Centroids of effective interactions from measured single-particle energies: An application

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Centroids of the effective nucleon-nucleon interaction for the mass region $A=28-64$ are extracted directly from experimental single-particle spectra, by comparing single-particle energies relative to different cores. Uncertainties in the centroids are estimated at approximately 100 keV, except in cases of exceptional fragmentation of the single-particle strength. The use of a large number of inert cores allows the dependence of the interaction on mass or model space to be investigated. The method permits accurate empirical modifications to be made to realistic interactions calculated from bare nucleon-nucleon potentials, which are known to possess defective centroids in many cases. In addition, the centroids can be used as input to the more sophisticated fitting procedures that are employed to produce matrix elements of the effective interaction.

I. INTRODUCTION

We have recently presented a simple technique for extracting useful information about effective interactions from experimental single-particle spectra.[1] Interaction centroids are determined from single-particle spectra by comparing single-particle energies relative to different inert cores. In this paper we present a significant extension of the formalism of Ref. 1, as well as a full-scale application of the technique to the mass range $A=28-64$. The aim is both to illustrate the technique and at the same time to provide reliable information about effective interactions near the top of the *sd* shell and bottom of the *fp* shell, thereby supplementing information available from other sources.

Values for individual two-body matrix elements of the interaction can, of course, be deduced directly from the spectra of nuclei with two nucleons relative to an inert core. For a recent example and full discussion of the difficulties inherent in such methods, see the review of Daehnick.[2] The present approach is in many cases complementary to that of Daehnick (since different data are involved—single-particle spectra rather than two-particle spectra). But, in addition, the technique allows information to be extracted from situations in which the method of Ref. 2 cannot be used, either due to a lack of necessary spectroscopic data or because of excessive fragmentation of the two-particle states through configuration mixing within the model space.

The reasons for using a purely empirical approach, such as those of Refs. 1 and 2, or even a full fit to experimental energy levels, are self-evident. It has proven impossible to construct an effective Hamiltonian which is both sufficiently accurate for use in spectroscopic calculations and also firmly based theoretically. The deficiencies of realistic interactions such as those produced by Kuo and Brown[3,4] are well known: when used in shell-model calculations for nuclei towards the middle of the *sd* shell, the Kuo interaction[3] shifts whole bands of levels relative to each other;[5-7] similarly the Kuo-Brown interaction for *fp*-shell nuclei[4] produces spectra in which levels involving nucleons not in the $f_{7/2}$ orbit appear too low in excitation relative to levels involving only $f_{7/2}$ nucleons.[8,9] These defects can be traced to incorrect centroids for the interaction between nucleons in different single-particle orbits and the consequences are particularly acute when the model space spans two major shells. The agreement between calculated spectra and data can be substantially improved by an empirical modification of these centroids. One result of the present work is that such adjustments can be made more reliably.

An alternative approach to effective interactions is favored by Wildenthal and co-workers:[10,11] matrix elements of the effective interaction are regarded as parameters to be adjusted in a fit to experimental energy levels. This approach has met with considerable success in the *sd* shell, where it is possible to perform shell-model calculations in which all Pauli-allowed *sd*-shell basis states are retained. However, it is difficult to extend this method to heavier nuclei. One of the problems is that the fitting procedure, which relies on an iterative technique, is both difficult and time consuming in large model spaces. It is therefore advantageous to use as input an interaction with accurate centroids, since this will greatly reduce the number of iterations required to produce convergence and should also help prevent convergence to a spurious solution. The technique illustrated here can thus be regarded as providing accurate starting values for large-scale fits to energy levels.

II. SUMMARY OF TECHNIQUE

The equations used to compute centroids from single-particle energies can be derived in several ways;[1,12] their structure is in any case intuitively obvious, so no further details of their derivation will be offered here.

Let A_0 represent a closed-shell nucleus with $N=Z$, and A_0' a second inert core which comprises A_0 and the

orbit p completely filled with $N_p=2j_p+1$ neutrons and an equal number of protons, i.e., $A_0'=A_0+2N_p$. Relative to the core A_0', the energy of a single neutron in any orbital q is

$$\epsilon_q^\nu(A_0')=\epsilon_q^\nu(A_0)+(2N_p-\delta_{pq})\bar{V}(pq)\ , \tag{1}$$

where $\epsilon_q^\nu(A_0)$ is the energy of the same neutron with respect to the core A_0. If q refers to an orbital below the Fermi energy of the core A_0', then $\epsilon_q(A_0')$ is to be interpreted as a suitably defined single-hole energy. The $\bar{V}(pq)$ are interaction centroids defined by

$$\bar{V}(pq)=\frac{\sum_{JT}(2J+1)(2T+1)V^{JT}(pq)}{\sum_{JT}(2J+1)(2T+1)}\ , \tag{2}$$

where $V^{JT}(ij)=\langle ij|V|ij;JT\rangle$ is a diagonal matrix element of the effective interaction for spin J and isospin T.

If we also consider cores with neutron or proton excesses, then further centroids can be extracted. These are defined by equations similar to Eq. (2):

$$\bar{V}^{\,T}(pq)=\frac{\sum_{J}(2J+1)V^{JT}(pq)}{\sum_{J}(2J+1)} \tag{3}$$

and

$$\bar{V}^{\,np}(pq)=\frac{\sum_{J}(2J+1)V^{J}(pq)}{\sum_{J}(2J+1)}\ , \tag{4}$$

the sums over J include matrix elements for fixed T and all T for the isospin and neutron-proton centroids, respectively. Any two of the centroids can be regarded as independent; the remainder are given by the following relationships, which can be derived by explicitly evaluating the sum in Eqs. (2)–(4):

$$\begin{aligned}\bar{V}(pq)&=[\bar{V}^{\,np}(pq)+\bar{V}^{\,T=1}(pq)]/2\\&=[\bar{V}^{\,T=0}(pq)+3\bar{V}^{\,T=1}(pq)]/4\end{aligned} \tag{5}$$

for $p\neq q$, and

$$\begin{aligned}(4j+1)\bar{V}(pp)&=(2j+1)\bar{V}^{\,np}(pp)+2j\bar{V}^{\,T=1}(pp)\\&=(j+1)\bar{V}^{\,T=0}(pp)+3j\bar{V}^{\,T=1}(pp)\end{aligned} \tag{6}$$

for $p=q$, with $j=j_p$.

Consider the case in which the cores A_0 and A_0' differ through N_p neutrons occupying the orbit p, so that $A_0'=A_0+N_p$ (neutrons), with $N\neq Z$ for one of the cores and $N=Z$ for the other. Relative to these two cores, a neutron in the orbital q has energies related by

$$\epsilon_q^\nu(A_0')=\epsilon_q^\nu(A_0)+(N_p-\delta_{pq})\bar{V}^{\,T=1}(pq)\ , \tag{7}$$

whereas a proton in orbit q has energies given by

$$\epsilon_q^\pi(A_0')=\epsilon_q^\pi(A_0)+N_p\bar{V}^{\,np}(pq)\ . \tag{8}$$

If, on the other hand, the excess nucleons occupying orbital p are all protons, so that $A_0'=A_0+N_p$ (protons), the roles of protons and neutrons are reversed, and Eqs. (7) and (8) are replaced by

$$\epsilon_q^\pi(A_0')=\epsilon_q^\pi(A_0)+(N_p-\delta_{pq})\bar{V}^{\,T=1}(pq) \tag{9}$$

and

$$\epsilon_q^\nu(A_0')=\epsilon_q^\nu(A_0)+N_p\bar{V}^{\,np}(pq)\ . \tag{10}$$

The basis of the proposed technique is that the energies $\epsilon_q(A_c)$ for the core A_c should be deduced from experimentally determined binding energies $B(A,J^\pi)$ using

$$\epsilon_q(A_c)=\pm[B(A_c\pm1,J^\pi=j_q^\pi)-B(A_c,0^+)] \tag{11}$$

for a single particle and a single hole, respectively. Interaction centroids may then be extracted using Eqs. (7)–(10). We prefer to use in Eq. (1) the binding energy of the state of the nucleus $(A_c\pm1)$ with largest single-particle strength, rather than the center of gravity of the single-particle strength; this point is further discussed in Sec. IV.

There are, however, a number of complications in the application of Eq. (11). One problem is the effect of the Coulomb force on the value of $\epsilon_q^\pi(A_c)$ deduced from Eq. (11). Since the intention is to extract centroids of the *nuclear* interaction, the effects of the Coulomb interaction must be eliminated. The method we have used to do this is discussed in the next section.

There is a further difficulty when the core has $N\neq Z$. Assuming that the excess nucleons are neutrons in orbit p, then single-particle states can be constructed from the core by adding a *neutron* to any orbit q above the *neutron* Fermi level or by removing a *neutron* from the orbital p. The single-particle energy of such a state can then be used directly in Eqs. (7) and (10). Further single-particle states are formed by removing a *proton* from below the *proton* Fermi level of the core or placing a *proton* in the orbital p; the single-particle energy of these states can be inserted directly into Eqs. (8) and (9). Other single-particle states can be constructed by adding *protons* above the *neutron* Fermi level or removing *neutrons* from below the *proton* Fermi level. In general, however, this process does not produce a state of unique isospin; the single-particle state is, in fact, a linear combination of a state of isospin $T_{\rm core}+\frac{1}{2}$ (the analog of a single-particle level of the neighboring nucleus) and a state of isospin $T_{\rm core}-\frac{1}{2}$ (the corresponding antianalog state). The energies of the two physical states, suitably combined with the appropriate isospin coupling coefficients, yield an estimate for the single-particle energy $\epsilon_q(A_c)$ which may then be used in Eqs. (7)–(10).

III. APPLICATION TO NUCLEI WITH $A=28-64$

As an extended application of the proposed technique, we discuss single-particle states in the mass range $A=28-64$, with the orbitals $2s_{1/2}$, $1d_{3/2}$, $1f_{7/2}$, and $2p_{3/2}$ considered active. The ground states of ^{28}Si, ^{32}S, ^{40}Ca, ^{56}Ni, and ^{64}Ge are therefore assumed to comprise closed shells; in addition, the ground states of ^{30}Si, ^{36}S, ^{48}Ca, and ^{60}Ni are regarded as neutron-rich inert cores. Furthermore, ^{34}Si is also taken to be good closed-shell nu-

cleus, with excess neutrons in both the $s_{1/2}$ and $d_{3/2}$ orbits; the formalism of the previous section, when suitably modified, also applies in this case. The possibility of the nucleus ^{34}Si being a doubly magic nucleus was the subject of a recent experimental investigation.[13] Its use as a core permits a significant extension to the range of values of (Z,N) that can be probed.

The single-particle and -hole energies computed using Eq. (11) and measured binding energies[14–17] are listed in Table I. The numbers in parentheses are uncertainties in the single-particle energies due to uncertainties in measured ground-state binding energies; they are shown only where they exceed 50 keV. In cases where no spectroscopic information is available as a guide, assumptions have been made as to which observed excited states correspond to single-particle excitations. In addition, there still exists an unfortunate lack of reliable data for nuclei around ^{34}Si. For some excited states in neighboring nuclei we have assumed spin-parity assignments which are, at best, tentative; some single-particle energies for $A_c=34$ must therefore be used with appropriate caution. The ground states of ^{30}S and ^{36}Ca are isobaric analogs of the ^{30}Si and ^{36}S ground states, respectively, and can therefore be used as proton-rich inert cores. The relevant single-particle energies are given in Table II. Analysis using these energies provides a second estimate of centroids found using the neutron-rich cores, and in a few instances provides additional information.

As explained above, use of Eqs. (8) and (9) requires the separation of the nuclear and Coulomb contributions to $\epsilon_q^\pi(A_c)$. In applications of Eq. (8) the cores A_0 and A_0' have the same Z, as do the two single-particle nuclei, so that their Coulomb energies cancel and no further correction is necessary. This assumes that the Coulomb energy is independent of mass, which is only approximately true; the same assumption is in any case made when extracting ϵ_q^ν. In the application of Eq. (9), however, the Coulomb energies do not cancel. For the case $N=Z$, removal of the Coulomb energy $E_C^q(A_c)$ from the experimentally determined values of $\epsilon_q^\pi(A_c)$ should produce the corresponding neutron energy $\epsilon_q^\nu(A_c)$. In other cases, with $N\neq Z$, we have assumed that the Coulomb energy is equal to $E_C^q(A_c)$ for the $N=Z$ core with the same Z. Again, this assumes mass independence of the Coulomb energy. A check on the results is provided by the fact that, if the Coulomb correction is calculated with sufficient accuracy, the value of ϵ_q^π for a neutron-rich core should be equal to the neutron energy ϵ_q^ν for the corresponding proton-rich core; an average difference of 100 keV is found for the four cases for which such a comparison may be made.

TABLE I. Values (in MeV) of single-particle energies $\epsilon_q^\nu(A_c)$ (upper) and $\epsilon_q^\pi(A_c)$ (lower) extracted using Eq. (11) and experimental binding energies from Refs. 14–17.

A_c	$s_{1/2}$	$d_{3/2}$	$f_{7/2}$	$p_{3/2}$
28	−8.47	−7.20	−4.85	−3.54
	−2.75	−1.36	0.70	1.60
30	−10.61	−6.59	−3.46	−3.05
	−7.30	−4.33[a]	−1.20[a]	−0.71(5)[a,b]
32	−15.04	−8.64	−5.71	−5.42
	−8.87	−2.28	0.41	0.57
34		−7.54	−2.43(5)	
	−12.19	−9.80		
36	−12.99[a,b]	−9.89	−4.30	−3.66
	−13.10	−8.39	−3.86[a]	
40	−18.11	−15.64	−8.36	−6.42
	−10.85	−8.33	−1.09	0.63
48	−13.67[a]	−13.69[a]	−9.95	−5.15
	−15.81	−16.17	−9.63	−5.60[a]
56	−19.83	−20.40	−16.43	−10.25
	−10.09	−10.73	−7.17	−0.74
60			−14.96[a]	−11.39
	−12.25	−12.70	−9.53	−4.80
64				−15.03(40)[c]
				−5.02(27)

[a]Averaged over states of different isospin (see text).
[b]Generated from the energy of an analog state and computed Coulomb energies (see text).
[c]Binding energy of ^{63}Ge from systematics (Refs. 14 and 15).

TABLE II. Values (in MeV) of single-particle energies $\epsilon_q^\nu(A_c)$ (upper) and $\epsilon_q^\pi(A_c)$ (lower) for proton-rich cores. Energies are extracted using Eq. (11) and experimental binding energies from Refs. 14–17.

A_c	$s_{1/2}$	$d_{3/2}$
30	−13.05	−10.13[a]
	−4.40	−0.28(5)
36	−18.96(7)	−14.79(5)
		−2.56

[a]Averaged over states of different isospin (see text).

Interaction centroids extracted using Eqs. (7)–(10) and the single-particle energies of Tables I and II are presented in Tables III and IV. Centroids for different pairs (A_0, A_0') are shown separately in order to illustrate any dependence on mass or model space. Where the same information is available from both neutron- and proton-rich cores, the two values agree extremely well and averages are tabulated. Two values listed in Table I have been computed from the binding energies of isobaric analog states, with Coulomb energies estimated as in Ref. 18, since the necessary information about the parent states is not available. Except in one instance, the value of $\bar{V}(pq)$ has been computed from the corresponding centroids $\bar{V}^{np}$ and $\bar{V}^{T=1}$ together with Eqs. (5) or (6), rather than extracted directly using Eq. (1). It is easily shown that the value of $\bar{V}(pq)$ from Eq. (1) with cores (A_1, A_3), where A_1 and A_3 are adjacent cores with $N=Z$, should be identical to $\bar{V}(pq)$ calculated using $\bar{V}^{T=1}$ with cores (A_1, A_2) and $\bar{V}^{np}$ with cores (A_2, A_3), where A_2 is the intervening neutron-rich core; this is confirmed by calculation.

It has previously been established[19,20] that particle-hole states of isospin $T_{\text{core}}-\frac{1}{2}$ (with $T_{\text{core}}\neq 0$) are not well described by models such as the weak-coupling model of Bansal and French.[21] One might therefore expect that

TABLE III. Centroids of the interactions $V(pq)$ with $p=q$ (in MeV).

A_0, A_0'	$\bar{V}(ss)$[a]	$\bar{V}^{np}(ss)$	$\bar{V}^{T=1}(ss)$
	$s_{1/2}$-$s_{1/2}$ interaction		
28,30	−2.23	−2.28	−2.14
30,32	−2.16	−2.23	−2.02
34,36			−1.36
	$d_{3/2}$-$d_{3/2}$ interaction		
A_0, A_0'	$\bar{V}(dd)$[a]	$\bar{V}^{np}(dd)$	$\bar{V}^{T=1}(dd)$
30,34	−0.92	−1.37	−0.32
32,36	−1.05	−1.53	−0.42
36,40	−0.95	−1.44	−0.30
	$f_{7/2}$-$f_{7/2}$ interaction		
A_0, A_0'	$\bar{V}(ff)$[a]	$\bar{V}^{np}(ff)$	$\bar{V}^{T=1}(ff)$
40,48	−0.68	−1.07	−0.23
48,56	−0.43	−0.84	0.03
	$p_{3/2}$-$p_{3/2}$ interaction		
A_0, A_0'	$\bar{V}(pp)$[a]	$\bar{V}^{np}(pp)$	$\bar{V}^{T=1}(pp)$
56,60	−0.74	−1.02	−0.37
60,64	−0.62(6)	−0.91	−0.24(13)

[a]Calculated from $\bar{V}^{np}(pq)$ and $\bar{V}^{T=1}(pq)$ using Eqs. (5) or (6).

centroids derived from the energies of levels of mixed isospin would be less reliable than centroids from states of isospin $T_{\text{core}}+\frac{1}{2}$ only; there is, however, no evidence of this in Tables III and IV.

IV. DISCUSSION OF THE RESULTS

The accuracy of the interaction centroids presented in Tables III and IV (and also their interpretation) depends on the extent to which the ground states of the cores approximate closed-shell configurations. The assumption of shell closure, as indicated by the excitation energy of the first excited state of the core nucleus, is apparently reasonable in cases such as ^{40,48}Ca, ^{34}Si, and ^{36}S where this energy exceeds 3 MeV, but certainly questionable for ^{28}Si, ^{60}Ni, and ^{64}Ge, where the energy is less than 2 MeV. A further indicator is the magnitude of the energy gap at the Fermi surface, which can be ascertained from Table I. This suggests that the $N=Z$ nuclei ^{32}S, ^{40}Ca, and ^{56}Ni form the best inert cores, and that the assumption of proton shell closure is also valid for ^{48}Ca.

The effect of core breaking is also evident from the degree of fragmentation of the single-particle states. In determining the single-particle energies of Table I, we have used the binding energy of the state of the nucleus $(A_c\pm1)$ with largest spectroscopic strength, rather than the center of gravity of the single-particle strength computed from measured spectroscopic factors. The latter energy might be considered more appropriate in some circumstances, but frequently it cannot be determined since the relevant transfer reactions are not possible. The center of gravity typically lies about 0.3 MeV above the energy used,[2] but since the centroids are determined by *differences* in single-particle energies this systematic error should, in most cases, cancel on average. Moreover, since the energy difference is ultimately divided by factors such as N_p and $2N_p$, the uncertainty in the centroids due to fragmentation is further reduced.

Nevertheless, core breaking leads occasionally to significantly larger uncertainties. In order to ascertain the extent to which fragmentation affects the interaction centroids, we have also calculated single-particle energies using the center of gravity of the single-particle strength in Eq. (11), whenever sufficient data exist. The results are unfortunately sensitive to states which carry only a few percent of the strength but lie several MeV above the state of largest strength. Since these states may be popu-

TABLE IV. Centroids of the interactions $V(pq)$ with $p\neq q$ (in MeV).

A_0, A_0'	$\bar{V}(sd)$[a]	$\bar{V}^{np}(sd)$	$\bar{V}^{T=1}(sd)$
	$s_{1/2}$-$d_{3/2}$ interaction		
28,30	−0.59	−1.47	0.31
30,32	−0.13	−1.01	0.74
34,36	−0.37	−1.18	0.44
30,34		−1.22	
32,36	−0.25	−1.01	0.51
36,40	−0.50	−1.28	0.29
	$d_{3/2}$-$f_{7/2}$ interaction		
A_0, A_0'	$\bar{V}(df)$[a]	$\bar{V}^{np}(df)$	$\bar{V}^{T=1}(df)$
30,34			0.26
32,36	−0.36	−1.07	0.35
36,40	−0.31	−1.02	0.40
40,48	−0.37	−0.98	0.24
48,56	−0.23	−0.84	0.39
	$f_{7/2}$-$p_{3/2}$ interaction		
A_0, A_0'	$\bar{V}(fp)$[a]	$\bar{V}^{np}(fp)$	$\bar{V}^{T=1}(fp)$
40,48	−0.31	−0.78	0.16
48,56	−0.17	−0.64	0.30
56,60	−0.09	−0.59	0.42
	$s_{1/2}$-$f_{7/2}$ interaction		
A_0, A_0'	$\bar{V}(sf)$[a]	$\bar{V}^{np}(sf)$	$\bar{V}^{T=1}(sf)$
28,30	−0.13	−0.95	0.70
30,32	−0.31	−1.13	0.52
34,36		−0.94	
40,48	−0.03	−0.62	0.56
48,56	−0.19	−0.77	0.40
	$d_{3/2}$-$p_{3/2}$ interaction		
A_0, A_0'	$\bar{V}(dp)$[b]	$\bar{V}^{np}(dp)$	$\bar{V}^{T=1}(dp)$
32,36			0.44
32,40	−0.13		
36,40		−0.69	
56,60		−0.49	
	$s_{1/2}$-$p_{3/2}$ interaction		
A_0, A_0'	$\bar{V}(sp)$[a]	$\bar{V}^{np}(sp)$	$\bar{V}^{T=1}(sp)$
28,30	−0.45	−1.15	0.24
30,32	−0.49	−1.18	0.21
56,60		−0.54	

[a]Calculated from $\bar{V}^{np}(pq)$ and $\bar{V}^{T=1}(pq)$ using Eqs. (5) or (6).
[b]Extracted directly using Eq. (1).

lated in nucleon transfer reactions through two-step processes rather than direct transfer, a DWBA analysis is inappropriate and the resulting spectroscopic factors are meaningless. The contribution of such states to the single-particle energy, which is often a few hundred keV, will, in these circumstances, be entirely spurious. This further analysis reveals that the single-particle strength is highly fragmented over several states in one case only. The $s_{1/2}$ hole energy relative to ^{32}S is shifted by approximately 1.7 MeV by this fragmentation and, as a result, several estimates of the $s_{1/2}$-$s_{1/2}$ centroids are altered by a considerable amount. Otherwise, fragmentation shifts the single-particle strength by up to 0.8 MeV, with the center of gravity being on average 280 keV above the level of largest spectroscopic factor. The resulting change in the interaction centroids is occasionally as large as 0.8 MeV, although the average change is only 120 keV. The lack of sensitivity to fragmentation is thus confirmed for the majority of centroids.

Interaction centroids with $p=q$ (displayed in Table III) are relatively insensitive to the choice of cores. One exception is $\bar{V}^{T=1}$ (ss); although there exists some uncertainty about single-particle energies for $A_c=34$, differences apparent in Table III are probably significant. The model-space dependence of $\bar{V}^{T=1}$ (ff) has already been noted,[22] although in a different context. The values of $\bar{V}^{np}(pq)$ for $p\neq q$, shown in Table IV, also display little dependence on (A_0, A_0'), apart from a general reduction in magnitude with increasing mass. This is particularly noticeable for the $s_{1/2}$-$p_{3/2}$ interaction, where the spread in values of (A_0, A_0') is largest. There is substantially more sensitivity in the results obtained for $\bar{V}^{T=1}(pq)$, where variations of up to 0.5 MeV are found; nonetheless, the average variation is considerably less than this, and does not appear to be correlated with mass.

For comparison, centroids of the realistic interactions of Kuo and Brown[3,4] are presented in Table V. Different centroids are appropriate to different model spaces, and should therefore be compared with values from Tables III and IV for an appropriate combination of cores. The purely *sd*-shell interaction of Kuo and Brown is calculated for an ^{16}O core, whereas *fp*-shell and cross-shell matrix elements include core-polarization corrections for a ^{40}Ca core and are intended for nuclei above $A=40$. Comparison reveals no strong trends, although the Kuo-Brown centroids $\bar{V}^{np}$ tend to be less attractive by about 200 keV, whereas the centroids $\bar{V}^{T=1}$ are on average 200 keV more attractive. There are, however, some differences which are substantially larger, illustrating the previously described defect of these realistic interactions.

The accuracy (and consequent utility) of the present technique is best illustrated by comparing with effective interactions from large-scale fits to energy levels. The centroids displayed in Table VI are restricted to fits in which matrix elements were allowed to vary without excessive constraint and in which the fitted data spanned a reasonable range of nuclei. The *sd* interaction of Wildenthal[11] should be compared with centroids from the present work averaged over $A=28-40$. The fitted interaction is mass dependent; we have computed centroids appropriate to $A=34$. Comparison with values from Tables III and IV shows excellent agreement, although $\bar{V}^{T=1}(sd)$ from the present work is consistently more repulsive by about 400 keV. There has been no comparable investigation of the *fp*-shell interaction. Centroids listed in Table VI were compiled from several sources in which fits were performed in smaller subspaces or for a limited range of nuclei in the full space. Values have been averaged where this is meaningful. Comparison with the present results for $(A_0, A_0')=(40,48)$ seems most appropriate, except for centroids from Refs. 29 and 28 which should be compared to (56,64) and perhaps an average for $A=40-56$, respectively. The agreement is once again impressive; the spread of fitted values of $\bar{V}^{T=1}(pp)$ highlights the sensitivity of the $T=1$ centroids to the model space (or simply to the choice of data to be fitted). Information for interaction matrix elements connecting different major shells is even more limited. Values taken from Ref. 27 should be compared to present

TABLE V. Centroids (in MeV) of the interactions of Kuo and Brown (Refs. 3 and 4).

pq	$\bar{V}(pq)$	$\bar{V}^{np}(pq)$	$\bar{V}^{T=1}(pq)$
ss	−2.57	−2.88	−1.95
sd	−0.65	−1.18	−0.12
dd	−0.74	−1.25	−0.07
ff	−0.44	−0.71	−0.13
fp	−0.32	−0.54	−0.10
pp	−0.86	−1.12	−0.52
df	−0.42	−0.79	−0.05
sf	−0.19	−0.53	0.14

TABLE VI. Centroids of fitted interactions (in MeV). The interaction matrix elements are from the references listed in the footnotes.

pq	$\bar{V}(pq)$	$\bar{V}^{np}(pq)$	$\bar{V}^{T=1}(pq)$
ss	−2.24[a]	−2.47[a]	−1.77[a]
sd	−0.59[a]	−1.15[a]	−0.03[a]
dd	−0.95[a]	−1.40[a]	−0.35[a]
ff	−0.69[b]; −0.56[c]	−1.06[b]; −0.84[c]	−0.21[d]; −0.24[c]
fp	−0.31[c]	−0.71[c]	−0.13[e]; +0.10[c]
pp	−0.45[f]	−0.93[f]	−0.23[g]; −0.71[h]; +0.19[f]
df	−0.28[k]	−1.01[k]; −0.98[l]	+0.45[k]
dp		−0.97[i]; −0.64[j]	
sf		−0.75[i]	

[a]Reference 11.
[b]Reference 23.
[c]Reference 28.
[d]References 8 and 23–26.
[e]References 8 and 24–26.
[f]Reference 29.
[g]Reference 24.
[h]Reference 26.
[i]Reference 25.
[j]Reference 26.
[k]Reference 27.
[l]References 25 and 26.

results averaged over $A=32-40$; for the remaining centroids, use of $(A_0, A_0')=(40,48)$ is more appropriate. Where comparison is possible, excellent agreement is found.

V. CONCLUSION

We have shown how average values for two-body matrix elements of the effective interaction can be obtained empirically from single-particle spectra: centroids of the interaction are deduced by comparing single-particle energies with respect to different inert cores. In the mass range under investigation there is a large number of assumed inert cores; this often allows a given centroid to be extracted using more than one combination of cores, so that the model-space dependence of the effective interaction can be studied. The interaction apparently becomes less attractive on average with increasing mass, although the $T=1$ centroid is evidently more sensitive to the effects of core polarization.

The centroids are estimated to be accurate to about 100 keV, in general, although the uncertainty may exceed this by a significant amount in special circumstances. One particular difficulty, whose effect cannot be quantified, is the incorrect identification of the single-particle states due to a lack of spectroscopic information. Two further sources of error are inaccuracies in the extraction of the Coulomb contribution to nuclear binding energies and exceptional fragmentation of the single-particle strength. Fragmentation is unfortunately difficult to avoid. Use of centers of gravity calculated with measured spectroscopic factors, as in the previous section, is, in fact, inconsistent. We should also use the center of gravity of the closed-shell configuration in place of the ground state of the core and, moreover, include single-particle strength built on excited states of the core nucleus; these two effects will tend to cancel. Comparison of the present results with centroids of fitted interactions shows that surprising accuracy is achieved without the need to correct for fragmentation of the single-particle strength—in fact, the overall agreement is better if fragmentation is ignored.

The interaction centroids, which are responsible for the gross properties of nuclei, are an important measure of the effective interaction. Realistic interactions, which are calculated from bare nucleon-nucleon potentials, are known to suffer from inaccurate centroids in many cases. Nonetheless, one can greatly enhance the predictive power of realistic interactions by performing *ad hoc* adjustments to the defective centroids. One aim of the present work is to allow such adjustments to be made reliably, by providing, in a straightforward manner, empirical estimates for the centroids. The success which can be achieved is demonstrated by the shell-model calculations of Refs. 8 and 9.

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