table II uses  $b_{\min}$  given by Hill *et al.*  $14-16$  as

$$
b_{\min} = r_0 \left[ A_p^{1/3} + A_T^{1/3} - X \left( A_p^{-1/3} + A_T^{-1/3} \right) \right],
$$
 (8)

where  $r_0$  = 1.34 fm and  $X=0.75$ . (Note that there is a small difference between some of my WW calculation and those of Hill et  $al$ .<sup>14-16</sup> due to a small term which they had inadvertently forgotten.<sup>19,20</sup>)

There are several features readily apparent from Table II.

(i)  $\sigma_{E1} + \sigma_{E2}$  is always larger than  $\sigma_{WW}$ . However, for nucleon emission from <sup>12</sup>C, <sup>16</sup>O, and <sup>18</sup>O this difference is never larger than about 4%, but for neutron emission from  ${}^{59}Co$ ,  ${}^{89}Y$ , and  ${}^{197}Au$  the difference is much larger varying between about  $7-15\%$ .

(ii) For nucleon emission from  ${}^{12}C$  and  ${}^{16}O$  both  $\sigma_{E1} + \sigma_{E2}$  and  $\sigma_{WW}$  agree with experiment for both choices of  $b_{\min}$ .

(iii) For nucleon emission from <sup>18</sup>O both  $\sigma_{E_1} + \sigma_{E_2}$  and  $\sigma_{\text{WW}}$  disagree with experiment for both choices of  $b_{\text{min}}$ .  $\sigma_{WW}$  actually gives slightly better agreement but not by a significant amount.



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(iv) For neutron emission from  $^{197}$ Au,  $\sigma_{E1} + \sigma_{E2}$  is significantly closer to experimental values than  $\sigma_{ww}$  is, although for most cases it still lies outside the error bars. An exception, however, is a much poorer agreement for  $139$ La (see also Refs. 19 and 20). Significant discrepancies with  $^{197}$ Au data have been noted previously for WW theory.

(v) For neutron emission from <sup>89</sup>Y,  $\sigma_{E1} + \sigma_{E2}$  is in much better agreement with experiment than  $\sigma_{WW}$  is. This is especially true for the  $40Ar$  and  $56Fe$  projectiles

(vi) For  $^{59}Co$ ,  $\sigma_{E1} + \sigma_{E2}$  is again better for <sup>20</sup>Ne, although slightly worse for  $56$ Fe. As above, the agreement for the  $^{139}$ La projectile is significantly poorer.

Finally, the earlier results of Bertulani and Baur can be compared for single neutron emission from  ${}^{59}Co, {}^{89}Y,$ and  $^{197}$ Au targets with  $^{12}$ C,  $^{20}$ Ne,  $^{40}$ Ar, and  $^{56}$ Fe project tiles (see Table II and Ref. 6). Surprisingly the results of Ref. 6 give better agreement with experiment than Table II for <sup>12</sup>C and <sup>20</sup>Ne on <sup>197</sup>Au and also for <sup>40</sup>Ar on <sup>89</sup>Y. However, for <sup>40</sup>Ar and <sup>56</sup>Fe on <sup>197</sup>Au and <sup>56</sup>Fe on <sup>59</sup>Co, Table II gives far superior agreement with experiment. Otherwise other comparisons are comparable. However, it should be emphasized that there are substantial differences between Ref. 6 and Table II. In particular, all dipole and quadrupole cross-section values are significantly larger than the present work.

### IV. SUMMARY AND CONCLUSIONS

Calculations have been made for nucleon emission via EM dissociation in relativistic nucleus-nucleus collisions. Results are presented for the Weizsacker-Williams theory and also for separate electric dipole and quadrupole components. The theories have been compared to an extensive data set. It is found that electric quadrupole  $(E2)$ effects are not significant for proton and neutron emission from  $^{12}C$ ,  $^{16}O$ , or  $^{18}O$ . However,  $E2$  contributions are substantial for neutron emission from  ${}^{59}Co$ ,  ${}^{89}Y$ , and  $197$ Au, generally leading to improved agreement between theory and experiment. Notable disagreements occur for <sup>139</sup>La projectiles (1.26 GeV/nucleon) where the theoretical  $\sigma_{E1} + \sigma_{E2}$  are too big. Quadrupole effects improv the theoretical results for  $^{16}$ O projectiles at 60 and 200 GeV/nucleon, although the theoretical cross sections are still too small.

In general, it has been found that electric quadrupole effects are an important component in nucleus-nucleus collisions and that these effects can be calculated accurately.

Note added in proof. Some additional references on electric quadrupoles are R. Fleischhauer and W. Scheid, Nucl. Phys. A 493, 583 (1989); 504, 855 (1989); A. Goldberg, ibid. 420, 636 (1984). Also note that Eq. (4) of Ref. 9 should have  $E_{\text{GOR}}$  in the numerator instead of E.

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