Asymmetry in inclusive polarized electron scattering from polarized nuclei: Sum rule approach

W. Leidemann, E. Lipparini, and S. Stringari
Dipartimento di Fisica and Istituto Nazionale di Fisica Nucleare, Gruppo Collegato di Trento, Università di Trento, 38050 Povo, Italy
(Received 29 January 1990)

The asymmetry of the inelastic cross section for the scattering of polarized electrons from polarized targets is investigated in the quasielastic region by using sum rule techniques. The sum rule method provides for the deuteron and ^{3}He an elegant and direct way to extract the dependence of the asymmetry on the neutron electric form factor. The nuclear structure ingredients entering the expressions for the asymmetry are the nonspherical components of the ground state wave functions and the structure functions. The sum rule predictions for the asymmetry have been compared with the results of microscopic calculations for the deuteron and ^{3}He. Application of the method to heavier nuclei shows that the asymmetry is particularly sensitive to core polarization effects.

I. INTRODUCTION

Inelastic electron scattering of polarized electrons from polarized targets is expected to provide new exciting insights on the structure of nuclei and nucleons.[1] In particular the possibility of measuring the neutron electric form factor through such reaction has recently stimulated several theoretical investigations[2-4] and proposals of experimental measurements in light nuclei (d, ^{3}He).[5]

A key quantity in the above context is given by the asymmetry of the inelastic cross section:

$$A=\frac{\left[\dfrac{d\sigma}{d\Omega}\right]_{fi}^{+}-\left[\dfrac{d\sigma}{d\Omega}\right]_{fi}^{-}}{\left[\dfrac{d\sigma}{d\Omega}\right]_{fi}^{+}+\left[\dfrac{d\sigma}{d\Omega}\right]_{fi}^{-}}\ , \tag{1}$$

where $(d\sigma/d\Omega)_{fi}^{\pm}$ is the cross section relative to the sign $\pm$ of the electron helicity. From an experimental point of view the asymmetry A is of particular interest since most classes of systematic errors, which limit the accuracy in measuring $d\sigma/d\Omega$ itself, cancel in taking the ratio.

In the present work we propose an investigation of the asymmetry for inclusive electron scattering based on the use of sum rules. The sum rule method has proven to be useful in the study of the symmetric part of the cross section in inclusive electron scattering.[6] Recently sum rules for the asymmetric components have also been explicitly investigated.[7] With the help of these sum rules we will define an average asymmetry, which allows us to study various effects without greater theoretical effort. In particular we study in a direct and attractive way the influence of the neutron electric form factor.

The paper is organized as follows. In Sec. II we develop the formalism introducing the sum rules for the asymmetric and symmetric part of the inelastic electron cross sections entering Eq. (1). In terms of these sum rules we define an average asymmetry $\overline{A}$.

In Sec. III we investigate the problem for the deuteron. In particular we compare the predictions of the sum rule approach for the asymmetry with the results of the microscopic calculations of Ref. 3.

In Sec. IV we focus on ^{3}He. In this case the sum rule method provides for the asymmetry an expression which generalizes the one for elastic scattering from polarized neutrons. The nuclear structure ingredients enter through the P_D and P_S' percentages in the wave function of the ground state and through the two-body structure function. Different choices for the neutron electric form factor are discussed and predictions for the asymmetry as a function of the momentum transfer q are given at various polarization angles.

In Sec. V we briefly discuss the behavior in heavier nuclei, with special emphasis to core polarization effects. In Sec. VI we draw our final conclusions.

II. ASYMMETRY AND SUM RULES

The cross section for the inclusive scattering of polarized electrons from polarized targets can be written as[1]

$$\left[\frac{d\sigma}{d\Omega}\right]_{fi}^{h}=\Sigma_{fi}+h\,\Delta_{fi}\ , \tag{2}$$

where h is the electron helicity. In terms of the nuclear response functions

$$R_{fi}^{L}=|\langle f|\rho(\mathbf{q})|i\rangle|^2\ , \tag{3a}$$

$$R_{fi}^{T}=|\langle f|j_+(\mathbf{q})|i\rangle|^2+|\langle f|j_-(\mathbf{q})|i\rangle|^2\ , \tag{3b}$$

$$R_{fi}^{TT}=2\,\mathrm{Re}[\langle i|j_+^\dagger(\mathbf{q})|f\rangle\langle f|j_-(\mathbf{q})|i\rangle]\ , \tag{3c}$$

$$R_{fi}^{TL}=-2\,\mathrm{Re}[\langle i|\rho^\dagger(\mathbf{q})|f\rangle(\langle f|j_+(\mathbf{q})-j_-(\mathbf{q})|i\rangle)]\ , \tag{3d}$$

$$R_{fi}^{T'}=|\langle f|j_+(\mathbf{q})|i\rangle|^2-|\langle f|j_-(\mathbf{q})|i\rangle|^2\ , \tag{3e}$$

$$R_{fi}^{TL'}=-2\,\mathrm{Re}[\langle i|\rho^\dagger(\mathbf{q})|f\rangle(\langle f|j_+(\mathbf{q})+j_-(\mathbf{q})|i\rangle)]\ , \tag{3f}$$

the quantities Σ_{fi} and Δ_{fi} can be written as

$$\Sigma_{fi}=\sigma_{\rm Mott} f_{\rm rec}^{-1}\{v_L R_{fi}^{L}+v_T R_{fi}^{T}+v_{TT} R_{fi}^{TT}+v_{TL} R_{fi}^{TL}\} , \tag{4}$$

and

$$\Delta_{fi}=\sigma_{\rm Mott} f_{\rm rec}^{-1}\{v_{T'} R_{fi}^{T'}+v_{TL'} R_{fi}^{TL'}\} , \tag{5}$$

respectively. In Eqs. (4) and (5) $\sigma_{\rm Mott}$ denotes the Mott cross section, $f_{\rm rec}^{-1}$ is the nuclear recoil correction, and

$$v_L=\left[\frac{Q^2}{q^2}\right]^2 , \tag{6a}$$

$$v_T=-\frac{1}{2}\frac{Q^2}{q^2}+\tan^2\frac{\theta}{2} , \tag{6b}$$

$$v_{TT}=\frac{1}{2}\frac{Q^2}{q^2} , \tag{6c}$$

$$v_{TL}=\frac{1}{\sqrt{2}}\frac{Q^2}{q^2}\left[-\frac{Q^2}{q^2}+\tan^2\frac{\theta}{2}\right]^{1/2} , \tag{6d}$$

$$v_{T'}=\tan\frac{\theta}{2}\left[-\frac{Q^2}{q^2}+\tan^2\frac{\theta}{2}\right]^{1/2} , \tag{6e}$$

$$v_{TL'}=\frac{1}{\sqrt{2}}\frac{Q^2}{q^2}\tan\frac{\theta}{2} , \tag{6f}$$

with $Q^2=\omega^2-q^2$. The quantities $\omega=\epsilon'-\epsilon$ and $\mathbf{q}=\mathbf{k}'-\mathbf{k}$ are the energy and momentum transferred by the electron, respectively, while θ is its scattering angle (see Fig. 1). In Eqs. (3) $\rho(\mathbf{q})$ and $j_\pm(\mathbf{q})$ are the Fourier transforms of the charge and current density operators, respectively, with

$$j_\pm=\mp\frac{1}{\sqrt{2}}(j_x\pm i j_y) .$$

The asymmetry of the cross section for the scattering of polarized electrons is defined by

$$A=\frac{\left[\dfrac{d\sigma}{d\Omega}\right]_{fi}^{+}-\left[\dfrac{d\sigma}{d\Omega}\right]_{fi}^{-}}{\left[\dfrac{d\sigma}{d\Omega}\right]_{fi}^{+}+\left[\dfrac{d\sigma}{d\Omega}\right]_{fi}^{-}}=\frac{\Delta_{fi}}{\Sigma_{fi}} . \tag{7}$$

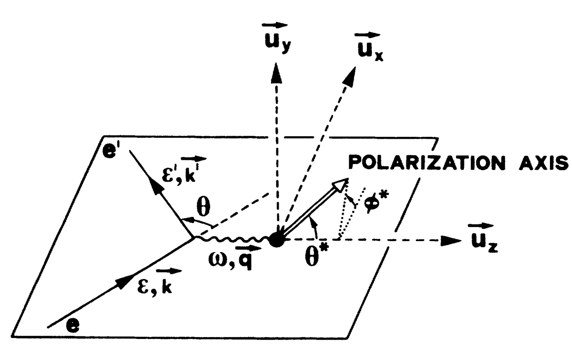

FIG. 1. Kinematics and coordinate systems for the scattering of polarized electrons from polarized nuclei.

In the following we employ sum rules techniques in order to evaluate the average asymmetry

$$\overline{A}=\frac{\sum_f \Delta_{fi}}{\sum_f \Sigma_{fi}} \tag{8}$$

giving the ratio between the energy integrated asymmetric and symmetric cross sections for a fixed momentum transfer. For a rather accurate experimental determination of $\overline{A}$ it should be sufficient to measure Δ_{fi} and Σ_{fi} in the quasielastic peak region since this region gives most of the contribution to the cross section. For the same reason the quantity $\overline{A}$ is expected to be close to the value of the asymmetry at the quasielastic peak.

Sum rules for the symmetric part Σ_{fi} have been well known for a long time,[6] whereas the ones for the asymmetric part Δ_{fi} have been only recently derived.[7] Since the kinematic factors (6) do not significantly vary with ω in the quasielastic peak region, they have only a minor influence on the energy dependence of A. For simplicity we will evaluate them at the peak. One then obtains the following result for $\overline{A}$:

$$\overline{A}=\frac{v_{T'}R_i^{T'}+v_{TL'}R_i^{TL'}}{v_L R_i^{L}+v_T R_i^{T}+v_{TT}R_i^{TT}+v_{TL}R_i^{TL}} , \tag{9}$$

where

$$R_i^L=\sum_f R_{fi}^L=\left\langle i\left|\sum_{k,l} e_k(q)e_l(q)\cos q(z_k-z_l)\right|i\right\rangle , \tag{10a}$$

$$R_i^T=\sum_f R_{fi}^T=\frac{q^2}{2m^2}\left\langle i\left|\sum_{k,l}\mu_k(q)\mu_l(q)\sigma_k^x\sigma_l^x\cos q(z_k-z_l)\right|i\right\rangle+\frac{2}{m^2}\left\langle i\left|\sum_{k,l}e_k(q)e_l(q)p_k^x p_l^x\cos q(z_k-z_l)\right|i\right\rangle , \tag{10b}$$

$$\begin{aligned}R_i^{TT}=\sum_f R_{fi}^{TT}=&-\frac{q^2}{2m^2}\left\langle i\left|\sum_{k,l}\mu_k(q)\mu_l(q)(\sigma_k^x\sigma_l^x-\sigma_k^y\sigma_l^y)\cos q(z_k-z_l)\right|i\right\rangle\\&-\frac{2}{m^2}\left\langle i\left|\sum_{k,l}e_k(q)e_l(q)(p_k^x p_l^x-p_k^y p_l^y)\cos q(z_k-z_l)\right|i\right\rangle ,\end{aligned} \tag{10c}$$

$$R_i^{TL}=\sum_f R_{fi}^{TL}=0 , \tag{10d}$$

$$R_i^{T'}=\sum_f R_{fi}^{T'}=-\frac{1}{\sqrt{2}}\frac{q^2}{m^2}\left\langle i\left|\sum_k \mu_k^2(q)\sigma_k^z\right|i\right\rangle , \tag{10e}$$

$$R_i^{TL'}=\sum_f R_{fi}^{TL'}=\sqrt{2}\frac{q}{m}\left\langle i\left|\sum_{k,l}e_k(q)\mu_l(q)\sigma_l^x\cos q(z_k-z_l)\right|i\right\rangle . \tag{10f}$$

Note that the sum $R_i^{TL}=\sum_f R_{fi}^{TL}$ identically vanishes due to parity invariance of the ground state. In the derivation of Eqs. (10) we have summed over all possible final states including the elastic contribution. At low momentum transfer such a contribution can become important and has to be subtracted explicitly from Eqs. (10).

In the calculation of the sum rules (10) we have assumed the following expressions for the density and current operators:

$$\rho(q)=\sum_{k=1}^{A} e_k(q)e^{iqz_k} , \tag{11a}$$

$$\mathbf{j}(q)=\sum_{k=1}^{A}\left[\frac{e_k(q)}{m}\frac{1}{2}(\mathbf{p}_k e^{iqz_k}\mathbf{p}+e^{iqz_k}\mathbf{p}_k) + i\frac{\mu_k(q)}{2m}\mathbf{q}\times\boldsymbol{\sigma}_k e^{iqz_k}\right] , \tag{11b}$$

which ignore subnuclear currents and relativistic effects. The role of meson exchange and isobar currents will be investigated in Sec. III and shown to be negligible. In Eqs. (10) and (11) $e(q)$ and $\mu(q)$ are the nucleon electric and magnetic form factors, respectively. The dependence of the form factors on the four momentum transfer cannot be easily taken into account by the sum rule approach. However, the form factors do not vary much in the peak region, which gives most of the contribution to the sum rules (10). This allows us to take their value at the peak.

The nuclear ground state $|i\rangle$ entering Eqs. (10) has quantum numbers J_i and $M_i(|i\rangle\equiv|J_i,M_i\rangle)$. If the target nucleus is polarized the magnetic subspaces M_i are populated in a nonuniform way with probability p_{M_i} and hence one has to consider the quantity

$$\bar{R}=\sum_{M_i} p_{M_i}R_i . \tag{12}$$

The operators of Eqs. (10) apply to a system where the axis of quantization is the z axis defined by the momentum transfer $\mathbf{q}$. However, the states $|J_iM_i\rangle$ are quantized with respect to the quantization axis specified by the angles θ^* and ϕ^* of Fig. 1. To evaluate Eqs. (10) one must consequently express the state $|J_iM_i\rangle$ in terms of the state vectors defined with respect to the z axis (see Ref. 1 for explicit details). In particular we note that Eq. (10e) is proportional to $\cos\theta^*$. Conversely, apart from minor corrections due to nonspherical components in the $k\neq l$ terms, Eq. (10f) is proportional to $\cos\phi^*\sin\theta^*$, while Eqs. (10a) and (10b) are independent of the polarization angles θ^* and ϕ^*.

III. DEUTERON

In the case of the deuteron, Eqs. (10) yield

$$\bar{R}^{L}=e_p^2(q)+e_n^2(q)+2e_p(q)e_n(q)T(q) , \tag{13a}$$

$$\bar{R}^{T}=\frac{q^2}{2m^2}\{\mu_p^2(q)+\mu_n^2(q)+2\mu_p(q)\mu_n(q)T(q) \times[\tfrac{1}{3}+\sqrt{2}p_2^d(\sin^2\theta^*\cos^2\theta^*-\tfrac{1}{3})]\} , \tag{13b}$$

$$\bar{R}^{T'}(q)=-\frac{1}{\sqrt{6}}\frac{q^2}{m^2}p_1^d\cos\theta^*[\mu_n^2(q)+\mu_p^2(q)](1-\tfrac{3}{2}P_D) , \tag{13c}$$

$$\bar{R}^{TL'}(q)=\frac{2}{\sqrt{3}}\frac{q}{m}p_1^d\cos\phi^*\sin\theta^* \times\{[e_p(q)\mu_p(q)+e_n(q)\mu_n(q)](1-\tfrac{3}{2}P_D) +[e_p(q)\mu_n(q)+e_n(q)\mu_P(q)]T(q)\} . \tag{13d}$$

In Eqs. (13) P_D is the D-state probability in the ground state,

$$p_1^d=\sqrt{3/2}(p_1-p_{-1}) ,$$

$$p_2^d=\frac{1}{\sqrt{2}}(p_1+p_{-1}-2p_0),$$

and

$$T(q)=\int d\mathbf{s}\rho^{(2)}(s)\cos\mathbf{q}\cdot\mathbf{s} ,$$

where $\rho^{(2)}(s)$ denotes the two-body density matrix $[\int d\mathbf{s}\rho^{(2)}(s)=1]$. In the calculation of the off-diagonal terms ($k\neq l$) of Eqs. (10b) and (10f) we have neglected the D-state contribution and, furthermore, ignored the last term of Eq. (10d), which is negligibly small. For the same reasons the contribution of the sum rule R^{TT} has been ignored. The results reveal that the sum rules $R^{T'}$, $R^{TL'}$, R^{L}, and R^{T} depend very weakly on the nuclear structure ingredients since P_D is small and $T(q)$ quickly decreases with q. As a consequence, in the momentum range explored in the present work, the above sum rules are mostly determined by the q dependence of the nucleon form factors.

In Fig. 2 we report the resulting predictions for the asymmetry $\bar{A}$ at two different polarization angles θ^*, employing the dipole model

$$e_n(q)=-\frac{\tau}{1+p\tau}\mu_n(q)$$

with $\tau=-Q^2/4m^2$ and $p=5.6$ (Ref. 8) for the neutron electric form factor. The variable $q_{\text{c.m.}}$ is the momentum transfer in the c.m. system. The dotted curve gives $\bar{A}$

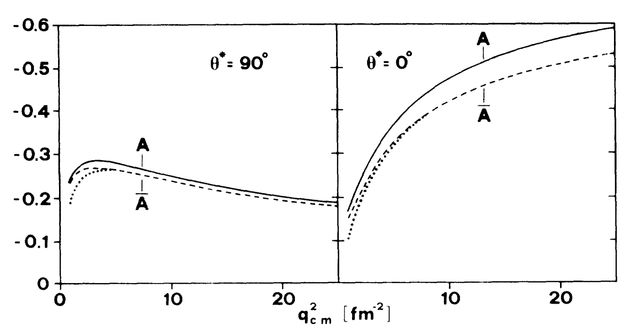

FIG. 2. The asymmetries A from Ref. 3 (full curves) in comparison to $\bar{A}$ with (dashed curves) and without (dotted curves) subtraction of the elastic contribution for $\vec{d}(\vec{e},e')np$ at $\theta^*=90°$ (left) and 0° (right) (quasifree kinematics with $\theta=60°$, $\phi^*=0°$, $p_1^d=1$, $p_2^d=0$). For $e_n(q)$ the dipole fit (Ref. 8) is used.

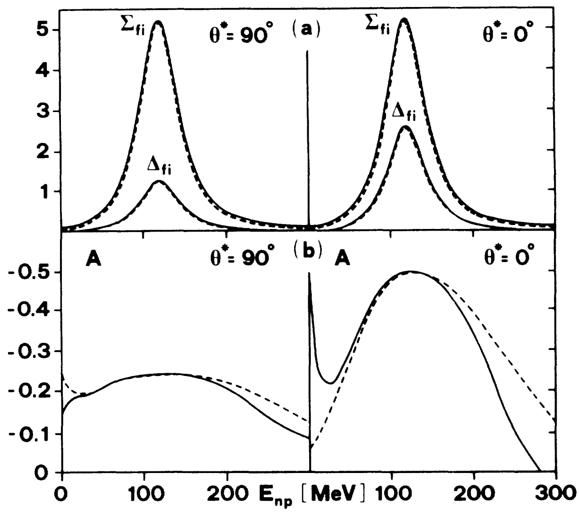

FIG. 3. (a) Σ_{fi} and Δ_{fi} from Ref. 3 in units of $10^{-4}\mu$b/sr MeV for $\vec{d}(\vec{e},e')np$ at $q^2_{\rm c.m.}=12$ fm^{-2} ($\theta=60°$) with (full curves) and without (dashed curves) MEC and IC contribution at $\theta^*=90°$ (left) and 0° (right) [further target polarization variables and $e_n(q)$ as in Fig. 2]. The lower part (b) shows the corresponding asymmetries.

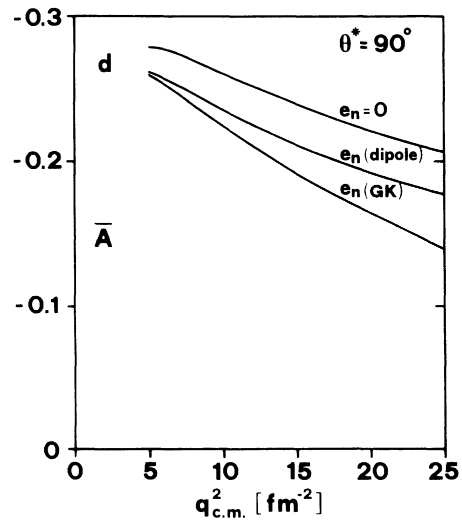

FIG. 4. The asymmetry $\bar{A}$ for $\vec{d}(\vec{e},e')np$ at $\theta^*=90°$ with various neutron electric form factors: $e_n(q)=0$, dipole fit (Ref. 8), and GK fit (Ref. 9) (kinematics and further target polarization variables as in Fig. 2).

without subtraction of the elastic contribution. It is evident that such a contribution is only important below 5 fm^{-2}. In the same figure we report the predictions for the asymmetry A at the quasielastic peak from Ref. 3. The calculation of Ref. 3 is based on a nonrelativistic description of the n-p system using a realistic NN potential and including subnuclear degrees of freedom via meson exchange currents (MEC) and isobar configurations (IC). Here we show the results obtained with the Paris potential. The figure shows that A and $\bar{A}$ differ in the whole momentum range by only 5% at $\theta^*=90°$ and by about 10% at $\theta^*=0°$.

In order to study the effect of MEC and IC and to understand better the similarities of A and $\bar{A}$ we show in Fig. 3 the results of Ref. 3 for Σ_{fi}, Δ_{fi}, and A at $q^2_{\rm c.m.}=12{\rm fm}^{-2}$. The angles θ^* and ϕ^* have been chosen in a way that once only $R^{T'}_{fi}(\theta^*=0°,\phi^*=0°)$ and once only $R^{TL'}_{fi}(\theta^*=90°,\phi^*=0°)$ is contributing to Δ_{fi}. It is readily seen in Fig. 3(a) that both cross sections are peaked in the quasielastic region. Furthermore, the influence of MEC and IC is not only small in the peak region but also of minor importance for the integrated cross sections $\sum_f\Sigma_{fi}$ and $\sum_f\Delta_{fi}$. Coming to the asymmetries [Fig. 3(b)], one sees that they are rather different for the two values of θ^*. In particular at 90° A is only weakly dependent on the relative kinetic energy E_{np} of the outgoing n-p pair and consequently turns out to be rather close to the average asymmetry $\bar{A}$.

Figure 4 shows the effect of the neutron electric form factor on $\bar{A}$. Besides the use of the dipole model,[8] we take the recent Gari-Krümpelmann fit[9] (GK) and consider also the case $e_n(q)=0$. In spite of the inclusive nature of the reaction the influence of the neutron electric form factor on the average asymmetry $\bar{A}$ is rather strong, e.g., at $q^2_{\rm c.m.}=15{\rm fm}^{-2}[e_n(q)=0.050$ (dipole) and 0.087 (GK)] $\bar{A}$ is reduced by 12% (dipole) and 20% (GK) with respect to the choice $e_n(q)=0$.

IV. ^{3}He

The ^{3}He nucleus is of special interest because the asymmetry A is expected to be dominated by the neutron.[4] One consequently hopes to get unique information on the neutron form factors. In the case of ^{3}He the dynamic structure function cannot yet be calculated with the same accuracy as in the case of the deuteron. Therefore the sum rule evaluation of the average asymmetry $\bar{A}$, which only requires the evaluation of a few sum rules, is of particular relevance. For ^{3}He the sum rules [Eqs. (10) and (12)] become

$$\bar{R}^L=2e_p^2(q)+e_n^2(q)+2[e_p^2(q)+2e_p(q)e_n(q)]T(q)\ , \tag{14a}$$

$$\bar{R}^T=\frac{q^2}{2m^2}[2\mu_p^2(q)+\mu_n^2(q)-2\mu_p^2(q)T(q)]\ , \tag{14b}$$

$$\bar{R}^{T'}(q)=-\frac{q^2}{2m^2}p_1\cos\theta^*\{\mu_n^2(q)-\tfrac{2}{3}P_D[\mu_p^2(q)+2\mu_n^2(q)]+\tfrac{2}{3}P_{S'}[\mu_p^2(q)-\mu_n^2(q)]\} \tag{14c}$$

$$\bar{R}^{TL'}(q)=\sqrt{2}\frac{q}{m}p_1\cos\phi^*\sin\theta^*\{e_n(q)\mu_n(q)-\tfrac{2}{3}P_D[e_p(q)\mu_p(q)+2e_n(q)\mu_n(q)] +\tfrac{2}{3}P_{S'}[e_p(q)\mu_p(q)-e_n(q)\mu_n(q)]+2e_p(q)\mu_n(q)T(q)\}\ , \tag{14d}$$

where $T(q)=\int ds\rho_S^{(2)}(s)\cos\mathbf{q}\cdot\mathbf{s}$ is the Fourier transform of the two-body density matrix calculated ignoring S' and D components $[T(0)\simeq 1]$ and $p_1=p_{1/2}-p_{1/2}$. In deriving Eqs. (14) we have neglected possible P components in the ground state wave function. Furthermore, in the calculation of the nondiagonal terms of Eqs. (10b)–(10f) we have included only the S wave component of the ground state and, finally, we did not consider the last term of Eq. (10b) and the contribution of the sum rule R^{TT} which is vanishingly small. The nuclear structure effects in the sum rules (14) enter through the P_D and $P_{S'}$ percentages and through the structure function $T(q)$. At not too low momentum transfers such effects only lead to small corrections of the sum rules which are mainly determined by the nucleon form factors. In particular the same rules $R^{T'}$ and $R^{TL'}$ are dominated by the neutron contribution and therefore the resulting average asymmetry $\overline{A}$ [Eqs. (9)] generalizes the expression for the asymmetry of the elastic polarized electron neutron scattering cross section

$$A=p_1\frac{-v_{T'}\frac{q^2}{2m}\mu_n^2(q)\cos\theta^*+v_{TL'}\sqrt{2}\frac{q}{m}e_n(q)\mu_n(q)\sin\theta^*\cos\phi^*}{v_L e_n^2(q)+v_T\frac{q^2}{2m^2}\mu_n^2(q)} \; . \tag{15}$$

Looking at Eqs. (9), (14), and (15) one readily sees that due to the presence of the proton contribution the asymmetry for ^{3}He is smaller than the one of the neutron.

In Fig. 5 we show our results for the asymmetry $\overline{A}$. For P_D and P_S' we have used the values reported in Ref. 4, while for $T(q)$ we take the results of Ref. 10 (see Fig. 6). We have considered two choices for $e_n(q)$, i.e., dipole model of Sec. III with $p=1$, which is rather similar to the GK fit, and the case $e_n(q)=0$. Again quasifree kinematics are chosen with $Q^2=-31.8$ fm^{-2} and $\theta=60°$. Note that β is the angle between the directions of incoming electron beam and target polarization ($\theta^*\simeq\beta+30°$ for the kinematics above). In the same figure we plot the predictions for A at the quasielastic peak given in Ref. 4. Similarly to the deuteron we find that A and $\overline{A}$ differ by small amounts. Figure 7 compares A and $\overline{A}$ in the quasifree region varying the momentum transfer and keeping the electron angle fixed ($\theta=60°$). Both asymmetries behave in a similar way.

Figure 8 shows the influence of e_n on $\overline{A}$ at the polarization angle $\theta^*=90°$, where its effect is maximal, and taking the same form factor models as for the deuteron case in Fig. 4. With a vanishing e_n the asymmetry is almost 0. Such a result shows that the two protons of ^{3}He have only a minor influence on $\overline{A}$ in the quasielastic peak region. On the contrary the asymmetry depends very much on the neutron electric form factor. With the e_n of the dipole model $\overline{A}$ is increased by about a factor of 4 between 10 and 20 fm^{-2}. Of course, the effect becomes more pronounced if one takes the even stronger e_n of the GK fit.

V. HEAVIER NUCLEI

In this section we discuss some implications of our results (10) on the asymmetry of the inclusive electron scattering cross section in heavier nuclei. In particular we focus on the magnetic effects due to the core polarization. We consider odd nuclei with an $N=Z$ core. Typical examples are ^{7}Li, ^{9}Be, ^{11}B, ^{11}C, etc. The magnetic moments of such nuclei deviate from the Schmidt value by an amount which can be qualitative explained in terms of the polarization of the nucleons in the $1p_{3/2}$ shell. The

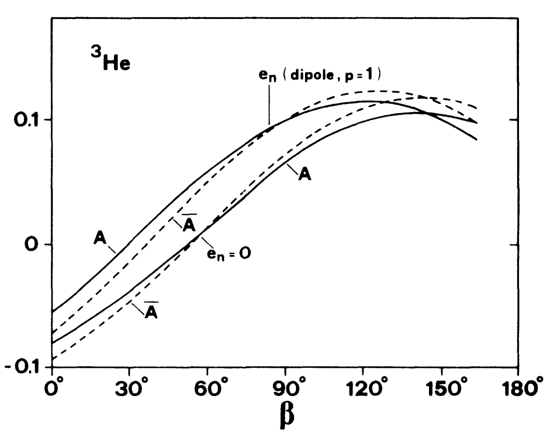

FIG. 5. The asymmetries A from Ref. 4 (full curves) and $\overline{A}$ (dashed curves) for $^3\vec{\mathrm{He}}(\vec{e},e')X$ with $e_n(q)=0$ and $e_n(q)$ of dipole fit (Ref. 8) with $p=1$ at $-Q^2=31.8$ fm^{-2} (quasifree kinematics with $\theta=60°$, $\phi^*=0°$, $p_1=1$).

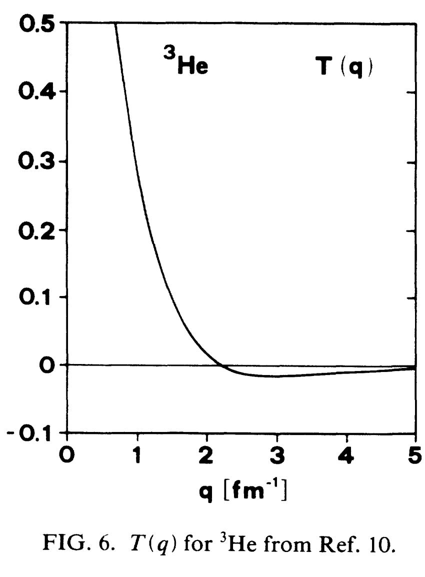

FIG. 6. $T(q)$ for ^{3}He from Ref. 10.

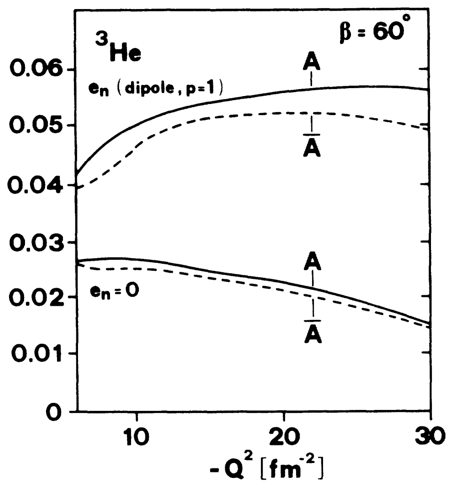

FIG. 7. As in Fig. 5 but β fixed to 60° while $-Q^2$ varies from 6 to 30 fm^{-2}.

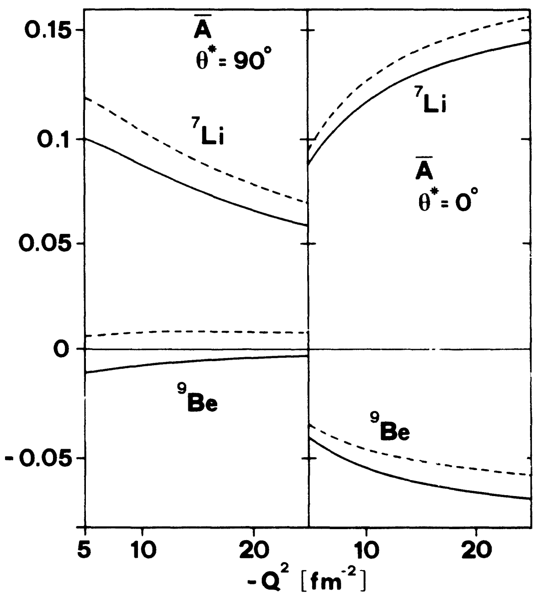

FIG. 9. The asymmetry $\bar{A}$ for $^7\vec{\mathrm{Li}}(\vec{e},e')X$ and $^9\vec{\mathrm{Be}}(\vec{e},e')X$ at $\theta^*=90°$ (left) and 0° (right) with (full curves) and without (dashed curves) core polarization effects. For $e_n(q)$ the dipole fit (Ref. 8) is used (kinematics and further target polarization variables as in Fig. 8).

same polarization effects affect the evaluation of the sum rules (10e) and (10f), which determine the asymmetry of the cross section. Assuming that the polarization effects are of isovector nature (this is justified by the weakness of the nuclear interaction in the isoscalar spin-spin channel[11,12] we get the following expression for $R^{T'}$ and $R^{TL'}$:

$$\bar{R}^{T'}=-\frac{q^2}{2m^2}p\cos\theta^*\{\mu_{\rm ext}^2(q)\langle\sigma\rangle+\chi_{\sigma\tau}[\mu_n^2(q)-\mu_p^2(q)]\}\ , \tag{16}$$

$$\bar{R}^{TL'}=\sqrt{2}\frac{q}{m}p\cos\phi^*\sin\theta^*\{e_{\rm ext}(q)\mu_{\rm ext}(q)\langle\sigma\rangle+\chi_{\sigma\tau}[e_n(q)\mu_n(q)-e_p(q)\mu_p(q)]\}\ , \tag{17}$$

where $e_{\rm ext}$ and $\mu_{\rm ext}$ are the nucleon electric and magnetic form factors of the valence nucleons, respectively, and $\langle\sigma\rangle$ is equal to 1 if the valence nucleon occupies a $J=l+\frac{1}{2}$ state and equal to $-J/(J+1)$ if the valence nucleon occupies a $J=l-\frac{1}{2}$ state. In Eqs. (16) and (17)

$$p=\frac{1}{J}\sum_i M_i p_{M_i}$$

gives the polarization of the system, while

$$\chi_{\sigma\tau}=\frac{\delta\mu_{\sigma\tau}}{\mu_n(0)-\mu_p(0)+\frac{1}{2}}$$

is the contribution of the core polarization to the matrix element $\langle\sum_i\sigma_i^z\tau_i^z\rangle$. The quantity $\delta\mu_{\sigma\tau}$ usually leads to the main correction to the Schmidt value for the magnetic moments. In Eq. (16) we have neglected the contributions due to the $k\neq l$ terms of the matrix element (10f).

Equation (16) shows that in nuclei where the valence particle is a neutron (e.g., ^{9}Be) the core polarization gives an important relative contribution to $\bar{R}^{TL'}$ and hence to the asymmetry $\bar{A}$. Such a contribution is much more important than the corresponding contribution to the static value of the magnetic moment. The above behavior is well illustrated in Fig. 9 where we plot the asymmetry $\bar{A}$ for ^{9}Be and ^{7}Li with and without core polarization effects. In this calculation the dipole neutron form factor of Ref. 8 has been used. Note that in the case of ^{9}Be the results of Fig. 9 depend sizably on the choice for the neutron electric form factor. The quantity $\delta\mu_{\sigma\tau}$ has been extracted from the experimental values for the magnetic moments ($\delta\mu_{\sigma\tau}\simeq\mu_{\rm exp}-\mu_{\rm Schmidt}=0.7$ in ^{9}Be and -0.6 in

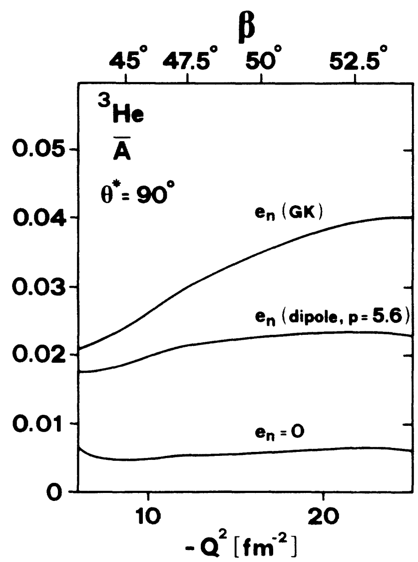

FIG. 8. The asymmetry $\bar{A}$ for $^3\vec{\mathrm{He}}(\vec{e},e')X$ at $\theta^*=90°$ (corresponding β is given on upper scale) with the various $e_n(q)$ of Fig. 4 (quasifree kinematics with $\theta=60°$, further target polarization variables as in Fig. 5).

^{7}Li). While in ^{7}Li the relative effects of core polarization are similar to the ones for the magnetic moments, in ^{9}Be at $\theta^*=90°$ the relative effect of core polarization is significantly enhanced as a consequence of the fact that the "Schmidt" contribution is quenched by the neutron electric form factor.

VI. CONCLUSIONS

In this work we have used sum rules for the asymmetric and symmetric parts of the inclusive electron scattering cross section to calculate the asymmetry in polarized nuclei due to the helicity of the electron. The main results emerging from our analysis are the following.

(1) The average asymmetry $\overline{A}$ is rather similar to the asymmetry A at the quasielastic peak. Therefore the sum rule method provides an elegant and practical generalization of the expression for the asymmetry of elastic electron nucleon scattering. In particular it reveals in an explicit way the dependence of the asymmetry on the neutron electric form factor. Such a dependence is, as expected, particularly important for ^{3}He. The nuclear structure ingredients entering our expressions for the asymmetry are the nonspherical components of the ground state wave function and the structure function.

(2) Meson exchange and isobar currents affect in a minor way the asymmetry in the quasielastic region. This has been explicitly shown in the case of the deuteron.

(3) In heavier nuclei the asymmetry of the cross section is particularly sensitive to core polarization effects.

[1]T. W. Donnelly and A. S. Raskin, Ann. Phys. (N.Y.) **169**, 247 (1986).
[2]C. Y. Cheung and R. M. Woloshyn, Phys. Lett **127B**, 147 (1983).
[3]H. Arenhövel, W. Leidemann, and E. L. Tomusiak, Z. Phys. A **331**, 123 (1988); *ibid.* **334**, 363(E) (1989) and paper in preparation.
[4]B. Blankleider and R. M.Woloshyn, Phys. Rev. C **29**, 538 (1983).
[5]R. Madey *et al.* (unpublished); E. W. Otten and H. G. Andresen (unpublished).
[6]K. W. MacVoy and L. Van Hove, Phys. Rev. **125**, 1034 (1962); F. Dellagiacoma, R. Ferrari, G. Orlandini, and M. Traini, Phys. Rev. C **29**, 777 (1984), and references therein.
[7]E. Lipparini and S. Stringari, Phys. Rev. C **40**, R19 (1989).
[8]S. Galster, H. Klein, J. Moritz, K. H. Schmidt, D. Wegener, and J. Bleckwenn, Nucl. Phys. **B32**, 221 (1971).
[9]M. Gari and W. Krümpelmann, Z. Phys. A **322**, 689 (1985).
[10]R. Schiavilla, D. S. Lewart, V. R. Pandharipande, S. C. Pieper, R. B. Wiringa, and S. Fantoni, Nucl. Phys. **A473**, 267 (1987); R. Schiavilla, private communication.
[11]E. Lipparini and A. Richter, Phys. Lett. **114B**, 13 (1984).
[12]H. Toki, G. Bertsch, and D. Cha, Phys. Rev. C **28**, 1398 (1983).