Test of distorted wave kinematic coupling approximation calculations for knockout reactions

Arun K. Jain
Nuclear Physics Division, Bhabha Atomic Research Centre, Bombay 400 085, India
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A test has been devised to check the validity of conventional distorted-wave impulse approximation (DWIA) treatment of knockout reactions. The conventional DWIA formalism separates the three-body final state Schrödinger equation for a knockout reaction into two two-body Schrödinger equations by assuming an asymptotic constant value for the three-body coupling term commonly known as the kinematic coupling approximation (KCA). In the test case, which consists of an extreme asymmetric situation where one of the distorting optical potentials is assumed to vanish, the three-body final state Schrödinger equation can be solved exactly as a product of two two-body solutions using one particular set of relative coordinates. Large influence of the three-body coupling term is seen in the comparison of the exact and KCA results for $(\alpha,2\alpha)$ and $(p,p\alpha)$ knockout reactions when the distorting optical potentials are weakly absorbing.

I. INTRODUCTION

In the recent past increasing evidence has been found for the failure of the conventional distorted-wave impulse approximation (DWIA) calculations for the knockout reactions in reproducing the experimental data.[1-7] As the inputs to these DWIA calculations are generally reasonably well defined it appears that the reason for this failure lies in some simple-minded approximations made within the framework of the DWIA formalism.[1,8-10] Although many of the approximations are empirically justified[1,10,11] it has been found that the approximation leading to the decoupling of the three-body final state into two two-body systems has never been tested properly.[8,10,12] It is the aim of the present investigation to shed some light on this aspect. Some discussion can be found on this three-body final state problem in the form of a diproton model (DPM (Ref. 13) or in the comparison of results from the kinematic coupling approximation (KCA) and the potential coupling approximation (PCA),[10,14] however, there exists no estimate of the amount of discrepancy due to these approximations with respect to some exact results. As the validity and the applicability of the diproton model are very much restricted,[13,15] generally the calculations employ the kinematic coupling approximation,[1-10] or in some cases even the potential coupling approximation has been employed.[10,14]

II. FORMALISM

The final state Hamiltonian H_f for a knockout reaction $A(a,ab)B$ having three free particles in the final state is generally taken as[1,8]

$$H_f=\mathcal{H}_a+\mathcal{H}_b+\mathcal{H}_B+T_a+T_b+T_B + V_{aB}(r_{aB})+V_{bB}(r_{bB}) , \quad (1)$$

where T's and $\mathcal{H}$'s represent the kinetic energy operator and internal Hamiltonian, respectively, for a, b, and B, and $V_{xy}(r_{xy})$ is the interaction between x and y which is commonly replaced by the corresponding optical potential. The interaction V_{ab} is supposed to be taken care of by the impulse approximation.[8,9]

In terms of relative coordinates $\mathbf{r}_{aB}$ and $\mathbf{r}_{bB}$ (see Fig. 1) the final state Hamiltonian H_f can be written as[1,8,10]

$$H_f=\mathcal{H}_a+\mathcal{H}_b+\mathcal{H}_B+T_{aB}+T_{bB} + V_{aB}(r_{aB})+V_{bB}(r_{bB})-\frac{\hbar^2}{m_B}\nabla_{aB}\cdot\nabla_{bB} . \quad (2)$$

Here the term $-\hbar^2/m_B\nabla_{aB}\cdot\nabla_{bB}$ couples the relative motion of the three-particles in the final state. When this coupling term is replaced by $\mathbf{k}_{aB}\cdot\mathbf{k}_{bB}/m_B$ (where $\mathbf{k}_{aB}$ and $\mathbf{k}_{bB}$ are the asymptotic momentum vectors conjugate to the relative coordinates $\mathbf{r}_{aB}$ and $\mathbf{r}_{bB}$, respectively) it amounts to the kinematic coupling approximation. The same final state Hamiltonian H_f can be written in terms of another set of relative coordinates $\mathbf{r}_{aA}$ and $\mathbf{r}_{bB}$ (see Fig. 1) as[10]

$$H_f=\mathcal{H}_a+\mathcal{H}_b+\mathcal{H}_B+T_{aA}+T_{bB} + V_{aB}(r_{aB})+V_{bB}(r_{bB}) . \quad (3)$$

It is to be remarked that the coupling shows up here through the interaction $V_{aB}(r_{aB})$ which is a function of r_{aB} and not of r_{aA}. When $V_{aB}(r_{aB})$, where

$$\mathbf{r}_{aB}=\left[\frac{m_A}{m_B}\mathbf{r}_{aA}-\frac{m_b}{m_B}\mathbf{r}_{ab}\right]$$

is approximated by $V_{aB}[(m_A/m_B)r_{aA}]$, it amounts to the potential coupling approximation.[10]

Although the representation Eq. (2) of the three-body final state in terms of $(\mathbf{r}_{aB},\mathbf{r}_{bB})$ coordinates is symmetric, it is seen that in a situation where the interaction $V_{aB}(r_{aB})$ is weakly distorting the coupling still shows up in Eq. (2) through the $\nabla_{aB}\cdot\nabla_{bB}$ term while the effect of the coupling decreases with the weakening of the interaction $V_{aB}(r_{aB})$ in Eq. (3) for the potential coupling formal-

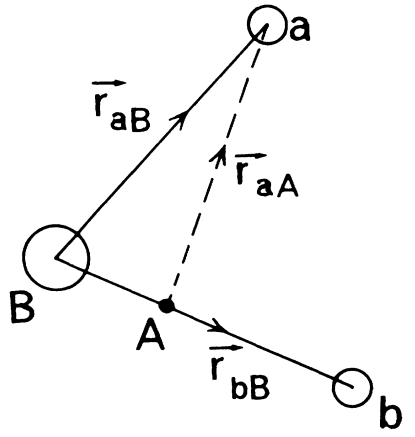

FIG. 1. Relative coordinates r_{aB} and r_{bB} used for kinematic coupling formalism and r_{aA} and r_{bB} used for potential coupling formalism of the three-body final state treatment of the $A(a,ab)B$ reaction.

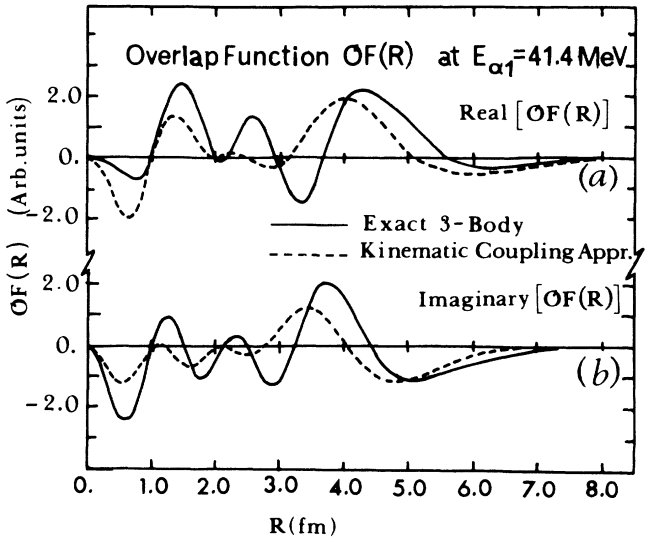

FIG. 2. Overlap function $\mathcal{O}F(R)$ vs intercluster (α-^{12}C) separation; R for 90 MeV kinematics of $^{16}O(\alpha,2\alpha)^{12}C_{g.s.}$ reaction for zero recoil momentum position, $E_{\alpha_1}=41.423$ MeV using α-^{12}C optical potential only in the final state and no imaginary parts of the optical potentials. (——) exact three-body calculations and (— — —) kinematic coupling approximation calculations. (a) Real part and (b) imaginary part.

ism. It reminds us of the situation existing in the $A(e,e'p)B$ and $A(p,p\alpha)B$ reactions where the distortions are comparatively weak in the $e-B$ and $p-B$ channels, respectively. In the ideal situation where the interaction $V_{aB}(r_{aB})$ is not present at all, the solution for the final state Hamiltonian H_f in the form of Eq. (3) will be exact, while in the form of Eq. (2) it will have the approximation associated with the replacement of the coupling term $-(\hbar^2/m_B)\nabla_{aB}\cdot\nabla_{bB}$ by its asymptotic value. We thus arrive at a test case where one can estimate the amount of error involved in making the kinematic coupling approximation in a situation where one of the optical potentials [say $V_{aB}(r_{aB})$] is absent in the final state. Using these final state solutions in the evaluation of the transition amplitudes and overlap functions, DWIA calculations have been performed for some representative knockout reactions, and these results are discussed in the following section.

III. RESULTS AND DISCUSSION

Calculations have been performed for the kinematics of the 90 MeV $^{16}O(\alpha,2\alpha)^{12}C_{g.s.}$ reaction[16] and the 101.5 MeV $^{16}O(p,p\alpha)^{12}C$ reaction.[6] One of the optical potentials, $\alpha_1-^{12}C$ for the $(\alpha,2\alpha)$ reaction and $p-^{12}C$ for the $(p,p\alpha)$ reaction, has been equated to be zero in the final state three-body Schrödinger equation. In order to study the region of localization for the influence of the coupling term, all the imaginary components of the optical potentials are also neglected. For the case of $(\alpha,2\alpha)$ kinematics the results of the overlap function $\mathcal{O}F(R)$ [where $\mathcal{O}F(R)$ is the product of the bound intercluster wave function, $u(R)$ and the three distorted waves integrated over all the coordinates except the intercluster separation $-R$] are presented in Fig. 2 for the zero recoil momentum condition. It is observed in this figure that both the real and imaginary parts of the $\mathcal{O}F(R)$ differ widely for the KCA and the exact three-body treatment of the final state. It is particularly interesting to note that the contributions to $\mathcal{O}F(R)$ from the surface region ($R\approx 2-3$ fm) are invariably suppressed [both real and imaginary parts of $\mathcal{O}F(R)$]. Similar suppression of the surface contributions is witnessed in the $\mathcal{O}F(R)$ at other recoil momentum points; Fig. 3 shows this behavior for recoil momentum, $p_B=95$ MeV/c corresponding to $E_{\alpha_1}=51.5$ MeV. The components comprising the $\mathcal{O}F(R)$, i.e., the bound wave function $u(R)$ and the product of the three distorted waves, $\chi(R)$, shown in Fig. 4, corroborate this suppression of the surface contributions due to the neglect of the three-body contributions.

Energy sharing distributions for the 90 MeV $^{16}O(\alpha,2\alpha)^{12}C_{g.s.}$ reaction kinematics for the exact and KCA treatment of the three-body final state are presented in Fig. 5. The main impact of the correct treatment of the three-body coupling term as seen in Fig. 5 is to enhance the cross section by about one order of magnitude near the zero recoil momentum position, $E_{\alpha_1}=41.423$ MeV. Besides this a comparison of the exact and KCA results in this figure indicates that the coupling increases the cross section faster with the increase of E_{α_1} (where E_{α_1} is the energy of the particle wave which is not distorted by the optical potential in the final state). It is interesting to note that a similar behavior is required to improve the DWIA fits to the $(p,p\alpha)$ reaction data. Energy sharing distributions have also been calculated for the 101.5 MeV $^{16}O(p,p\alpha)^{12}C_{g.s.}$ kinematics[6] using the KCA and the exact three-body treatment of the final state and are presented in Fig. 6. All the imaginary components of the optical potentials have been neglected for these calculations also. As the optical potential for the $p-^{12}C$ channel is weakly distorting, a comparison between our model calculations and experimental data[6]

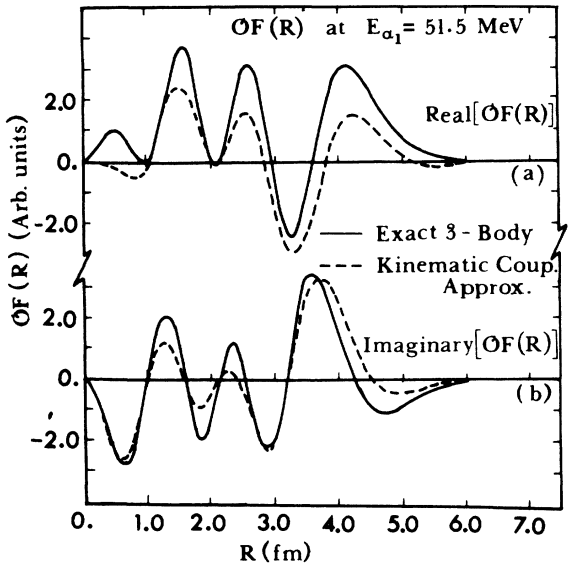

FIG. 3. (a) and (b) Same as Fig. 2 but for $E_{\alpha_1}=51.5$ MeV corresponding to 95 MeV/c recoil momentum.

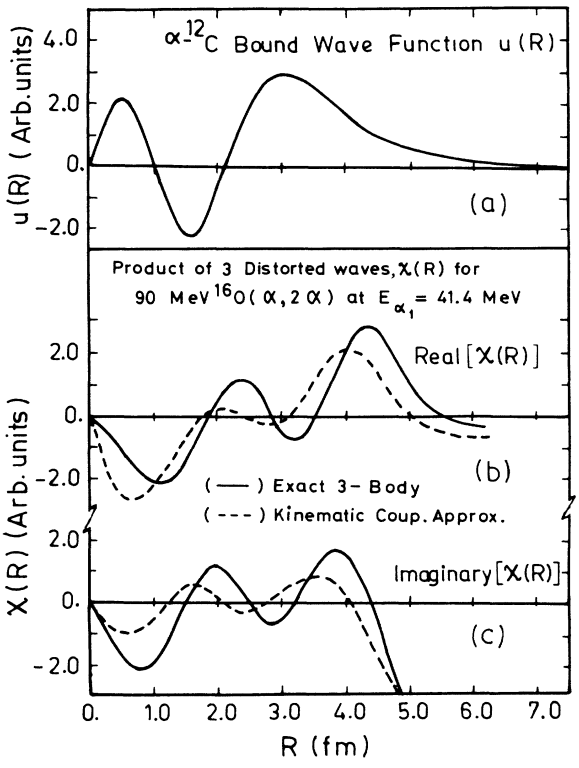

FIG. 4. (a) Intercluster wave function $u(R)$ for α-^{12}C of ^{16}O using Saxon-Woods well radius parameter $r_0=1.09$ fm. (b) Product of three distorted waves, $\chi(R)$ for the test case of 90 MeV ^{16}O(α,2α) at $E_{\alpha_1}=41.423$ MeV real part, (——) exact three-body, and (— — —) KCA. (c) Same as (b) but imaginary part.

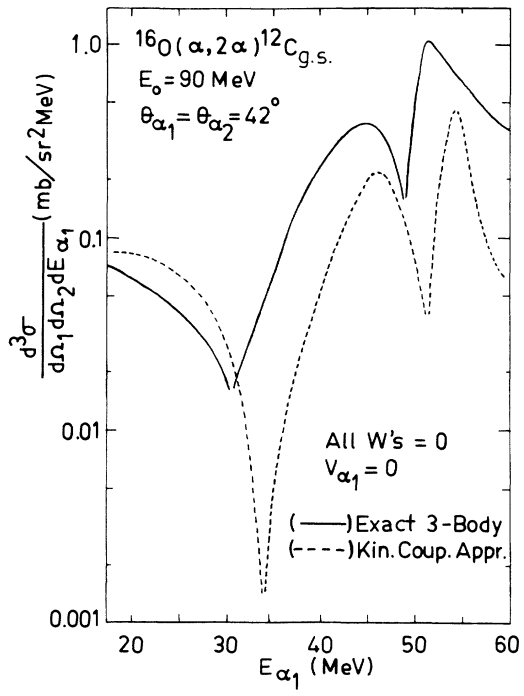

FIG. 5. Calculated energy sharing distributions for 90 MeV ^{16}O(α,2α) reaction kinematics for the test case when the V_{α_1}-^{12}C optical potential is taken as zero for the final state and all imaginary potentials are neglected; (——) exact three-body coupling, and (— — —) conventional kinematic coupling approximation.

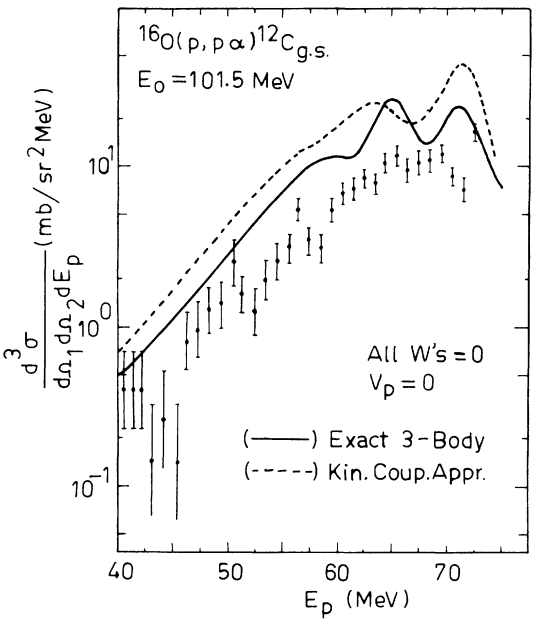

FIG. 6. Comparison of the 101.5 MeV ^{16}O(p,$p\alpha$)^{12}C$_{g.s.}$ reaction results for the test case (final state p-^{12}C optical potential zero as also all the imaginary potentials are neglected) for the exact three-body coupling (——) and conventional kinematic coupling approximation (— — —), and the experimental data.

may not be very unrealistic for this case. It is very encouraging to see that the exact three-body calculations without the imaginary components of the optical potentials are in very good agreement with the experimental data. Even the structures around the zero recoil momentum position ($E_p \cong 65$ MeV) are nicely reproduced by the exact three-body final state treatment of this reaction.

For the 101.5 MeV $^{16}O(p,p\alpha)^{12}C_{g.s.}$ reaction the absolute cross section at the zero recoil momentum position ($E_p \cong 65$ MeV) calculated with the exact final state treatment in the DWIA is 26 μb/sr^2 MeV as compared to the experimental value of 12 μb/sr^2 MeV. It is, however, to be noted that all of our calculations used the bound state Saxon-Woods parameter of 1.09 fm and a diffuseness parameter of 0.65 fm. A calculation of the cross section for this reaction as a function of the bound state Saxon-Woods well radius in Ref. 6 indicates that when r_0 increases from 1.09 to 1.3 fm the cross section increases by a factor of ~ 2.5. Taking this factor into account, one gets a spectroscopic factor in very good agreement with the shell model estimate of 0.23 for ^{16}O. Thus both shape as well as magnitude for the 101.5 MeV $^{16}O(p,p\alpha)^{12}C_{g.s.}$ are in excellent agreement with the exact three-body test case calculations when there are no imaginary potential components. When the imaginary parts of the optical potentials in both the initial and the final state are included in the calculations, the structures around the zero recoil momentum position disappear and the absolute cross section is also reduced by a factor of 4.

In context with the changes in optical potentials it is worth noting that in the study of the A dependence of the $A(e,e'p)$ reaction in the quasifree region[17] the attenuation length is almost three times longer than expectations based on the free nucleon-nucleon cross section. From our $(p,p\alpha)$ and $(\alpha,2\alpha)$ calculations also we tend to infer that the attenuation length (hence the imaginary part of the optical potential in the reverse sense) for α particles in nuclei is significantly longer (hence W is smaller) than expected from other considerations. The neglect of the imaginary component of optical potentials in our calculations for the test case situation also gets indirect support from some other considerations such as anomalous back-angle scattering of α particles and transfer reactions involving light heavy ions.[18]

IV. CONCLUSIONS

A test case has been found where the influence of the three-body final state coupling term present in the conventional DWIA treatment of the knockout reaction can be estimated. This test case is very asymmetric because one of the distorting optical potentials in the final state is completely neglected. The three-body coupling term has been found to have a large influence in the calculated DWIA knockout cross sections when the imaginary part of the optical potential is neglected. The main influence of the neglect of the dynamic three-body coupling term has been shown to suppress the surface contributions to the knockout reaction matrix element. Comparison of the experimental data for the 101.5 MeV $^{16}O(p,p\alpha)$ reaction with the calculations favor the inclusion of the three-body coupling term. Similar to the conclusion drawn from the $(e,e'p)$ reaction[17] analyses our calculations for $(\alpha,2\alpha)$ and $(p,p\alpha)$ reactions also indicate that the nuclear attenuation lengths are large not only for protons but also for the α particles.

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