Rapid massive cluster formation

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The process of rapid massive cluster formation at low densities is associated with the process of *multifragmentation*. This phenomenon is studied by comparing the free energy for internally formed clusters with the free energy of the homogeneous system. The importance of considering *two* freeze-out conditions (entropy-freeze-out and force-freeze-out) in the dynamics leading to multifragmentation is discussed. Calculations of the yield of intermediate mass fragments are made with the expanding-evaporating source model and compared with data. The consequences of distributions in excitation energy, on the observation of multifragmentation, are discussed.

I. INTRODUCTION

Recently, there has been much discussion of a process in the disassembly of excited nuclear systems referred to as "multifragmentation." This is a process in which several particles, each more massive than an α particle, are produced in the decay of single system. It is useful to distinguish the "multifragmentation" process from a conventional evaporation process, even when the latter might also yield several particles.

Consider, for example, the results of an evaporation calculation of particle yields from a system of mass 190 which is excited by 400 MeV. This calculation predicts a mean multiplicity of 0.2 for intermediate mass fragments (IMFs), i.e., fragments with mass number greater than 5. The decay of this system is not one of "multifragmentation" even though one predicts probabilities of $2\times10^{-2}, 1\times10^{-3}, 6\times10^{-5}, 2\times10^{-6}, \ldots$ for IMF multiplicities of 2,3,4,5, ...

This illustration indicates that nearly every decay process can provide events in which many intermediate mass fragments are produced. It thus shows why it is useful to reserve the term "multifragmentation" for a process which is qualitatively different from traditional evaporative decay. In this paper we will examine such a process, one in which there is rapid massive cluster formation (RMCF). Such a process is well suited to be designated by the term "multifragmentation."

We have previously constructed a simple schematic model for nuclear disassembly which involves an expanding-evaporating source of fragments. That model is outlined in Refs. 1. The model has been very useful in gaining insight into the RMCF process. In particular, we have studied the evolution of a source which loses mass by evaporation while it expands. In these studies, the instantaneous mass of the emitting source was determined as a function of density. Figure 1 shows the results of calculations for several nuclear systems which have as initial conditions the mass and charge of ^{197}Au and six different temperatures ranging in 1 MeV steps from 10 to 15 MeV. In each of the calculations the energy of the ground state of the instantaneous sources is assumed to be given by the simple form:

$$E_{\text{g.s.}}/A = 8[(1-\rho/\rho_0)^2 - 1] , \quad (1)$$

which varies parabolically from a value of -8 MeV, at normal density, to zero, at vanishing density. These are the two physical limits.

Notice in Fig. 1 that, for $T=15$ MeV, the model provides a monotonically decreasing density, while for lower temperatures the restoring force eventually takes over and brings the system back toward normal density. The precipitous drop in the mass of the source at densities near, or below, $0.4\rho_0$ can be readily seen in the cases with high temperatures. It is found that these sharp declines correspond to instantaneous temperatures in the range of 5–6 MeV (independent of the initial temperature), and that the mean mass of fragments emitted under these conditions rises abruptly despite the fact that the temperature there is lower than the starting temperature. The rise in the mean mass of the fragments is shown in Fig. 2. Here we plot the mean mass of emitted fragments for the $T=12$ and $T=15$ cases. These are plotted versus the

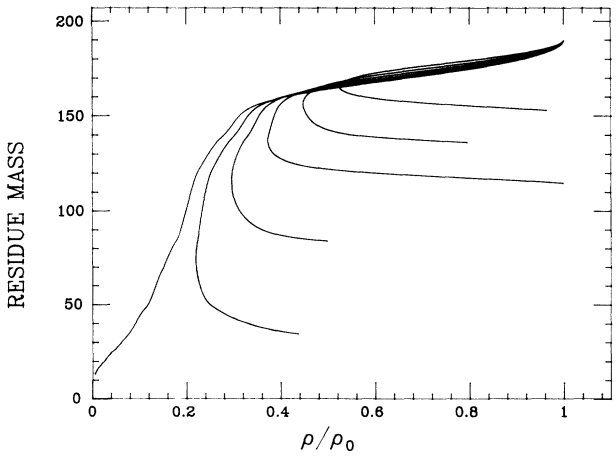

FIG. 1. Instantaneous residue mass as a function of the instantaneous density, calculated during the evolution of ^{197}Au starting from temperatures of 10 to 15 MeV in 1 MeV steps. The lowest densities were achieved for highest starting temperatures.

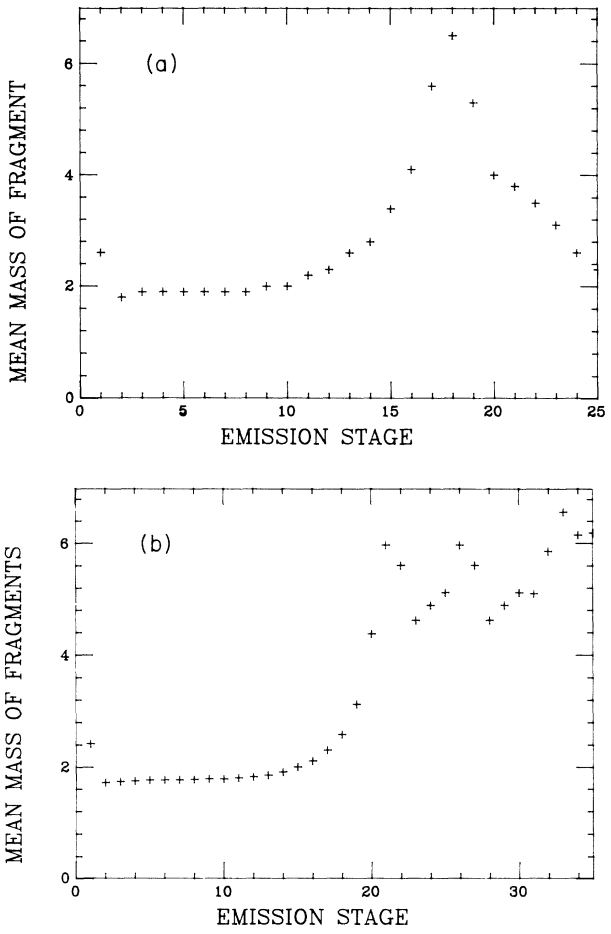

FIG. 2. Mean mass of fragments emitted at each stage in the evolution of an excited ^{197}Au nucleus beginning with a temperature of (a) 12 MeV; (b) 15 MeV.

emission stages which follow a time ordered sequence. The initial values of approximately 2 mass units reflect the dominance of the emission of neutrons, hydrogen, and α particles. At a specific stage (time) the mean mass rapidly triples. This is precisely the stage at which the mass of the source drops sharply. We suggest that this rise accompanies a special condition, associated with lower densities, for which rapid massive cluster formation (RMCF) occurs. When the dynamics of the expansion in our model precludes such densities, RMFC does not occur. This is illustrated in the behavior of the system with initial temperature of 10 MeV in Fig. 1 which emits predominantly light fragments while its density oscillates.

In what follows we explore the features of the reaction which permit this process to occur. In particular, we will explore the threshold conditions for this process. In Sec. II we discuss rapid massive cluster formation from the point of view of the thermodynamic free energy. Section III introduces two freeze-out conditions which must be considered in the case of each multifragmentation process. Section IV provides some specific calculations which are applied to cases under experimental investigation. Finally in Sec. V we summarize and draw conclusions.

II. FREE ENERGY

In this section we use thermodynamic concepts to explore the conditions under which RMCF occurs. We will be concerned with the qualitative features of the production of the IMFs.

For simplicity, we treat the excited system according to the same assumptions used in the expanding-evaporating source model. That is, we take the heated expanded system to be a nucleus of density ρ excited above its ground state. The energy (collective compressional energy) of that ground state depends on ρ. The density of energy states of the system at the (thermal) excitation above the ground state is assumed to be the same as that of a finite temperature Fermi gas. The Fermi energy, ϵ_f, associated with this gas varies with the density of the system as $\rho^{2/3}$. Therefore, it is only necessary to specify ϵ_f at one density ρ_0 (normal nuclear density). Experience with evaporation studies suggests that it is reasonable to use 30 MeV for ϵ_{f_0} with large systems (mass of 160 or greater), and 25 MeV with lighter systems (mass of the order of 100 or less).

Using this model we can determine the free energy F for any density and excitation energy (temperature) of the expanding system. This is given by

$$F = E - TS \ , \tag{2}$$

where E is the total energy (ground state plus excitation), T is the temperature, and S the entropy. All of the functions associated with excitation are treated in the finite temperature Fermi gas model. We find that the quantity F is negative and ranges from large absolute values, at normal density and high temperatures, to zero, at low densities and low temperatures.

To study the tendency for a low density system to condense into normal density clusters (IMFs), we compare the free energy per particle of the homogeneous system with the free energy per particle of the cluster. The latter quantity is found from

$$F_c = -T \ln Z + E_{\text{g.s.}} \ . \tag{3}$$

Here Z is the partition associated with the motion of the cluster center of mass:

$$Z = V \times g_c \times (M_c T / 2\pi\hbar^2)^{3/2} \ . \tag{4}$$

The ground state energy of the cluster is typically -8 MeV times the number of nucleons. The quantities M_c and g_c are the mass and degeneracy factors of the cluster. We take the value of temperature T for the cluster motion, in Eq. (4), to be equal to the temperature of the homogeneous system from which the cluster forms.

The crucial factor in the free energy for the cluster is the volume V in the partition function of Eq. (4). For cluster formation on the surface of the system, with motion outward with respect to the surface, we have shown[2] that this volume is intimately related to the evaporation decay rate itself. In Ref. 2 we demonstrated, us-

ing Weisskopf detailed balance, that the volume required for the free energy of the evaporated cluster to equal to the free energy of its constituent nucleons in the source is precisely the volume occupied by that flux of the clusters which come from the surface of the source during the mean lifetime for the statistical emission of the cluster. If we assume that clusters will form when their free energy is equal or less than that of the constituents in the source, then, subject only to conservation laws, IMF emission by surface evaporation can occur at any excitation energy.

What is the situation with regard to *internal* clustering? In order to explore this process we again must consider the volume V which enters calculation for the cluster free energy. In the case of internal clustering we take as the volume in Eq. (4), that volume occupied by the A nucleons which join to form the cluster. This would also be the volume of the cavity produced in the homogeneous system by the formation of the cluster. With this choice of volume the cluster free energy may be calculated at each density.

We now can calculate the crossover condition (density, temperature) at which the free energy of the cluster becomes lower than the free energy of particles in the homogeneous source. Shown as an illustration in Fig. 3 is the case for isentropic expansion from an initial temperature of 15 MeV at density ρ_0. The Fermi energy is taken to be 30 MeV, and the cluster assumed to be ^{16}O. The schematic density dependence of Eq. (1) is used for the energy of the ground state of the source. When this procedure is repeated for different entropies we obtain the locus of crossover conditions shown in Fig. 4.

The formation of fragments due to the crossover of free energies for *internal* clustering is a phenomenon qualitatively different from evaporation. This is the condition we associate with RMCF, and with true "multifragmentation." Indeed, the observed RMCF behavior evident in Fig. 1 is found to occur when conditions approach those of the crossover. Any system which reaches the crossover conditions might be expected to demonstrate RMCF.

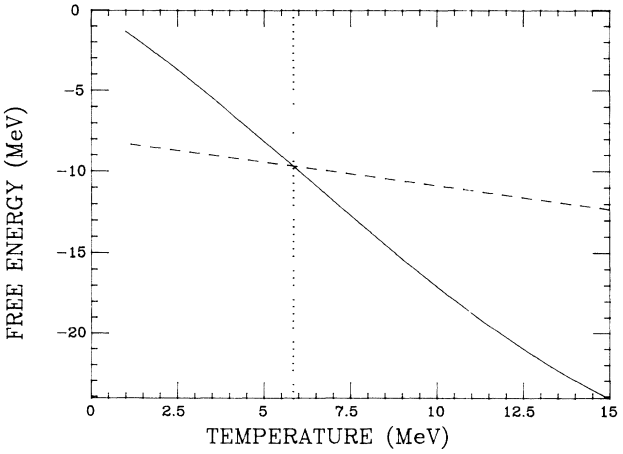

FIG. 3. Free energy per particle versus temperature for an isentropic expansion. Source as Fermi gas (solid); ^{16}O cluster (dashed).

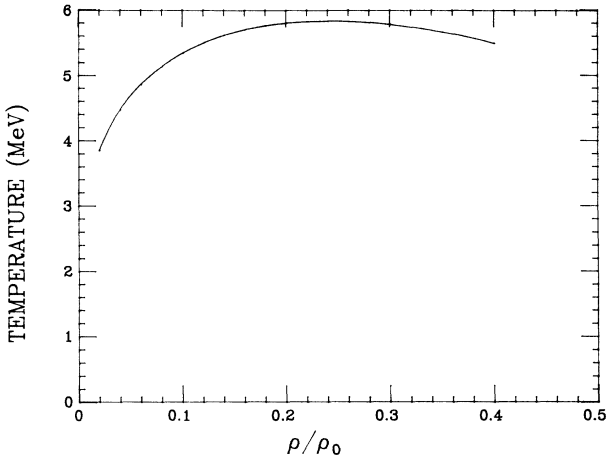

FIG. 4. Locus of conditions where the free energy for clusters (^{16}O) becomes less than the free energy for the homogeneous system.

III. TWO FREEZE-OUT CONDITIONS

In order to understand the physics leading to RMCF, and at the same time to appreciate the differences among various models which currently attempt to explore multifragmentation, we must carefully consider the concept of "freeze-out." This term is widely mentioned in a variety of models. It generally refers to the condition under which the fragments cease interacting strongly with one another and progress to detection, influenced only by Coulomb forces. We might associate the situation of RMCF with such a stage of disassembly.

As an important ingredient of the multifragmentation process, it is crucial to introduce a second, different, "freeze-out" condition. We shall call this one the *entropy*-freeze-out condition. By contrast, we will refer to the previously mentioned condition as the *force*-freeze-out condition. These distinct conditions are roughly analogous to the "saddle" and "scission" conditions in transition-state fission theory. In that context there is statistical balance among the states at the former and the end of strong interactions at the latter. An appreciation of both of these two conditions in the multifragmentation process is essential when comparing the different models.

In the pure microcanonical approaches exemplified by the extensive work of Gross,[3] Bondorf,[4] and Koonin, Randrup, and Fai,[5] the two freeze-out conditions are the same. That is, entropy is assumed to increase as the system expands to the large volume (between 4 and 10 times normal nuclear volume) in which the breakup is considered. In the expanding-evaporating source model, on the other hand, the two conditions are very different. The entropy-freeze-out condition is taken at normal nuclear density, whereas the force-freeze-out, for example the condition for RMCF, is at a much lower density. This choice is motivated by microscopic calculations which suggest a rapid rise in entropy during the initial compressional stages of a nuclear collision, with saturation of entropy at nearly constant value as the system expands to densities below normal density. Intermediate

models, in which the entropy increases up to a given density between normal nuclear density and the force freeze out, are possible, but have received little attention to date.

As an illustration, we next consider three sample cases, all with the same force-freeze-out condition, but with different entropy-freeze-out conditions and different situations between the freeze-out conditions. We take a density of $0.2\rho_0$ and a temperature of 5.8 MeV as the force-freeze-out condition for all three cases. This condition corresponds to a crossover condition in Fig. 4 and is associated with a thermal energy per particle of 5.18 MeV, and entropy per particle of 2.07.

We next work backwards in time for each of the cases to determine the conditions which prevailed at normal nuclear density, prior to the expansion which results in the force-freeze-out condition. For case I let us assume that the entropy-freeze-out point is at normal density, ρ_0. Then for this case the entropy per particle at ρ_0 is 2.07 (the same as at the force-freeze-out condition), while the temperature is 16.96 MeV, and the thermal energy per particle is 15.16 MeV. For cases II and III, let us assume that the entropy-freeze-out is at the force-freeze-out condition given above, i.e., with $\rho=0.2\rho_0$, etc. For case II we assume that the system must work against a force associated with the density dependence of the ground state given in Eq. (1). We then find that at ρ_0 the entropy per particle would be 1.75 (less than 2.07); the energy per particle would be 10.305 (if there is no collective kinetic energy at ρ_0), and the temperature would be 13.1 MeV. Finally for case III we assume the system does not work against any attractive potential in progressing from normal density to the force-freeze-out condition. Then we find that at ρ_0 the entropy per particle would be 1.28, with the thermal energy 5.18 MeV (the same as at the force-freeze-out condition), and the temperature would be 8.654.

This illustration, with cases having the same force-freeze-out conditions, clearly demonstrates the importance of considering two freeze-out conditions, and also the importance of the conditions leading to the freeze-out points. If case III were to hold, an energy of about 5 MeV/particle would lead to the same type of final freeze out, as would require 15.16 MeV per particle if case I were to hold. This big difference leads to big differences in predictions of the conditions which must be met in order to achieve multifragmentation or RMCF.

In the preceding example no particle loss from the source, between the freeze-out conditions, was considered. If one is to quantitatively determine what excitation energy is required for a system to achieve multifragmentation, one must also take into consideration the particles which are emitted between the two conditions. Besides the work which must be done against the attractive nuclear force, this evolution introduces additional energy considerations.

With the emission of particles, presumably from the surface of the expanding system, their kinetic energy and separation energy is removed from the source. A portion of the original excitation energy is also taken by the collective compressional (subnormal density) energy corresponding both to the reduction of the density (positive) and to the loss of source mass (negative). Finally, the expanding source acquires a collective kinetic energy. All of these components of energy must be supplied by the initial excitation energy, in addition to the minimal excitation energy required to achieve RMCF at the force-freeze-out condition.

The expanding-evaporating source model is especially well suited for studying the evolution of the system between the two freeze-out conditions. We have performed illustrative calculations with this model to examine the magnitude of each of the components of energy loss. For this purpose we considered a system with $A=190$ and $Z=76$ which has an entropy-freeze-out at normal nuclear density, ρ_0, with a temperature, at that point, of 15 MeV. Let us take the condition for RMCF to be a density of $0.30\rho_0$ with a temperature of 5.89 MeV. We assume this to be the force-freeze-out condition. During the time the system progresses from the entropy-freeze-out to the force-freeze-out a calculation shows that it loses about 45 units of mass and about 17 units of charge. These carry away about 836 MeV of energy in kinetic energy, and about 182 MeV in separation energy. The compressional energy of the system grows from 0, at the entropy-freeze-out point, to 574 MeV, at the force-freeze-out point. The collective kinetic energy (expansion) goes from 0 to 136 MeV. From an original excitation energy of about 2400 MeV ($E^*/A=12.6$ MeV) the excitation energy at the force-freeze-out has dropped to about 675 MeV ($E^*/A=4.66$). This is clearly a significant change, and further demonstrates that the dynamics of the process between the two freeze-out conditions has a crucial bearing on determining what excitation energy is required to achieve multifragmentation.

IV. CALCULATIONS AND DATA

We now compare the expanding-evaporating source model to data recently reported by a group from Gesellschaft Schwerionenforschung Darmstadt (GSI), Frankfurt, and Heidelberg.[6] These data were specifically analyzed to search for the onset of multifragmentation.

The experimental group[6] defined as an intermediate mass fragment (IMF) a cluster of charge three or greater. They measured the mean multiplicity $\langle M' \rangle$ of IMFs in coincidence with an additional trigger IMF. Finally they defined the "mean multiplicity" of IMFs M by $M=1+\langle M' \rangle$. A plot of M vs the mean energy is given in Ref. 6, for reactions involving different projectiles and different beam energies on targets of Au and Ag.

The notation which is used in Ref. 6 is, unfortunately, confusing, since the true IMF multiplicity, and the multiplicity in coincidence with one trigger fragment, are expected to be nearly equal, and not to differ by one, as implied. In the Appendix we provide a proof that two multiplicities would be exactly equal if the multiplicity probability follows a Poisson distribution.

We have determined the actual distribution P_n for the calculations with the expanding-evaporating source. While it is not exactly a Poisson distribution, it is very well approximated by such a distribution, even when the

mean multiplicity is only a small fraction of one. We find that the full multiplicity and the coincident multiplicity differ by less than 5% for excitations of 500 MeV or more in mass 190 systems.

We disagree, therefore, with the suggestion in Ref. 6 that a signature for the onset of multifragmentation is provided when $(\langle M' \rangle + 1)$ acquires values greater than one. Our analysis suggests that this feature only indicates that the mean multiplicity here is beginning to deviate from zero. Instead a full mean multiplicity approaching 1, i.e., $(\langle M' \rangle + 1)$ approaching 2, would reflect a sizeable probability for there being more than one fragment.

We have used our model to calculate the mean multiplicities for fragments with charge 3 or greater. These calculations were performed for systems with starting masses of 190 units ($Z=76$) and 100 units ($Z=44$), to reflect reactions involving gold and silver targets.

In order to compare with the discussion of Ref. 6, we plot in Figs. 5 and 6 $(\langle M \rangle + 1)$ vs the excitation energy used in the calculation. The symbols indicate the data presented in Ref. 6, without showing the error bars. We find that the calculations for *both* the light and heavy targets represent the data remarkably well. The discussion in Ref. 6 indicated that the models the authors used were not successful in accounting for both light and heavy targets. In particular, the model of Bondorf anticipated a sharp rise in both sets of data at relatively low excitation energy, a feature not found in the data. This prediction appears as the dotted line in our figures.

We suggest that this anticipation may stem from the fact that the Bondorf model is one in which the entropy- and force-freeze-out conditions are the same. It thus explicitly considers none of the physics occurring between the two. In another microcanonical calculation[3] Gross predicts that the cracking mode (multifragmentation) energies at 4–5 MeV/nucleon, consistent with the sharp rise predicted by Bondorf model. In the calculations of Gross both freeze-out conditions have the density about $0.2\rho_0$ and the temperature is about 5–6 MeV. These conditions are indeed similar to those at RMCF conditions but not to the initial conditions of our calculations.

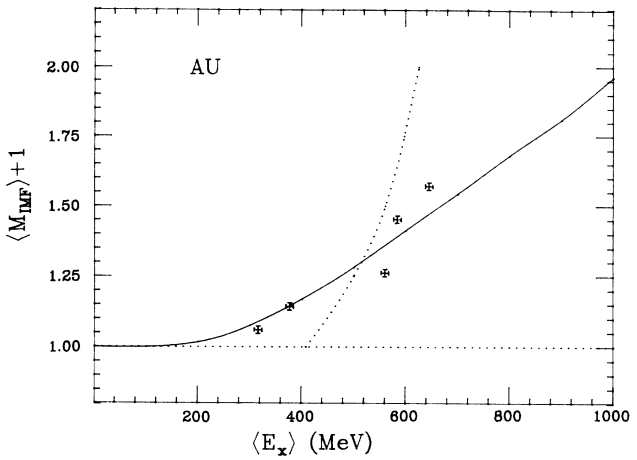

FIG. 6. $(\langle M_{\rm IMF} \rangle + 1)$ vs excitation energy, calculated with the expanding-evaporating source model for an Au target. The data indicated and the rising dotted line is the Bondorf model prediction from Ref. 6.

We calculated the mean multiplicity for both systems at energies above those reported in the experiments. We have also explored the sensitivity of these calculations to the stiffness of the density dependence of the ground state of the source. These results are shown in Figs. 7 and 8. In one set of calculations for each target we held the source density fixed at normal density throughout the emission period. This represents infinite stiffness. For another set of calculations we used the form in Eq. (1). This corresponds to a compression constant of 144 MeV. In addition, calculations were made with compress constants of 120 and 200 MeV for the mass 190 case.

We find that, in the region explored by the experiments in Ref. 6, the results are independent of the density

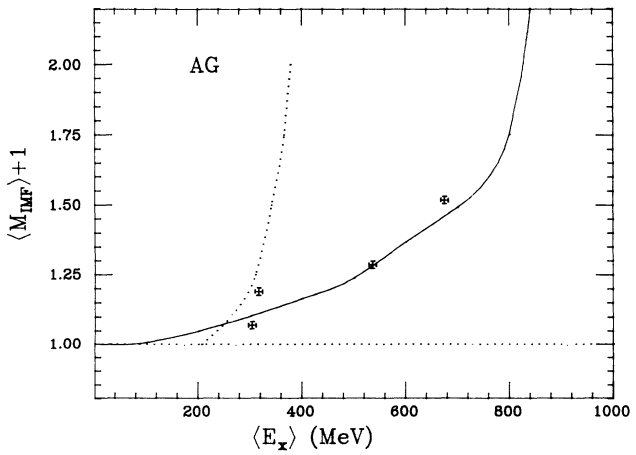

FIG. 5. $(\langle M_{\rm IMF} \rangle + 1)$ vs excitation energy, calculated with the expanding-evaporating source model for an Ag target. The data indicated and the rising dotted line is the Bondorf model prediction from Ref. 6.

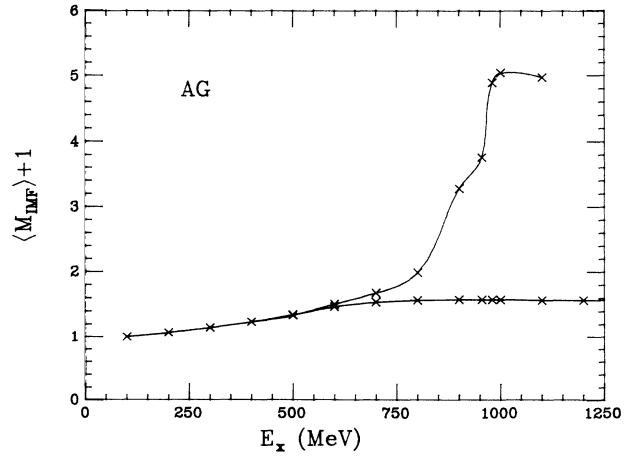

FIG. 7. $(\langle M_{\rm IMF} \rangle + 1)$ vs excitation energy, calculated with the expanding-evaporating source model for an Ag target over an extended energy range. The lower curve has no expansion; the upper has expansion with Eq. (1).

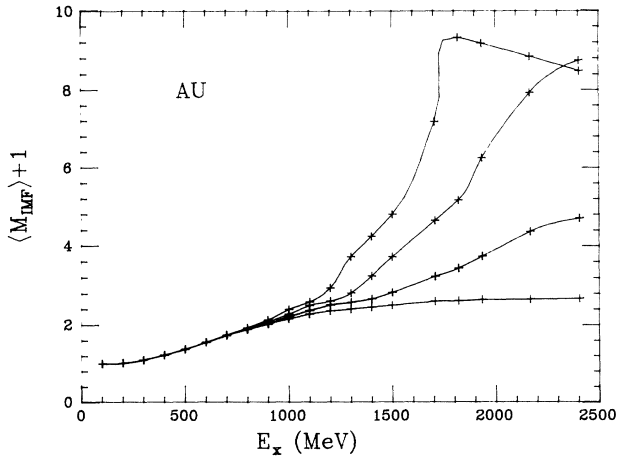

FIG. 8. $(\langle M_{\rm IMF}\rangle+1)$ vs excitation energy, calculated with the model for an Au target over an extended energy range. The lower curve has no expansion; the upper curves have expansion with compressibility constants of 120, 144 [Eq. (1)], and 200 MeV, respectively.

dependence. This is because, for the energies involved, conditions of low density are not found. The results of calculations performed without permitting the source to expand provide essentially the same multiplicities as the calculations which involve oscillations of the density about normal conditions. Only at higher excitation energies is the expansion substantial enough to probe the regions of RMCF. According to the model, those IMFs which are produced in the range of the experimental energies appear to come from surface evaporation.

In the energy region above the experimental one we do find a sharp rise, evidence for RMCF, at an excitation energy of the order of 1500 MeV for the mass 190 system. Furthermore the exact location of this rise is sensitive to the restoring force (equation of state).

It thus appears that a sharp rise with energy in IMF multiplicity would be a good and simple feature to signal the onset of multifragmentation. The slope of this rise should, however, be greater than that associated with the surface evaporation process. It appears, at this time, that such a signature has not been seen.

Let us now consider the fact that, in collisions with a beam of given energy, there is a distribution of energy deposited in the targets that are struck. The decays which are detected thus arise from sources which have a *range* of energies. The excitation energies that are extracted from experiments are average energies. Clearly some of the systems contributing to the yield have energies above and some below this value. The mean multiplicities obtained in most *calculations* represent values which come from a definite excitation energy. Let us define this mean multiplicity (for a given excitation energy) as $M(E^*)$. The experimentally measured mean multiplicity would then correspond to the average of $M(E^*)$ over the distribution of energies represented by the collection of sources.

We should compare experimental data to this average, which we designate $M_{\rm exp}$,

$$M_{\rm exp}=\int M(E)P(E)dE \Big/ \int P(E)dE \ .$$

Here $P(E)$ gives the probability that the source has excitation energy E. The average energy, $\langle E\rangle$, follows from the same probability distribution. Only for the case that $M(E)$ is linear in E do we obtain $M_{\rm exp}$ equal to $M(\langle E\rangle)$. Thus a quantitative prediction of multiplicity versus the average energy requires both $M(E)$ and $P(E)$.

This is especially important for the situation involving the onset of multifragmentation. We would expect some of the sources of the distribution to have sufficient energy to undergo multifragmentation even though the *average* energy is below this threshold. Since the multiplicity of IMFs is expected to rise rapidly with multifragmentation, such systems provide more than their share of fragments. Thus, depending on the distribution in energy, one might expect the influence of multifragmentation to be experimentally detected at relatively lower average energies.

For data in which the center of mass beam energy is less than that required to probe multifragment densities, the influence of the energy distribution would not be great, since the dependence of the multiplicity on energy is not far from linear. This seems the case for the data in Ref. 6. For cases where the beam energy would permit multifragmentation the situation is different.

One set of data[7] that is currently being examined for signatures of multifragmentation is that of 800 MeV/nucleon α-particle collisions with Au. Here the maximum possible energy is 3200 MeV, sufficiently high to produce low densities.

It has been suggested[8] that for very high energy protons one should expect an excitation energy distribution which is exponential in form. We have examined the effect of folding the calculated multiplicity of IMFs (mass 6 or greater) over such a distribution which is cutoff at the maximum energy:

$$P(E)=\Theta(E_{\rm max}-E)\times\exp(-E/E_c) \ . \qquad (5)$$

We take $E_{\rm max}=3200$ MeV and vary the slope coefficient, E_c, in the distribution to produce mean energies of 700 and 900. In each of these distributions the probability of there being systems with energy in excess of 1500 MeV is not large. It is at these higher energies, however, where the yield of IMF fragments is expected to be large. Our results do, indeed, show an influence from the energy distribution. They indicate that the multiplicity for an exponential distribution with mean energy of 700 MeV is 1.160 while that of a system at 700 MeV is 0.73. Similarly a distribution with an average energy of 900 MeV provides a mean multiplicity of 1.72 while a system at 900 MeV has a multiplicity of only 1.08.

V. CONCLUSIONS

We have shown that there is some probability that a decaying system will emit several intermediate mass fragments, even in the case of normal surface evaporation. Such a process should not be classified a true "multifragment" event. Rather this term should be reserved for events which are qualitatively different. We have introduced the concept of rapid massive cluster formation

(RMCF) which we found to occur in systems at low density. We have examined this process, and have associated it with the conditions which prevail when the free energy of a cluster formed inside a source becomes lower than the free energy of the homogeneous system from which it forms.

We have introduced the concept of *two* freeze-out conditions, both of which are necessary to characterize the dynamics leading to multifragmentation. One of these is the condition at which the entropy becomes fixed, and one the condition at which the strong forces cease to act. The latter is the condition most commonly used when reference is made to "freeze-out." The importance of *each* has been discussed, and the differences between different models have been related to differences in the entropy-freeze-out assumptions. In particular, for microcanonical calculations of multifragmentation this condition is the same as the force-freeze-out, while for the expanding evaporating source model they are very different. We have explored predictions in the latter model. That model is especially well suited for studying the passage from entropy-freeze-out to the force-freeze-out when the two conditions differ. In that case it was shown that the physics associated with this evolution has a great effect on predictions of the conditions necessary to achieve multifragmentation.

We have used the expanding-evaporating model to study some systems involving both Au and Ag targets which were recently studied experimentally in a search for signatures of multifragmentation. In these cases we are able to well reproduce the experimental data for both targets. Other models had not been able to do this. We also find results which indicate that the data are consistent with a low value of mean multiplicity. In addition there seems no indication in these systems that there has been expansion to low densities. Thus, for those cases, there seems no strong evidence for multifragmentation. However, the calculations do suggest that a sharp rise in the multiplicity of IMFs is indeed expected but at energies higher than observed. Such a rise would seem a good indicator of multifragmentation by the RMCF process. Further, the rise would provide information concerning the compressibility of finite nuclei.

Finally we discussed the fact that in collisions with a beam of projectiles having a given energy there is, nonetheless, a distribution in the energy deposited in the excited systems. We illustrated the influence of this distribution on the yield of IMFs. Because of this influence we suggest that the onset of multifragmentation may be detectable through the rise in the multiplicity of IMFs when the average energy of the excited ensemble of systems is less than the threshold.

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APPENDIX

We prove here that when the multiplicity probability follows a Poisson distribution then the mean multiplicity of IMFs, and the mean multiplicity in coincidence with a trigger IMF, have the same value.

Let ν be the mean multiplicity of IMFs and the $P_n(\nu)$ be the probability that the multiplicity is n. The mean multiplicity is defined by

$$\langle M \rangle = \sum_n nP_n \Big/ \sum_n P_n = \nu \ .$$

Next consider the case of multiplicity in coincidence with a trigger IMF. For the events which create n fragments, any one of these may become the trigger and the remaining $(n-1)$ will be in coincidence with that one. Similarly, the events in which n coincident fragments are seen with the trigger are those in which a total of $(n+1)$ fragments are produced, any one of which can be the trigger. The probability that n fragments be seen in coincidence with the trigger, P'_n, is then to be proportional to $(n+1)\times P_{n+1}$, where the factor of $(n+1)$ comes from the various possible triggers. It immediately follows that the mean number of coincident fragments is

$$\langle M' \rangle = \sum_{n=0} nP'_n \Big/ \sum_{n=0} P'_n$$
$$= \sum_{n=0} n(n+1)P_{n+1} \Big/ \sum_{n=0} (n+1)P_{n+1} = \nu \ ,$$

and

$$\langle M' \rangle = \langle M \rangle \ .$$

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