Neutron hole strength distribution of the $2f_{7/2}$ state of ^{207}Pb

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The depletion of the neutron hole strength of $2f_{7/2}$ state at 2.34 MeV excitation energy in ^{207}Pb has been explained on the basis of the core polarization effect after retrieving the energies of the shell-model neutron hole states in ^{208}Pb from a numerical solution of the Schrödinger equation. The present findings are discussed in the light of other existing theoretical calculations.

I. INTRODUCTION

Numerous stripping and pickup reaction experiments on ^{208}Pb have been carried out to understand the shell-model strength distributions of the neutron and proton states of ^{208}Pb around the Fermi surface. The fractional occupation probabilities of the neutron or proton states are the basic ingredient to test the validity of the nuclear shell-model. For the doubly closed ^{208}Pb nucleus, the spectroscopic factors of the discrete states lying within the few MeV excitation energies of the Fermi surface should have been unity. But in recent years, we have observed that the single particle or hole strengths of the discrete states of ^{208}Pb deviate from unity.[1,2] The microscopic calculations on short-range and tensor correlations show a depletion of the normally completely filled shell-model orbitals lying in the vicinity of the Fermi surface of ^{208}Pb. The quenching have been shown to be as large as 30%. The numerical calculation, based on the Dyson equation with an energy dependent mass operator within the theory of finite Fermi systems,[3] exhibits the depletion of a few shell-model orbitals within 4 MeV excitation energy of the Fermi surface of ^{208}Pb (Ref. 3). Moreover, the microscopic shell-model calculation[4] involving 1p-1h and 2p-2h excitations (seniority quantum number restricted to ≤ 3) reproduce the quenching of the discrete shell-model states of ^{208}Pb. On the experimental side, the high resolution $(e,e'p)$ knock-out experiment[1] and the $(d,{}^3\mathrm{He})$ reaction experiment[5] show the reduction of the knock-out strength of the ground state $3s_{1/2}$ proton orbitals in both the ^{206}Pb and ^{208}Pb nuclei.

In the present paper we will show the neutron strength distribution of the $2f_{7/2}$ neutron hole state of ^{207}Pb which loses its shell-model identity from the 2.34 MeV state. Recently,[6] the spectroscopic factors of the four $3p_{1/2}$, $3p_{3/2}$, $2f_{5/2}$, and $2f_{7/2}$ states have been calculated from the proton decay of the ^{208}Bi isobaric analog state. From the escape width of the isobaric analog state in ^{208}Bi into the neutron hole states of ^{207}Pb, the single hole strengths of the aforementioned four discrete neutron hole states have been estimated. This experimental result indicates that 30% of the total shell-model strength of the $2f_{7/2}$ state is distributed in the higher excitation energy region. To account for this, we have solved the Schrödinger wave equation with a potential well consisting of the Woods-Saxon (WS) and spin-orbit terms vouching upon Numerov's method. This is necessary because the exact distribution pattern of all the neutron hole states down to the deep region is still not known from the experimental findings. As a result, centroid energies of the shell-model neutron orbitals cannot be calculated. The deduced shell-model energies of the neutron hole states have been utilized to diagonalize the $\frac{1}{2}^-$, $\frac{3}{2}^-$, $\frac{5}{2}^-$, and $\frac{7}{2}^-$ Hamiltonian matrices within the framework of the hole-core vibrational coupling model.

II. ESTIMATION OF THE ENERGY OF THE $2f_{7/2}$ STATE IN ^{207}Pb

The energy of the neutron states are calculated within the Woods-Saxon potential including a spin orbit part[7]

$$V(r)=\left\{V_0\left[1+\exp\left[\frac{r-r_0A^{1/3}}{a_0}\right]\right]^{-1}+V_s\frac{1}{r}\frac{d}{dr}\left[1+\exp\left[\frac{r-r_sA^{1/3}}{a_s}\right]\right]^{-1}\mathbf{L}\cdot\mathbf{S}\right\} \quad (1)$$

The potential parameters are depicted in Table I. With these sets of potential parameters, we have obtained excellent agreement of the calculated energies of the $3p_{1/2}$, $3p_{3/2}$, and $2f_{5/2}$ discrete neutron states with the experimental ones. Moreover, the energies of the deeply bound neutron $2g_{7/2}$ and $2g_{9/2}$ states have also been reproduced accurately with these WS potential parameters. The calculated energies of the neutron hole states are also shown in Table I.

III. CORE-POLARIZATION EFFECT

In the hole-core coupling model calculation,[8,9] the hole states are coupled with the collective vibrational states of the core nucleus and this is the vital aspect for the fragmentation of the single hole strength which we want to investigate. The reason is the proximity of the energy of the hole state and the core-coupled hole states, as well as the appreciable value of the matrix elements of the in-

TABLE I. Woods-Saxon potential parameters along with the unperturbed location of the neutron shell-model states of ^{207}Pb.

$V_0=48.479$ MeV; $V_s=27.74$ MeV;		$r_0=1.31$ fm; $r_s=1.246$ fm;		$a_0=0.718$ fm $a_s=0.391$ fm		
State	$3p_{1/2}$	$2f_{5/2}$	$3p_{3/2}$	$1i_{13/2}$	$2f_{7/2}$	$1h_{9/2}$
Energy (MeV)	−7.521	−8.145	−8.331	−8.646	−10.411	−10.882
State	$3s_{1/2}$	$2f_{3/2}$	$1h_{11/2}$	$2d_{5/2}$	$1g_{7/2}$	$1g_{9/2}$
Energy (meV)	−14.985	−15.188	−14.617	−16.674	−17.857	−20.287

teractional Hamiltonian between the non-spin-flip states. In the present study we have coupled $3p_{1/2}$, $2f_{5/2}$, $3p_{3/2}$, $1i_{13/2}$, $2f_{7/2}$, $1h_{9/2}$, $3s_{1/2}$, $2d_{3/2}$, $1h_{11/2}$, $2d_{5/2}$, $1g_{7/2}$, and $1g_{9/2}$ neutron states with the 2_1^+, $2^{+'}$, 4^+, 6^+, 8^+, 3_1^-, 5_1^-, 5_2^-, $3^{-'}$, 7^-, and $1^{-'}$ vibrational states of ^{208}Pb to frame the matrices of the $\frac{7}{2}^-$, $\frac{5}{2}^-$, $\frac{3}{2}^-$, and $\frac{1}{2}^-$ states. The Hamiltonian of the physical system can be written as

$$H=H_n+H_{\rm vib}+H_{\rm int} , \tag{2}$$

where $\langle H_n\rangle$ and $\langle H_{\rm vib}\rangle$ denote energies of the neutron hole states and the energies of the vibrational state of ^{208}Pb, respectively. For the small distortion of the collective core due to the core polarization effect, the leading order coupling term for the hole-core interaction Hamiltonian $H_{\rm int}$ in (2) is given by

$$\begin{aligned} H_{\rm int}&=(2\lambda+1)^{1/2}K(r)\left[\sum_\mu \alpha_{\lambda\mu}Y_{\lambda\mu}(\theta,\phi)\right]_0 \\ &=(2\lambda+1)^{1/2}K(r)[\alpha_\lambda Y_\lambda(\theta,\phi)]_0 , \end{aligned} \tag{3}$$

where $(\alpha_\lambda)_0$ represent the zero-point amplitude for the individual mode of vibration. The $(\alpha_\lambda)_0$ is

$$\begin{aligned} (\alpha_\lambda)_0&\equiv[\langle n_\lambda=0|\alpha_\lambda^2|n_\lambda=0\rangle]^{1/2} \\ &=\frac{[4\pi B(E\lambda)]^{1/2}}{(\lambda+3)Z} , \end{aligned} \tag{4}$$

where $|n_\lambda=0\rangle$ represents the ground state of ^{208}Pb. The radial function $K(r)$ is

$$K(r)=-r\frac{dV}{dr} . \tag{5}$$

The Hamiltonian H in Eq. (2) is diagonalized by writing the wave function for $J=j_1$ spin as

$$\psi_J=\sum_{\lambda j_2} a_{\lambda j_2}|n_\lambda;\lambda j_2,j_1\rangle , \tag{6}$$

where j_2 and λ are the angular momenta of the hole state and the vibrational state, respectively. In addition, $\boldsymbol{\lambda}+\mathbf{j}_2=\mathbf{j}_1$. The $H_{\rm int}$ in this basis [Eq. (6)] is

$$\begin{aligned} &\langle n_\lambda=1;\lambda j_2,j_1|H_{\rm int}|n_\lambda=0;0j_1,j_1\rangle \\ &\quad=\langle j_2|K(r)|j_1\rangle\langle j_2\|Y_\lambda\|j_1\rangle(2j_1+1)^{-1/2}\langle\alpha_\lambda\rangle , \end{aligned} \tag{7}$$

where the amplitudes of the λ-mode vibrational states $\langle\alpha_\lambda\rangle$ (Refs. 10 and 11) and $\langle H_{\rm vib}\rangle$ are listed in Table II.

IV. RESULTS AND DISCUSSIONS

The results of the diagonalization of the Hamiltonian matrix (H) for the $\frac{7}{2}^-$ state have been depicted in Table III. The amplitudes of the collective configuration of the weak fragmented states that bear more than 50% of the unity have been shown in Table III. In order to show the sharp contrasting situation of the $2f_{7/2}$ state with that of the $3p_{1/2}$, $3p_{3/2}$, and $2f_{5/2}$ states, we have also diagonalized H [Eq. (2)] for the $\frac{1}{2}^-$, $\frac{3}{2}^-$ and $\frac{5}{2}^-$ states. The optimized theoretical results supporting the available experimental values[6] that involve χ^2 minimization, have been shown in Table IV.

The most interesting result of our calculation is that the single hole strength of the main fragment of the $3p_{1/2}$, $3p_{3/2}$, and $2f_{5/2}$ states bear more than 80% of the shell-model sum rule strength. However, a sizeable 40% of the total neutron hole strength is lost from the main fragment of the $2f_{7/2}$ state of ^{207}Pb (2.34 MeV) and is dis-

TABLE II. The vibrational states of ^{208}Pb (Refs. 10 and 11). The dash over λ^π indicates states arising from the giant resonances.

λ^π	2_1^+	$2^{+'}$	4^+	6^+	8^+
Energy (MeV)	4.08	10.50	4.32	4.42	4.61
$\langle\alpha_\lambda\rangle$	0.025	0.037	0.024	0.015	0.010
λ^π	3_1^-	$3^{-'}$	5_1^-	5_2^-	
Energy (MeV)	2.61	17.50	3.20	3.71	
$\langle\alpha_\lambda\rangle$	0.046	0.011	0.017	0.010	
λ^π	7^-	$1^{-'}$			
Energy (MeV)	4.04	13.60			
$\langle\alpha_\lambda\rangle$	0.100	0.010			

TABLE III. Fragmentation of the single hole strengths of the $2f_{7/2}$ neutron state of ^{207}Pb. E is the energy in MeV, S is the spectroscopic factor, and $a_{\lambda j_2}(\lambda j_2)$ is the collective configuration. Numbers in parentheses indicate experimental value (Ref. 6).

E	S	$a_{\lambda j_2}(\lambda j_2)$
2.128 (2.340)	0.627 (0.500)	
4.598	0.044	-0.873 $(3_1^-, i_{13/2})$
4.981	0.014	0.673 $(5_1^-, i_{13/2})$
6.145	0.006	-0.924 $(2_1^+, p_{3/2})$
6.384	0.013	0.910 $(4^+, p_{3/2})$
6.635	0.055	0.693 $(2_1^+, f_{5/2})$
6.724	0.055	0.712 $(2_1^+, f_{5/2})$
6.950	0.010	0.864 $(4^+, f_{5/2})$
7.013	0.059	-0.825 $(2_1^+, p_{3/2})$
8.712	0.006	0.969 $(2_1^+, f_{7/2})$
8.946	0.005	-0.971 $(4^+, f_{7/2})$
10.721	0.005	-0.979 $(3_1^-, s_{1/2})$
12.645	0.005	0.975 $(2^{+'}, p_{3/2})$
15.487	0.036	0.878 $(7^-, g_{7/2})$
16.065	0.012	-0.951 $(3_1^-, g_{9/2})$
17.563	0.014	0.997 $(5_1^-, g_{9/2})$
	$\sum ES/\sum S = 4.173$ MeV	

tributed among various fragmented $\frac{7}{2}^-$ states extending up to 17.563 MeV excitation energy (Table III). The excited state at 12.645 MeV exclusively arises from the $2^{+'}$ giant resonance oriented vibrational state. As a result of this core polarization effect, the $2f_{7/2}$ hole state gets deeply bound and is shifted to -4.173 MeV from the ground state. The basic reason for the spreading of the $2f_{7/2}$ neutron strength can be physically explained as follows.

From Table I we observe that the unperturbed shell-model energy of the $2f_{7/2}$ state lies in the proximity of the 2_1^+ (4.08 MeV, Table II) excited vibrational state of ^{208}Pb. As a result, the probability of mixing of this particular shell-model state with the collective vibrational states of ^{208}Pb is large in comparison to the mixing of the $3p_{1/2}$, $3p_{3/2}$, and $2f_{5/2}$ discrete low-lying states with the same. Chiefly because of this aspect, the 2.34 Mev state of the $\frac{7}{2}^-$ configuration loses its basic shell-model identity and the quenching of the shell-model hole strength of this state is much pronounced. An exactly similar situation arises in the case of the $3s_{1/2}$ hole occupancy of the ^{205}Tl and ^{207}Tl (Ref. 1) nuclei. The dilution of the first excited $3s_{1/2}$ state ^{205}Tl is more pronounced than the same $3s_{1/2}$ state of ^{207}Tl. The lowest vibrational state of ^{206}Pb lies at a much lower energy level in comparison to that of ^{208}Pb. This allows the complex configuration mixing of the $3s_{1/2}$ state with the vibrational states of ^{206}Pb. As a consequence, the $3s_{1/2}$ state of ^{205}Tl also loses its basis shell-

TABLE IV. Fragmentations of the single hole strengths of the $3p_{1/2}$, $3p_{3/2}$, and $2f_{5/2}$ neutron states of ^{207}Pb. E is the energy in Mev; S is the spectroscopic factor. Numbers in parentheses indicate experimental value (Ref. 6).

$3p_{1/2}$ state		$3p_{3/2}$ state		$2f_{5/2}$ state	
E	S	E	S	E	S
0.000 (0.000)	0.945 (0.870)	0.633 (0.900)	0.805 (0.925)	0.850 (0.570)	0.815 (0.880)
6.213	0.004	6.508	0.071	6.158	0.010
6.733	0.007	6.706	0.011	6.252	0.035
8.343	0.007	7.047	0.043	6.506	0.025
8.974	0.006	8.412	0.005	6.737	0.018
12.438	0.004	8.733	0.008	6.957	0.013
12.651	0.004	8.954	0.007	8.085	0.006
13.159	0.004	10.947	0.006	8.684	0.004
17.719	0.016	11.141	0.006	10.710	0.004
		12.416	0.005	10.915	0.005
		12.637	0.005	13.593	0.004
		15.161	0.005	14.140	0.004
				17.077	0.035

model identity.

It has been shown[12] theoretically that the large depletion of the $3s_{1/2}$ neutron hole strength, particularly in ^{205}Tl, cannot be accounted for on the basis of random-phase approximation type correlations among the various shell-model states lying below the Fermi surface. This small depletion of the spectroscopic factor of the normally occupied shell-model states, like the $2f_{7/2}$ state of ^{207}Pb and the $3s_{1/2}$ state of ^{205}Tl, may be due to long-range, short-range, and tensor correlations of the shell-model orbitals. Therefore, in conclusion, we can advocate that the present calculation succeeds in two ways. Firstly, the core-polarization effect can explain the large depletion of the neutron hole strength of the occupied $2f_{7/2}$ state of ^{207}Pb which is as large as 50% of the shell-model sum rule strength. To the best of our knowledge, it can be predicted that no explicit theoretical calculation exists in the literature to support this large amount of depletion of the normally occupied shell-model orbital within the -3 MeV excitation region from the Fermi level of ^{208}Pb. Secondly, we are in a position to show the explicit fragmentation of the $2f_{7/2}$ state of ^{207}Pb, whereas a recent experiment[6] predicts only the single main fragment of the $2f_{7/2}$ state at 2.340 MeV excitation energy.

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