Effects of short range ΔN interaction on observables of the πNN system

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The inadequacy of standard few-body approaches in describing the πNN system has motivated searches for the responsible missing mechanism. In the case of πd scattering, it has recently been asserted that an additional short range ΔN interaction can account for essentially all the discrepancies between a few-body calculation and experimental data. This conclusion, however, has been based on calculations where a phenomenological ΔN interaction is added only in Born term to background few-body amplitudes. In the present work we investigate the effect of including such a ΔN interaction to all orders within a unitary few-body calculation of the πNN system. Besides testing the validity of adding the ΔN interaction in Born term in πd scattering, our fully coupled approach also enables us to see the influence of the same ΔN interaction on the processes $NN \to \pi d$ and $NN \to NN$. For πd elastic scattering, we find that the higher order ΔN interaction terms can have as much influence on πd observables as the lowest order contribution alone. Moreover, we find that the higher order contributions tend to cancel the effect obtained by adding the ΔN interaction in Born term only. The effect of the same ΔN interaction on $NN \to \pi d$ and $NN \to NN$ appears to be as significant as in $\pi d \to \pi d$, suggesting that future investigations of the short range ΔN interaction should be done in the context of the fully coupled πNN system.

I. INTRODUCTION

The study of the πNN system is of fundamental importance in intermediate energy nuclear physics. It provides the opportunity of studying many aspects of pion-nucleus interactions in a few-body system, which is amenable to exact solution. In the last ten years, there have been many attempts at describing the πNN system (for recent reviews see Refs. 1 and 2). Perhaps the most promising of these are models that describe the πNN system by the inclusion of pion absorption into an essentially three-body Faddeev description.[3–11] These models respect two- and three-body unitarity and describe, with the one set of coupled equations, the processes $\pi d \to \pi d$, $NN \to \pi d$, and $NN \to NN$. Despite the substantial amount of physics included in these models, there remain persistent discrepancies with data. The particularly well-known discrepancies are over-estimation of the backward angle $\pi d \to \pi d$ differential cross section at energies above the Δ resonance; the under-estimation of the $NN \to \pi d$ cross section; and the difficulty of describing most of the polarization quantities in all the three reactions $\pi d \to \pi d$, $NN \to \pi d$, and $NN \to NN$. This situation has led to a number of speculations concerning the nature of the underlying pieces of physics that are missing in the current calculations. One difficulty, recently pointed out by Jennings,[12] is the inadequacy of the current ways of including pion absorption. Lamot *et al.*[10] have emphasized the importance of off-shell effects. Afnan and Blankleider[13] propose an extension of the current models to describe the nucleon and delta on an equal footing.

At the same time, Dosch *et al.*[14–21] investigated the effect that a short range ΔN interaction may have on the observables of πd elastic scattering. Their procedure involves calculating the lowest order contribution, illustrated in Fig. 1(a), and adding this, in Born approximation, to partial wave πd elastic amplitudes coming from a

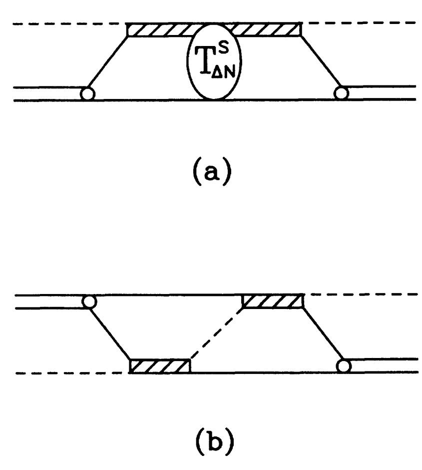

FIG. 1. The Born term contribution of the ΔN interaction to the πd scattering amplitude. (a) represents the short range ΔN interaction which is usually missing from most models; (b) is the corresponding one-pion exchange contribution which is included in the few-body calculations of the πNN system.

separate few-body calculation of the πNN system. Their ΔN interaction is described in a phenomenological way. By using dispersion techniques, they are able to express the diagram of Fig. 1(a) in the partial wave form

$$B^{J}_{l'l}(E)=\sum_{L'S'LS} F_{L'}(E)T^{L'S';LS}_{\Delta N\to\Delta N}F_{L}(E) , \tag{1.1}$$

where $T^{L'S';LS}_{\Delta N\to\Delta N}$ is an on-mass-shell ΔN t matrix defined for a zero-width delta. By varying the parameters of this effective ΔN t matrix, and by adding to the Faddeev amplitudes of Garcilazo's calculation,[11] they were able to obtain excellent agreement with all the currently available πd elastic scattering data.[19,21] Such a ΔN interaction should be distinguished from the "indirect" (u channel) one-pion exchange contribution already included in Faddeev calculations, see Fig. 1(b). Rather, the phenomenological ΔN interaction represents a shorter range interaction due to "direct" (t channel) meson exchanges and/or quark-exchange effects. Previously, the influence of a direct ΔN interaction on observables of the πd system was investigated in the context of dibaryon resonances.[22] Evidence for such a short range ΔN interaction has also come from other sources. In the description of inelastic pion-nucleus scattering within the Δ-hole model, a considerable improvement is obtained by introducing a strong zero-range ΔN interaction into the s-wave part of the spreading potential.[23] In addition, calculations at the quark level predict short range ΔN repulsion in some channels.[24]

Although the fitting procedure of Dosch *et al.* has been successful in describing πd elastic data, a closer analysis is needed before we can make conclusions about the significance of their results. A number of questions arise as to the reliability of their approach. Firstly, the ΔN interaction is added only in Born term. One may wonder how the results are modified if the short range ΔN interaction were included to all orders. Because of the coupling of πd elastic scattering to absorption channels, it is essential to see the effect that the short range ΔN interaction has on all the channels $\pi d\to\pi d$, $NN\to\pi d$, and $NN\to NN$. In addition, the expression of Eq. (1.1) does not make it clear how the off-shell behavior of the ΔN interaction enters the calculation.

In this paper we try to provide the answers to the above questions. In our approach, we construct a partial wave ΔN potential $V^{L'S';LS}_{\Delta N\to\Delta N}$ that gives rise to approximately the same t matrix $T^{L'S';LS}_{\Delta N\to\Delta N}$ as constructed by Dosch *et al.* As this t matrix is assumed to act between a nucleon and a zero-width Δ, it is further necessary to modify this interaction in order to take into account the physical width of the Δ. Dosch *et al.* do this by allowing the Δ mass to become complex in the kinematical relation for the on-shell momentum of the Δ, Eq. (3.2). We, however, choose a dynamical description by assuming that the potential $V^{L'S';LS}_{\Delta N\to\Delta N}$ acts between a nucleon and a *bare* delta; in this way, the delta obtains the proper mass and width by explicit dressing with one-pion loops. This potential is then added to the driving term of the few-body πNN equations of Afnan and Blankleider.[6] By solving the resulting set of equations, the ΔN potential is iterated to all orders. Indeed, all contributions involving cross terms between the short range ΔN potential and terms of the πNN multiple scattering series are included in the solution of the equations. Since pion absorption is included in the equations, we obtain results for all the reactions $\pi d\to\pi d$, $NN\to\pi d$, and $NN\to NN$. In our formalism it is also simple to investigate the sensitivity of our results to variations in the off-shell behavior of $V^{L'S',LS}_{\Delta N\to\Delta N}$.

We find that adding our ΔN interaction in Born term to the few-body πd scattering amplitude, gives effects of about the same magnitude as reported by Dosch *et al.* The energy dependence of these effects is however different in the two models. We show that this is probably due to the above mentioned different ways of taking into account the finite width of the Δ. We further find that the effect of including the ΔN interaction to all orders can be very significant for the differential cross section of πd scattering. The ΔN interaction is also found to be significant for the differential cross section and A_{y0} of $NN\to\pi d$ as well as the phase shifts and inelasticities of NN scattering. This suggests the importance of considering the short range ΔN simultaneously in all the reactions $\pi d\to\pi d$, $NN\to\pi d$ and $NN\to NN$.

This paper is organized as follows: In Sec. II we present the model and describe in detail the construction of our ΔN interaction. In Sec. III we discuss our results. Section IV contains a summary and conclusions.

II. CALCULATION

In a series of papers, Dosch *et al.* and in particular Ferreira, Andrade, and Dosch[19] (FAD) have demonstrated that the addition, in Born approximation, of a phenomenological ΔN interaction to πd Faddeev amplitudes, leads to a better agreement with experiment. In this work our goal is to study the effects of such a ΔN interaction incorporated consistently to all orders in the description of the πNN system. Since we are interested in comparing the results of our model to that of FAD, we use input that is similar (although not identical) to theirs. The details of the πNN model and the ΔN interaction are presented below.

A. πNN model

For the few-body description of the πNN system, not including the short range ΔN interaction, we use the model of Blankleider and Afnan[6] (BA). Apart from some modifications discussed shortly, all input and details of the model are as presented in Ref. 6. Here we therefore give only a brief discussion, concentrating on the differences between the present model and that of BA.

For the particular case of separable two-body interactions, the antisymmetrized πNN equations are given in operator form by

$$X_{d,d}=Z_{d,\Delta}\tau_\Delta X_{\Delta,d}+Z_{d,N}\frac{g_N}{2}X_{N,d}\ ,$$

$$X_{\Delta,d}=Z_{\Delta,d}+Z_{\Delta,d}\tau_d X_{d,d}+Z_{\Delta,\Delta}\tau_\Delta X_{\Delta,d}+Z_{\Delta,N}\frac{g_N}{2}X_{N,d}\ ,$$

$$X_{N,d}=Z_{N,d}+Z_{N,d}\tau_d X_{d,d}+Z_{N,\Delta}\tau_\Delta X_{\Delta,d}+Z_{N,N}\frac{g_N}{2}X_{N,d}\ , \tag{2.1}$$

$$X_{d,N}=Z_{d,N}+Z_{d,\Delta}\tau_\Delta X_{\Delta,N}+Z_{d,N}\frac{g_N}{2}X_{N,N}\ ,$$

$$X_{\Delta,N}=Z_{\Delta,N}+Z_{\Delta,d}\tau_d X_{d,N}+Z_{\Delta,\Delta}\tau_\Delta X_{\Delta,N}+Z_{\Delta,N}\frac{g_N}{2}X_{N,N},$$

$$X_{N,N}=Z_{N,N}+Z_{N,d}\tau_d X_{d,N}+Z_{N,\Delta}\tau_\Delta X_{\Delta,N}+Z_{N,N}\frac{g_N}{2}X_{N,N},$$

where $X_{d,d}$, $X_{N,d}$, and $X_{N,N}$ are amplitudes for the processes $\pi d\to\pi d$, $\pi d\to NN$, and $NN\to NN$, respectively. The equations also involve the off-shell amplitudes $X_{\Delta,d}$ and $X_{\Delta,N}$ describing intermediate state Δ formation via the processes $\pi d\to N\Delta$ and $NN\to N\Delta$. The Z amplitudes in Eqs. (2.1) are the corresponding lowest order contributions. We note that the Δ subscript in Eqs. (2.1) refers to a πN quasiparticle in *any* partial wave, although for the rest of this paper the symbol Δ is indeed intended to mean the P_{33} resonance. Similarly the d subscript in Eqs. (2.1) refers to an NN quasiparticle in any partial wave. We also note that, at this state, all amplitudes of Eqs. (2.1) are not properly normalized. However, only a further multiplication by two-body bound state normalization factors is needed to yield the physical amplitudes $T_{d,d}$, $T_{N,d}$, and $T_{N,N}$. The input to the πNN equations (2.1) consists of partial wave separable potentials for the reaction $\pi N\to\pi N$, as well as for $NN\to NN$ below pion production threshold. For the partial wave πN, and NN potentials (excluding the πN P_{11} pole term), we respectively write

$$v_{\pi N,\pi N}=|h_\Delta\rangle\lambda_\Delta\langle h_\Delta|,\quad v_{NN,NN}=|h_d\rangle\lambda_d\langle h_d|\ , \tag{2.2}$$

with the corresponding t matrices given by

$$t_{\pi N,\pi N}=|h_\Delta\rangle\tau_\Delta\langle h_\Delta|\ ,\quad t_{NN,NN}=|h_d\rangle\tau_d\langle h_d|\ . \tag{2.3}$$

Special attention needs to be given to the P_{11} channel because pion absorption takes place in this partial wave. We follow BA and parametrize the P_{11} interaction in terms of a two-term separable potential that describes the phase shift data up to $T_\pi\approx 300$ MeV, and that gives rise to a pole in the t matrix at the nucleon mass. Thus,

$$t_{P_{11}}=|h_{P_{11}}\rangle\tau_{P_{11}}\langle h_{P_{11}}|\ , \tag{2.4}$$

where $|h_{P_{11}}\rangle$ is the row matrix

$$|h_{P_{11}}\rangle=[|h_1\rangle|h_2\rangle] \tag{2.5}$$

and $\tau_{P_{11}}$ is a 2×2 matrix. The form factors $h_i(k)\equiv\langle k|h_i\rangle$ $(i=1,2)$, as indeed all our input form factors, are parametrized with the Yamaguchi form

$$h(k)=\frac{Ck^l}{(k^2+\beta^2)^n}\ . \tag{2.6}$$

Our fit to the P_{11} phase shift is shown in Fig. 2(a), the corresponding form factor parameters are given in Table I. The t matrix can be written in terms of pole (P) and nonpole (NP) parts

$$t_{P_{11}}=t_{P_{11}}^{P}+t_{P_{11}}^{\mathrm{NP}}\ . \tag{2.7}$$

Although the exact form of this "splitting of the P_{11}" is specified within the unitary theory, other forms are sometimes taken for the sake of simplicity (although at the expense of exact two- and three-body unitarity).[6] Here we depart from BA and adopt a prescription that is the nonrelativistic analog of the one used by Garcilazo:

$$t_{P_{11}}^{P}(e)\equiv\frac{(e+m_{N)}}{2e}t_{P_{11}}(e)\ , \tag{2.8}$$

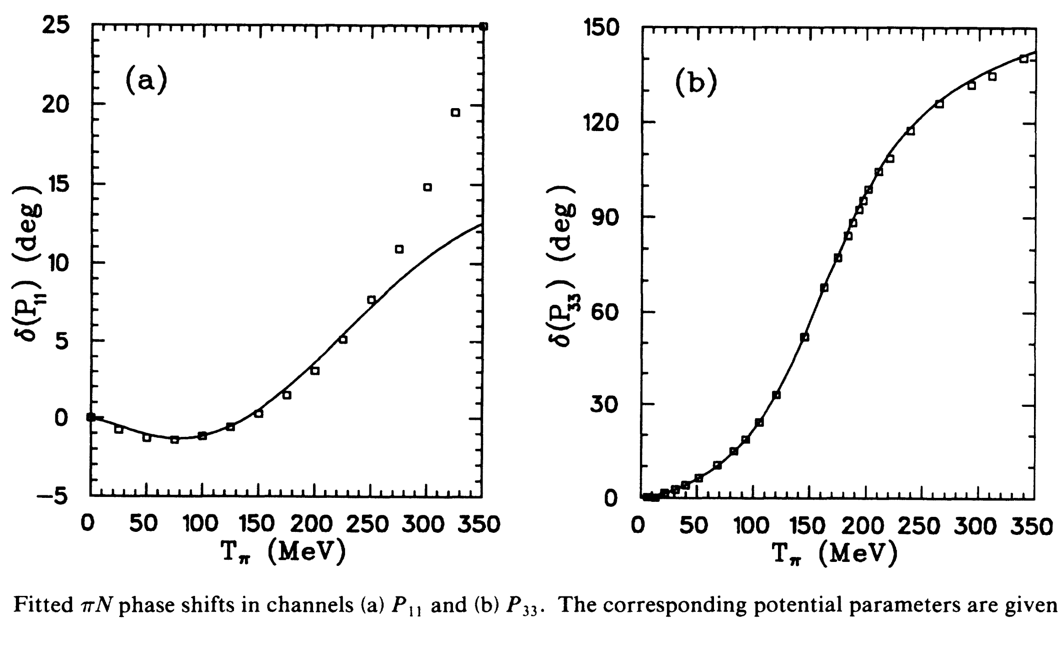

FIG. 2. Fitted πN phase shifts in channels (a) P_{11} and (b) P_{33}. The corresponding potential parameters are given in Table I.

$$t^{\mathrm{NP}}_{P_{11}}(e)\equiv\frac{(e-m_N)}{2e}t_{P_{11}}(e)\ . \tag{2.9}$$

Here e is the total energy available in the πN c.m. system and m_N is the nucleon mass. We deem this prescription more appropriate as Garcilazo's model was used to provide the background amplitudes in the work of FAD. A further advantage of this prescription is that it makes the separate contributions of the pole and nonpole pieces small—an empirically motivated choice that provides a better description of t_{20}.[25] In the notation of Eqs. (2.1), we have, in the P_{11} channel,

$$g_N(e)\equiv\frac{(e+m_N)}{2e}\tau_{P_{11}}(e)\ ,$$

$$\tau_\Delta(e)\equiv\frac{(e-m_N)}{2e}\tau_{P_{11}}(e)\ ,$$

$$|h_N\rangle\equiv|h_{P_{11}}\rangle\ ,$$

$$|h_\Delta\rangle\equiv|h_{P_{11}}\rangle\ .$$

We note that, with the form factors as defined above, the Z amplitudes of Eqs. (2.1) can be written in operator form as

$$Z_{\alpha,\beta}=F_{\alpha,\beta}\langle h_\alpha|G_0|h_\beta\rangle,\quad \alpha,\beta=\Delta,d,N\ , \tag{2.10}$$

where G_0 is the πNN free Green's function, and $F_{\alpha,\beta}$ is a constant arising from the antisymmetrization of the nucleons.[6]

We also depart from BA and describe the P_{33} interaction in terms of an elementary delta particle that is dressed by πN rescattering. In particular, we take the potential term as

$$v_{P_{33}}=|h_\Delta\rangle\frac{1}{e-m^0_\Delta}\langle h_\Delta|\ , \tag{2.11}$$

where m^0_Δ is the bare delta mass. The range and strength of the form factor $h_\Delta(k)$ are chosen in order to fit the P_{33} phase shifts as well as the Δ resonance width. At the same time, the bare mass is chosen to ensure that the resulting t matrix has the resonance pole at $m_\Delta=1211-i50$ MeV. The parameters of our best fit are given in Table I. By contrast, we note that BA used an energy independent P_{33} potential. Our present description is closer to the chiral bag interpretation of the Δ.[26] Within this interpretation, the elementary ΔN interaction is viewed to take place between the nucleon and the bare delta.

B. ΔN interaction

The second part of our model involves constructing the short range ΔN interaction. Let us recall that the potential of Eq. (2.11) leads to the off-shell πN c.m. t matrix

$$t_{P_{33}}(k'_\pi,k_\pi;e)=h_\Delta(k'_\pi)\tau_\Delta(e)h_\Delta(k_\pi)\ , \tag{2.12}$$

where

$$\tau_\Delta^{-1}(e)=e-m^0_\Delta-\int_0^\infty dk\,k^2\frac{h^2_\Delta(k)}{e^+-(k^2+m_\pi^2)^{1/2}-k^2/2m_N-m_N}\ . \tag{2.13}$$

We note that, as in Ref. 6, we use relativistic kinematics for the pion and nonrelativistic kinematics for the nucleons.

The lowest order contribution to πd elastic scattering involving an intermediate ΔN state, will be called the Born term $B_{d,d}$. Likewise, the lowest order contribution involving a short range ΔN interaction, illustrated in Fig. 1(a), will be called the ΔN Born term $B^\Delta_{d,d}$. Numerically they are given by

$$B_{d,d}(E)=N_d^2\int_0^\infty dp\,p^2Z_{d,\Delta}(p_\pi,p;E)\tau_\Delta(e)\times Z_{\Delta,d}(p,p_\pi;E)\ , \tag{2.14}$$

$$B^\Delta_{d,d}(E)=N_d^2\int_0^\infty dp'dp\,p'^2p^2Z_{d,\Delta}(p_\pi,p';E)\tau_\Delta(e')\times T^S_{\Delta N}(p',p;E)\tau_\Delta(e)Z_{\Delta,d}(p,p_\pi;E)\ , \tag{2.15}$$

where N_d is the deuteron wave function normalization factor, and $T^S_{\Delta N}(p',p;E)$ is the short range *off-shell* ΔN t matrix. The total two-body c.m. energy e is related to the total energy E and the relative ΔN momentum p by

$$e=E-\frac{p^2}{2m_N}-\frac{p^2}{2(m_N+m_\pi)}-m_N\ . \tag{2.16}$$

We assume that $T^S_{\Delta N}(p'p;E)$ results from an underlying energy-independent potential $V^S_{\Delta N}(p',p)$ between a *bare* delta and a nucleon. Furthermore, we assume that $V^S_{\Delta N}(p',p)$ is separable:

$$V^S_{\Delta N}(p',p)=h_{\Delta N}(p')\lambda_{\Delta N}h_{\Delta N}(p)\ . \tag{2.17}$$

We relate $V^S_{\Delta N}(p',p)$ and $T^S_{\Delta N}(p',p;E)$ by a Lippmann-Schwinger equation with a dressed Δ propagator defined as in Eq. (2.13),

TABLE I. Form factor parameters of Eq. (2.6) for the πN P_{11} and P_{33} separable potentials. The corresponding fits to πN phase shifts are shown in Fig. 2.

Channel	C_i (fm$^{l_i-2n_i+1}$)	β_i (fm^{-1})	λ_i	l_i	n_i	m_i^0 (MeV)
P_{11} $(i=1)$	759.3672	3.50	-1	1	3	
P_{11} $(i=2)$	1.1056	3.555	-1	3	2	
P_{33}	0.56684	1.076875	$(e-m^0_\Delta)^{-1}$	1	1	1391.08

$$T^S_{\Delta N}(p',p;E)=V^S_{\Delta N}(p',p)+\int_0^\infty dp''p''^2V^S_{\Delta N}(p',p'')\tau_\Delta(e'')\times T^S_{\Delta N}(p'',p;E)\ . \tag{2.18}$$

The separability assumption allows one to solve Eq. (2.18) algebraically. Thus

$$T^S_{\Delta N}(p',p;E)=h_{\Delta N}(p')\tau_{\Delta N}(E)h_{\Delta N}(p)\ , \tag{2.19}$$

where

$$\tau^{-1}_{\Delta N}(E)=\lambda^{-1}_{\Delta N}-\int_0^\infty dp\, p^2h^2_{\Delta N}(p)\tau_\Delta(e)\ . \tag{2.20}$$

Equation (2.19) enables one to write Eq. (2.15) in the simple form

$$B^\Delta_{d,d}(E)=F_{d,\Delta}(E)T^S_{\Delta N}(p_0,p_0;E)F_{\Delta,d}(E)\ , \tag{2.21}$$

where

$$F_{d,\Delta}(E)\equiv N_d\int_0^\infty dp\, p^2Z_{d,\Delta}(p_\pi,p;E)\tau_\Delta(e)\frac{h_{\Delta N}(p)}{h_{\Delta N}(p_0)} \tag{2.22}$$

and p_0 is some as yet unspecified ΔN relative momentum. At this stage we note that Eq. (2.21) is formally the same as the corresponding expression, Eq. (1.1), as used by FAD. However, the starting point for the two calculations is very different. FAD use a covariant formalism in which the F function, corresponding to our $F_{d,\Delta}(E)$, is calculated in terms of dispersion relation. Perhaps an even more fundamental difference is in the way the ΔN t matrix, corresponding to our $T^S_{\Delta N}(p_0,p_0;E)$ is treated in the two models. Here FAD describe the inherently off-shell ΔN t matrix in terms of an effective on-shell t matrix,

$$\tilde{T}^D_{\Delta N}(E)=\frac{1}{2i}(\eta e^{2i\delta}-1)\ , \tag{2.23}$$

where the on-shell ΔN momentum is taken to be complex (we elaborate on their approach in Sec. III). It is $\tilde{T}^D_{\Delta N}(E)$ that FAD treated as a complex parameter to fit πd elastic scattering data. By contrast, our F function is calculated nonrelativistically, and our t matrix $T^S_{\Delta N}(p_0,p_0;E)$ arises dynamically from an underlying ΔN potential. In addition, both $F_{d,\Delta}(E)$ and $T^S_{\Delta N}(p_0,p_0;E)$ are dependent on the form factor $h_{\Delta N}(p)$ and are thereby coupled.

We now would like to construct our potential $V^S_{\Delta N}(p',p)$ by fitting to the ΔN interaction of FAD. However, because of the above-discussed differences, there is no unique way in which this is to be accomplished. We choose to proceed in the spirit of the FAD model, and assume that the amplitude $\tilde{T}^D_{\Delta N}(E)$ can be described by the scattering of a nucleon and a stable Δ of mass $m_\Delta=1211$ MeV. The underlying potential will then be identified with $V^S_{\Delta N}(p',p)$. It is now natural to identify p_0 with the on-shell momentum of the stable Δ particle. Thus we solve the equation

$$T^D_{\Delta N}(p',p;E)=V^S_{\Delta N}(p',p)+\int_0^\infty dp''p''^2V^S_{\Delta N}(p,p'')G_{\Delta N}(E,p'')\times T^D_{\Delta N}(p'',p;E)\ . \tag{2.24}$$

where the parameters of $V^S_{\Delta N}(p',p)$ are varied in order to minimize the difference between $-\pi\mu_{\Delta N}p_0T^D_{\Delta N}(p_0,p_0;E)$ and $\tilde{T}^D_{\Delta N}(E)$. The $-\pi\mu_{\Delta N}p_0$ factor is needed as we use the convention of Goldberger and Watson;[27] here, as later, we use a tilde on quantities that are expressed in the conventions of FAD. In the above we use nonrelativistic kinematics for the nucleon and delta, thus

$$\mu_{\Delta N}=\frac{m_\Delta m_N}{m_\Delta+m_N}\ , \tag{2.25}$$

$$G^{-1}_{\Delta N}(E,p)=E^+-\frac{p^2}{2\mu_{\Delta N}}-m_N-m_\Delta\ , \tag{2.26}$$

is the free ΔN propagator, and the on-shell momentum p_0 is defined by

$$E=\frac{p_0^2}{2\mu_{\Delta N}}+m_N+m_\Delta\ . \tag{2.27}$$

We note that this way of constructing $T^D_{\Delta N}(E)$ gives no inelasticity (i.e., $\eta=1$), and because we do not introduce complex momenta, our on-shell t matrix is strictly zero below the ΔN threshold ($T_\pi\approx150$ MeV). Although neither of these restrictions are imposed in the FAD model, their ${}^5P_3$ amplitudes are purely elastic while the ${}^5S_2$ ones show substantial inelasticity only below $T_\pi=200$ MeV. We therefore make contact with their model only in the restricted energy region $200<T_\pi<300$ MeV, where the inelasticities of the FAD amplitudes are close to zero.

For the purposes of fitting to the FAD amplitudes, the form factors $h_{\Delta N}(p)$ are parametrized in terms of Yamaguchi forms, as in Eq. (2.6). It is found that one

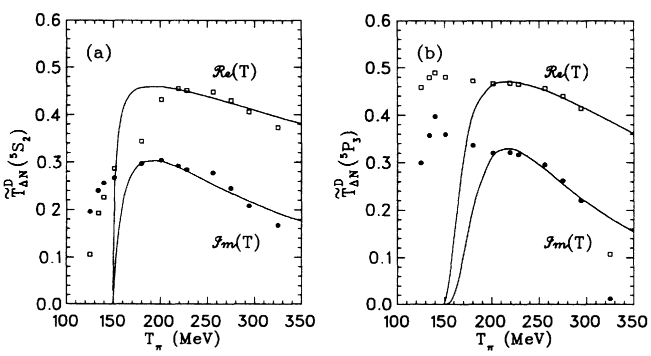

FIG. 3. The real and imaginary parts of the (zero width Δ) ΔN t-matrix $\tilde{T}^D_{\Delta N}$ in channels (a) ${}^5S_2$ and (b) ${}^5P_3$. The solid lines are our fits obtained by solving the scattering equation (2.24). The open squares and solid circles are the real and imaginary parts of $\tilde{T}^D_{\Delta N}$ as given in Ref. 19, which here are taken as data. We note that in the energy range 200–300 MeV, both ${}^5S_2$ and ${}^5P_3$ t matrices correspond to elastic ΔN phase shifts of approximately 30°.

TABLE II. Form factor parameters of Eq. (2.6) for the ΔN separable potentials corresponding to the fits shown in Fig. 3.

Channel	C_i (fm$^{l_i-2n_i+1}$)	β_i (fm^{-1})	λ_i	l_i	n_i
$^5S_2$	1.955 253	2.913 668	−1	0	1
$^5P_3$	5.989 537	2.130 922	−1	1	2

term separable potentials are sufficient to provide reasonable fits in the above-mentioned energy region. The fits for both $^5S_2$ and $^5P_3$ partial waves are shown in Fig. 3 where we compare with the ΔN amplitudes of FAD. The corresponding parameters are given in Table II.

Once the ΔN interaction is constructed, it may be included into the few-body calculation to all orders by simply making the replacement

$$Z_{\Delta,\Delta}(p',p;E)\rightarrow Z_{\Delta,\Delta}(p',p;E)+V^S_{\Delta N}(p',p;E) \quad (2.28)$$

in the Eqs. (2.1).

III. RESULTS AND DISCUSSION

In the work of FAD, the effect of the ΔN interaction was investigated for πd elastic scattering only. To make contact with their work, we first describe our results for πd elastic scattering, and then show the effect of this interaction on pion production and on the NN phase shifts.

A. $\pi d \rightarrow \pi d$

Having constructed our $^5S_2$ and $^5P_3$ ΔN potentials by fitting to the FAD phase shifts, it is interesting to compare the actual ΔN Born term, $B^{\Delta}_{d,d}$, used in the two models. The connection between the models can be most clearly obtained in the limit of a zero width Δ. In this limit the Born term in the FAD model is expressed as[19]

$$B^{\Delta}_{d,d}(E)=\tfrac{2}{3}g^2_{\Delta N\pi}f_J\frac{p^2_\pi}{m_\Delta m_N}\frac{\mu_{\Delta N}}{\mu_{\pi d}}(\tilde{F}^D_{d,\Delta})^2\frac{\tilde{T}^D_{\Delta N}}{-\pi\mu_{\Delta N}\mathrm{Re}(p_\Delta)}\ , \quad (3.1)$$

where p_Δ is the on-shell ΔN momentum given by

$$p_\Delta=\frac{1}{2E}[(E^2+m^2_\Delta-m^2_N)^2-4E^2m^2_\Delta]^{1/2} \quad (3.2)$$

with $m_\Delta=1211$ MeV. In Eq. (3.1), $g_{\Delta\pi N}$ is the $\Delta\pi N$ coupling constant ($g^2_{\Delta\pi N}=257$ GeV^{-2}) and f_J is a recoupling coefficient equal to $\frac{1}{3}$ for the $^5S_2$ channel and $\frac{2}{15}$ for the $^5P_3$ channel. Comparison with our formulation of Sec. II immediately gives the relationship between our F factor $F_{d,\Delta}$ and the one of FAD, $\tilde{F}^D_{d,\Delta}$:

$$F^2_{d,\Delta}\equiv\tfrac{2}{3}g^2_{\Delta N\pi}f_J\frac{p^2_\pi}{m_\Delta m_N}\frac{\mu_{\Delta N}}{\mu_{\pi d}}(\tilde{F}^D_{d,\Delta})^2\ . \quad (3.3)$$

We shall use Eq. (3.3) to compare the F factors of the two

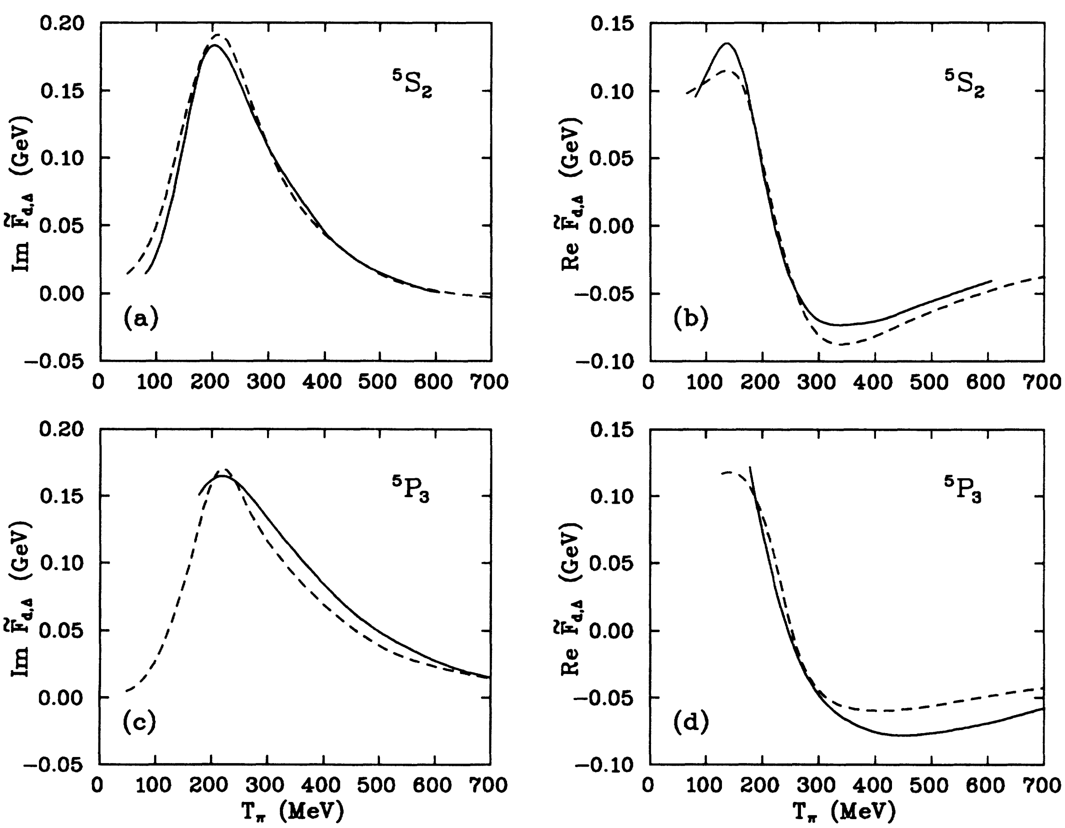

FIG. 4. Comparison of our $^5S_2$ and $^5P_3$ vertex functions $F_{d,\Delta}$ (solid lines), defined in Eq. (2.22), with the ones of Ferreira *et al.* (Ref. 19) (dashed lines). For this comparison, the results of Ref. 19 have been shifted by 25 MeV towards the higher energies, and Eq. (3.3) has been used to relate the different conventions.

models also in the case of a dressed Δ. This is reasonable as the effect of the dressing is expected to be contained not in the conversion factors, but inside the loop integral expressions for the F factors themselves. In Fig. 4 the F factors are shown where, for the purposes of comparison, the results of FAD were taken from Ref. 19 and shifted by 25 MeV in the positive T_π direction (although the origin of this shift is unclear, it is small on the energy scale of interest, and might for example be due to the different treatment of recoil effects). Apart from the overall 25 MeV shift, the real and imaginary parts of the F factors are very similar in the two models. For the $^5P_3$ channel we recall that our F factor, given by Eq. (2.22), goes to infinity as the on-shell (zero-width) Δ momentum, p_0, approaches zero ($T_\pi \approx 150$ MeV). As shown in Figs. 4(c) and 4(d), there is nevertheless a good agreement above 180 MeV with the F factors of FAD. The close similarity of the F factors in the two models enables us also to use Eq. (3.1) to compare the corresponding ΔN t matrices in the case of a dressed Δ. In our model we implement this dressing *dynamically*. As discussed in Sec. II, this means that we start with an underlying potential between a bare Δ and a nucleon, and then generate the dressing by explicitly calculating pion loops. By contrast, FAD implemented this dressing through a prescription where m_Δ in Eq. (3.2) is taken to be complex ($m_\Delta = 1211 - i50$ MeV). These two radically different ways to obtain the ΔN t matrix for a dressed Δ may now be compared using the relation

$$T^S_{\Delta N}(p_0,p_0;E) \equiv \frac{\tilde{T}^D_{\Delta N}(E)}{-\pi\mu_{\Delta N}\mathrm{Re}(p_\Delta)} , \tag{3.4}$$

which follows directly from Eqs. (2.21), (3.1), and the identification of Eq. (3.3). This comparison is presented in Fig. 5 where, for consistency with the F-factor comparison, we have added 25 MeV to T_π in the results of FAD. In contrast to the close similarity of the F factors, the ΔN t matrices differ markedly especially at the lower energies. Below 200 MeV this may be partly due to the already substantial differences in the zero-width ΔN t matrices (Fig. 3). Above 200 MeV, however, the differences can only be attributable to the different handling of the delta dressing. In this respect we conclude that the prescription used by FAD to take into account the dressing of the delta, is not compatible with a dynamical approach.

Having discussed the numerical differences between our calculation of the ΔN Born term and the one of FAD, we now turn to our main goal of investigating the effect of adding the ΔN interaction to all orders in the πNN system. At first, we follow FAD and add the $^5S_2$ and $^5P_3$ ΔN interactions only in Born approximation to

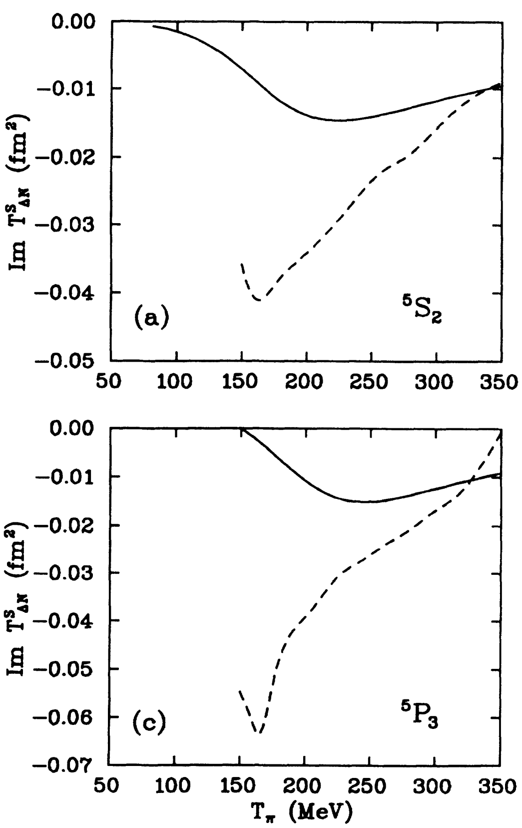

FIG. 5. Comparison of our $^5S_2$ and $^5P_3$ (dressed Δ) ΔN t matrices $T^S_{\Delta N}(p_0,p_0;E)$ (solid lines), calculated from Eq. (2.18), with the ones of Ferreira *et al.* (Ref. 19) (dashed lines). For this comparison, the results of Ref. 19 have been shifted by 25 MeV towards the higher energies, and Eq. (3.4) has been used to relate the different conventions.

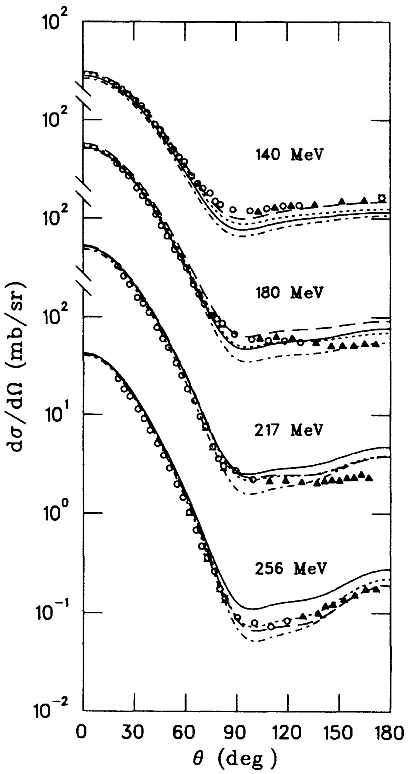

FIG. 6. The πd differential cross sections at pion lab kinetic energies of 140, 180, 217, and 256 MeV. The solid curves are the results of our unitary πNN calculation with no short range ΔN interaction. The results of adding the ΔN interaction in Born term are given by the dashed lines, while adding it to all orders is given by the dotted lines. The dash-dotted lines result when we use the ΔN t matrix of Ferreira *et al.* (Ref. 19) and add it in Born term to our background amplitudes. Data are from Refs. 28 and 29.

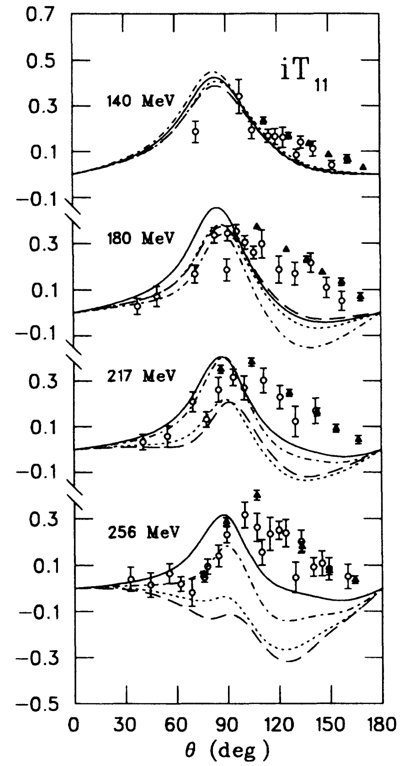

FIG. 7. The πd vector analyzing power iT_{11}. We use the same notation as in Fig. 6. Data are from Refs. 30 and 31.

our background few-body amplitudes. That is, $B^{\Delta}_{d,d}$ is numerically added to the few-body amplitude $T_{d,d}$ in both the $J=2^+$ and $J=3^-$ channels. The resulting effects for πd observables are presented in Figs. 6 and 7 where we show the differential cross section and the vector analyzing power iT_{11} for four energies spanning the delta resonance region. We shall not explicitly show the tensor polarizations since, in their case, the effect of the ΔN interaction is small, especially compared to the uncertainties in the current experimental data. In Figs. 6 and 7, the solid line corresponds to the calculation without ΔN interaction, and the dashed curve is the result of including the ΔN interaction in Born term. For lab pion kinetic energy $T_\pi = 140$ MeV the effect of adding $B^{\Delta}_{d,d}$ is to increase the differential cross section at backwards angles bringing it into agreement with the data. At $T_\pi=256$ MeV addition of $B^{\Delta}_{d,d}$ decreases the differential cross section at backward angles, and thus resembles the effect obtained by FAD. In other words in our case the ΔN interaction has the opposite effect at $T_\pi=140$ as compared to $T_\pi=256$ MeV. FAD do not obtain this behavior; in their case the effect of $B^{\Delta}_{d,d}$ always has the same sign, namely it always lowers the differential cross section at backward angles. This discrepancy with our energy dependence of the ΔN effect is not surprising considering the differences in the underlying ΔN interaction, as illustrated by Fig. 5. Indeed, if we add to our 140 MeV amplitudes the ΔN Born term of FAD, constructed directly from Eq. (3.1) with m_Δ complex in Eq. (3.2), we obtain the dash-dotted curves in Fig. 6, showing a very similar effect to that reported by FAD.

Looking at the influence of the short range ΔN interaction on iT_{11}, Fig. 7, we observe a very large effect at the higher energies. Further examination shows that this effect comes mainly from the change in the $J=3^-$ amplitude brought about by adding $B^{\Delta}_{d,d}$. Within their model, FAD obtained a much smaller effect. To clarify the origin of this behavior, we may again construct FAD's $B^{\Delta}_{d,d}$ and add it to our amplitudes. The result is shown as the dash-dotted line in Fig. 7. Once more this resembles the effect obtained by FAD (of course since we add the FAD Born term to *our* background amplitudes, we do not expect the effect to be completely the same as reported in Ref. 19). The conclusion to be drawn is that the observable iT_{11} can be very sensitive to relatively small changes in the 3^- amplitude, and therefore one should take care in attaching significance to the fitting of this observable within a given model.

The effect of including the constructed ΔN interaction to all orders inside the few-body calculation is now indicated by the dotted lines in Figs. 6 and 7. Comparing these results with the ones where the ΔN interaction has been added in Born term (dashed curves), it can clearly be seen that the addition in Born term generally overestimates the effect of the ΔN interaction. In fact, except for the cross section around 217 MeV, the effect of the iterations can be as important as the effect of the Born ΔN interaction itself. We therefore believe that if the ΔN interaction is to be included phenomenologically by fitting the data, the fully iterated amplitudes must be used for the fit.

As shown in Fig. 7, the effect of the iterations for the vector analyzing power iT_{11} appear to be smaller than for the differential cross section. On the other hand the iterations are not negligible, especially when compared with the small uncertainties in the new data of Ref. 31.

As compared to Ref. 19, one feature of our dynamical approach is that our Born term of Eq. (2.21) in principle depends on the off-shell behavior of the ΔN interaction. This can explicitly be seen in Eq. (2.22) for the F function, which contains $h_{\Delta N}(p)$, the form factor of the separable ΔN interaction. In the approach of FAD, however, the Born term depends only on the on-shell ΔN t matrix. To test if this underlying assumption is correct, we have tried various forms of the form factors $h_{\Delta N}(p)$ that result in the same fits to the ΔN data as shown in Fig. 3. We indeed find that different forms for $h_{\Delta N}(p)$, e.g., sum of two monopoles or rank-2 separable potential form factors, lead to no significant changes of the final results, and in this sense we find the FAD model consistent.

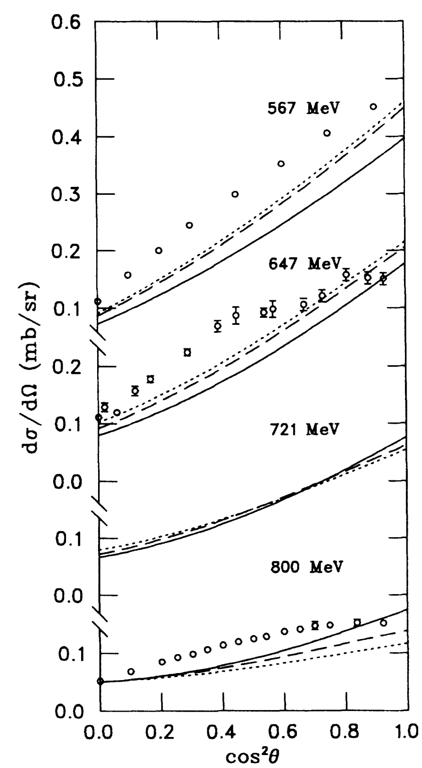

FIG. 8. The differential cross sections of $pp \rightarrow \pi^+ d$ at proton kinetic energies of 567, 647, 721, and 800 MeV. The solid curves are the results of our unitary πNN calculation with no short range ΔN interaction. The successive addition of the $^5S_2$ and $^5P_3$ ΔN interactions (both included to all orders) are given by the dashed and dotted lines, respectively. Data are from Refs. 32–34.

B. $NN \rightarrow \pi d$

The next part of this work consists of examining the effect of the ΔN interaction on pion production and on the NN phase shifts. Since these channels are coupled to the πd system, one should simultaneously consider them in judging the effect of any additional interaction on the πd system. Here we examine the effect of our constructed ΔN interaction on the observables of $\vec{p}\vec{p} \rightarrow \pi^+ d$. The deficiencies of conventional πNN calculations for $NN \rightarrow \pi d$ are well known.[13] The differential cross sections tend to fall below the data in most models, as do the polarization correlation parameters A_{xx}, A_{yy}, and A_{zz}. The discrepancy in the correlation parameters is thought to be indicative of a missing strength in the triplet NN channels. The analyzing power A_{y0} also tends to be badly reproduced, although in this case this is a particularly difficult task because of the many important interference terms that go to make up this observable. Considering the deficiences of the standard calculations, it is therefore particularly interesting to see the effect of the ΔN interaction on $NN \rightarrow \pi d$ observables.

In Figs. 8–13 we show the differential cross section and polarization observables for nucleon lab kinetic energies $T_N = 567$, 647, 721, and 800 MeV. We emphasize that these pion production results and the πd elastic scattering results discussed above have been calculated simultaneously from the same set of coupled equations,

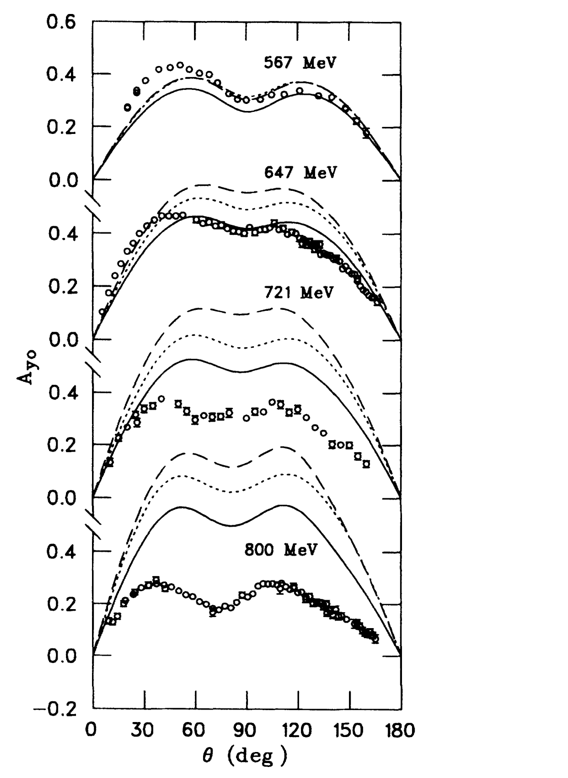

FIG. 9. The analyzing power A_{y0} in $\vec{p}p \rightarrow \pi^+ d$. We use the same notation as in Fig. 8. Data are from Refs. 35–37.

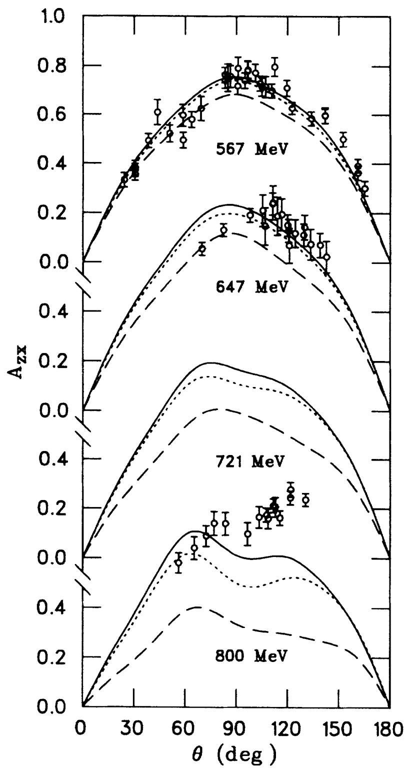

FIG. 10. The correlation coefficient A_{zx} in $\vec{p}\vec{p}\to\pi^+d$. We use the same notation as in Fig. 8. Data are from Refs. 38 and 39.

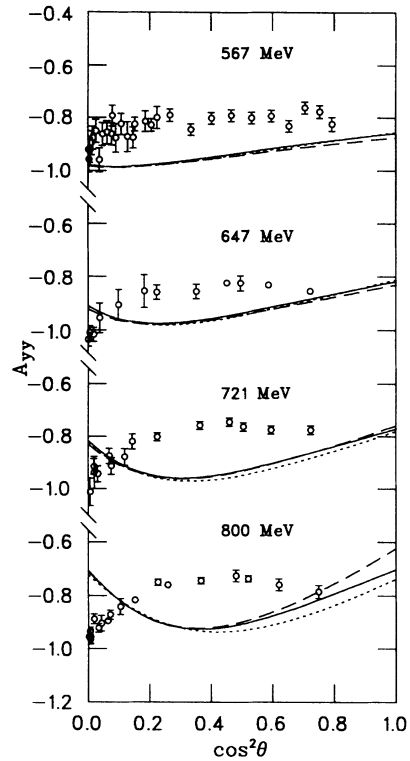

FIG. 12. The correlation coefficient A_{yy} in $\vec{p}\vec{p}\to\pi^+d$. We use the same notation as in Fig. 8. Data are from Refs. 40 and 41.

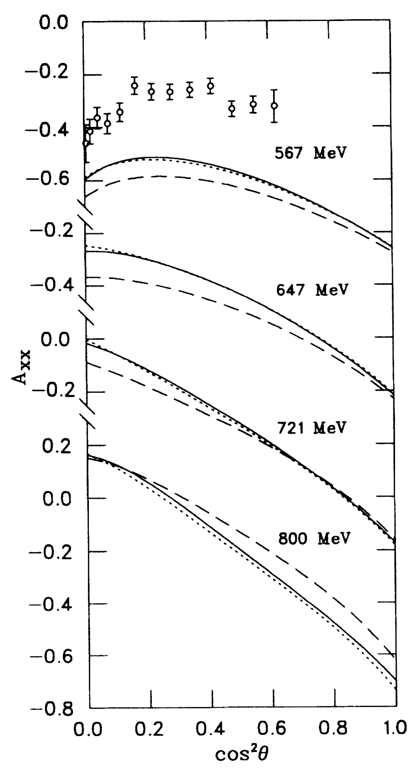

FIG. 11. The correlation coefficient A_{xx} in $\vec{p}\vec{p}\to\pi^+d$. We use the same notation as in Fig. 8. Data are from Ref. 40.

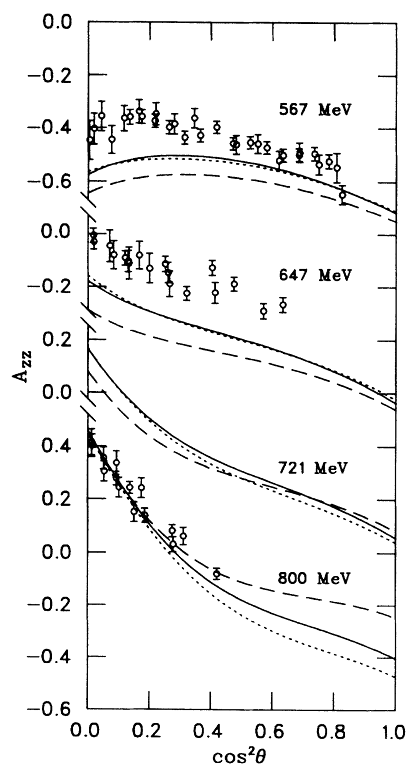

FIG. 13. The correlation coefficient A_{zz} in $\vec{p}\vec{p}\to\pi^+d$. We use the same notation as in Fig. 8. Data are from Refs. 39 and 40.

Eqs. (2.1). Thus the four energies for T_N correspond directly to the four energies for T_π in Figs. 6 and 7. In each case the solid curves are the results of the few-body calculation without the ΔN interaction, and the dotted curves are the results where both the $^5S_2$ and $^5P_3$ ΔN interactions are included to all orders. In Figs. 8–13 we also show the results of including just the $^5S_2$ ΔN interaction to all orders (dashed curves).

From Fig. 8 we see that the effect of the ΔN interaction on the differential cross sections is strongly energy dependent. As in the case of πd elastic scattering, below the Δ resonance the effect of the ΔN interaction is to increase $d\sigma/d\Omega$, bringing it close to the data, whereas at higher energies the effect is reversed. Although in general the effect of the ΔN interaction is substantial, this strong energy dependence leads to a "cross over" energy, at around $T_N = 721$ MeV, where the ΔN interaction has minimal effect. This cross over energy was also seen in $\pi d \to \pi d$, and moreover, it was at the same three-body c.m. energy (equivalent to $T_\pi = 217$ MeV). This is not altogether surprising since the energy dependence of both processes is basically determined by the Δ resonance. As an illustration of this underlying mechanism, consider the F factors of Fig. 4. When plotted in the argand plane, they display the characteristic looping behavior of a resonance. In this respect we note that the Born term contribution of the ΔN interaction to $NN \to \pi d$ is

$$B^{\Delta}_{d,N}(E) = F_{d,\Delta}(E) T^S_{\Delta N}(p_0 p_0; E) F_{\Delta,N}(E) , \tag{3.5}$$

which is the equivalent Eq. (2.21) for πd scattering, and has its behavior likewise influenced by the looping F factors. Figure 8 also displays the expected result that the contribution of the s-wave ΔN interaction to the cross section is substantially larger than the p-wave contribution.

The effect of the ΔN interaction on A_{y0} is shown in Fig. 9. The usual overestimation of A_{y0} at the higher energies is made worse by the addition of the ΔN interaction. This observable is very sensitive to fine details of a model, and, at this stage, not too much significance should be placed on the resulting large discrepancy with experimental data. Closely related to A_{y0} is the correlation parameter A_{xx} displayed in Fig. 10 (essentially they are, respectively, the imaginary and real parts of interfering scattering amplitudes). For both these observables we obtain substantial sensitivity to the ΔN interaction with a significant amount of cancellation between the $^5S_2$ and $^5P_3$ contributions.

A more serious discrepancy with data exists for the correlation parameters A_{xx}, A_{yy}, and A_{zz}. As seen from Figs. 11–13, for A_{xx} and A_{zz} there is a very large amount of cancellation between the $^5S_2$ and $^5P_3$ ΔN contributions, and A_{yy} appears very little affected by either of these. Since the original ΔN interactions were constructed phenomenologically, there is little reason to believe that these cancellations are in any way fundamental. On the other hand our results indicated that it is unlikely that ΔN interactions, not too different from ours, will resolve the long standing problem with the correlation parameters in $NN \to \pi d$.

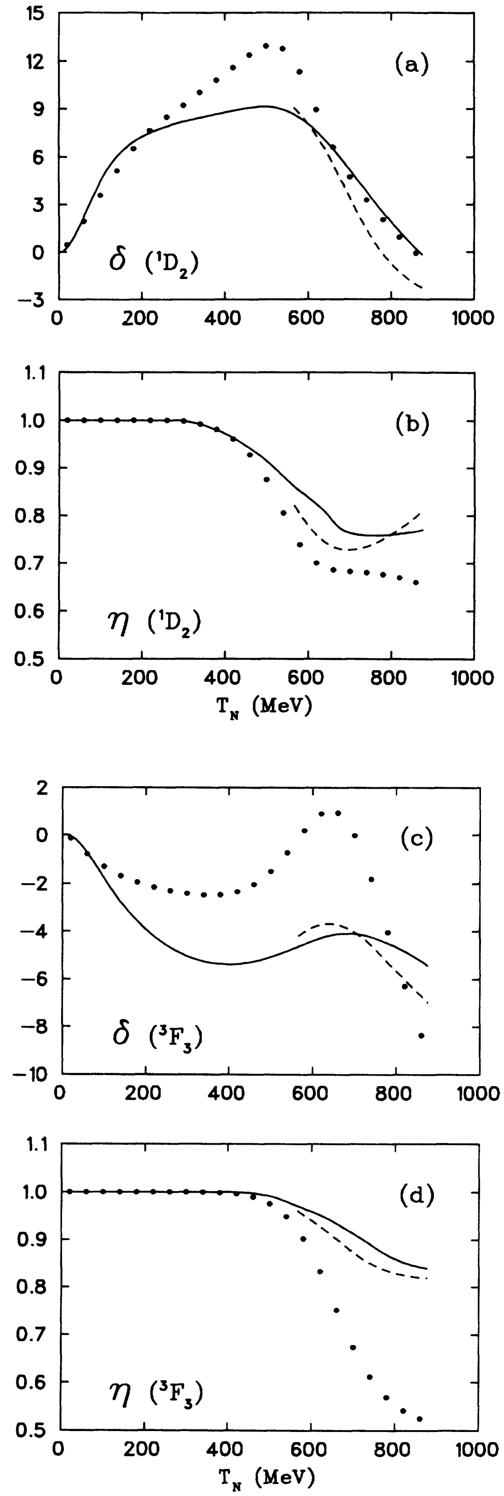

FIG. 14. The NN phase shifts δ and inelasticities η as a function of the nucleon lab kinetic energy. The solid curves are the result of our unitary πNN calculation with no short range ΔN interaction. Including the short range ΔN interaction (to all orders) results in the dashed curves—the $^5S_2$ interaction contributing to the $^1D_2$ NN channel, (a) and (b), and the $^5P_3$ interaction contributing to the $^3F_3$ NN channel, (c) and (d). The data are from Arndt *et al.*'s phase shift analysis program SAID (1988 version, Ref. 42).

C. $NN \to NN$

For NN elastic scattering we may take advantage of well established partial wave analyses in comparing our results with experiment. As we are interested to see the effect of the $^5S_2$ and $^5P_3$ ΔN interactions, we restrict the discussion to NN scattering in the $^1D_2$ and $^3F_3$ channels. In Fig. 14 we compare our phase shifts δ and inelasticities η with the results of the partial wave analysis of Arndt *et al.*[42] The standard few-body calculation, solid line, is only able to reproduce the gross features of the experimental results. At this stage it should be remembered that the present calculation generates the NN force purely through pion and nucleon exchanges. In particular we have not included the various heavy meson exchanges and therefore important contributions to the short and intermediate range NN interaction are missing from our calculation. It should also be noted that part of the inadequacy in the NN elastic channel, particularly the small inelasticities, originates in the missing strength in the $NN \to \pi d$ channel. This may, to some extent, be seen on the inclusion of the short range ΔN interactions. The results including both the $^5S_2$ and $^5P_3$ ΔN interactions are presented as the dashed curves in Fig. 14. Although the effects of this addition are small compared to the large discrepancy with the data, the sensitivity to the ΔN interaction is of similar magnitude to that seen in both $\pi d \to \pi d$ and $NN \to \pi d$. Although the inelasticities are mostly increased by the ΔN interactions, the $^5S_2$ ΔN interaction decreases the $^1D_2$ inelasticity above 800 MeV incident nucleon energy. This behavior may be a reflection of the energy dependence of the ΔN interaction seen in Fig. 8 for the $NN \to \pi d$ cross sections. This suggests a strongly coupled and significant role for the ΔN interaction in describing both $NN \to \pi d$ and NN elastic scattering.

IV. SUMMARY

We have investigated the effect of including a $^5S_2$ and $^5P_3$ ΔN interaction into a fully coupled few-body calculation of the reactions $\pi d \to \pi d$, $NN \to \pi d$, and $NN \to NN$. The ΔN interaction was taken to be separable and was constructed from the parametrization of Ferreira *et al.*[19] (FAD). In this sense our model endeavors to extend the work of FAD, who examined the effect of the lowest order ΔN interaction on the observables of πd scattering. In particular, we have examined the effect of including the ΔN interaction to all orders, and besides πd scattering, we have also investigated the effect of the same ΔN interaction on pion production and NN elastic scattering.

For πd elastic scattering, our expression for the lowest order ΔN contribution is given by Eq. (2.21) and agrees formally with the one of FAD. Moreover the F factor, which basically describes the $\pi d \to \Delta N$ amplitude, is numerically very similar in both models. However, despite similar input, our effective ΔN interaction differs significantly from the one of FAD. This is mainly due to the different ways the finite width of the Δ is taken into account in the two models. FAD parametrize the interaction between a stable Δ and a nucleon in terms of scattering $^5S_2$ and $^5P_3$ phase shifts. They then include the effects of an unstable Δ by allowing its mass to become complex in the kinematical relation of Eq. (3.2). In our model we use the FAD phase shifts to describe the interaction between a *bare* Δ and a nucleon. The effective ΔN interaction for a *physical* delta is then obtained by explicitly dressing the bare Δ with one pion loops.

For πd scattering where the ΔN interaction is added in Born term, this difference in the ΔN t matrices results in a different energy behavior for the ΔN effect in the two models. The magnitude of the effect, however, is similar. Both for πd differential cross sections, Fig. 6, and to a lesser extent for iT_{11}, Fig. 7, we find that the iteration of the ΔN interaction is important, particularly at energies away from about 217 MeV pion kinetic energy. It seems likely that an analysis, like the one of FAD, where one fits the ΔN interaction to observables of the πNN system, would yield significantly different results depending on whether the ΔN interaction is added in Born term or whether it is included to all orders.

For $NN \to \pi d$, we find that the effect of the ΔN interaction is particularly significant for the differential cross section, Fig. 8. Most standard few-body calculations have difficulty in describing this observable, and our results suggest that it might be important to include such a short range ΔN interaction in future calculations of pion production. Although the analyzing power A_{y0} is also sensitive to the ΔN interaction, other polarization observables appear to be less so. Thus the long standing problem with the correlation parameters A_{xx}, A_{yy}, and A_{zz} does not appear to be solvable by the introduction of a ΔN interaction, and some other mechanism needs to be sought.

The sensitivity of NN elastic scattering to the ΔN force was found to be small but also significant. However, in our calculation one needs to include major contributions, most likely heavy meson exchanges, before one can come closer to the results of experimental data.

Perhaps the most important conclusion of our study is that a ΔN interaction, if its magnitude is not too different from the one proposed by FAD, will significantly affect all the coupled processes of $\pi d \to \pi d$, $NN \to \pi d$, and $NN \to NN$. As mentioned in the introduction, there are a number of mechanisms, usually missing from current calculations, which can affect πNN observables, and which need to be carefully investigated before definite conclusions can be made about the true strength of the ΔN interaction. In this sense, the complete coupled description of πNN will be essential to discriminate among the possible additional mechanisms.

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