Energy dependence of the nuclear level density at energies above 100 MeV

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Level densities are calculated for the nucleus ^{40}Ca using an exact iterative method for noninteracting fermions. Various single-particle energies are tested. It is found that the conventional Fermi-gas energy dependence provides a good representation of the actual level density only up to energies about 100 MeV. Above this point, the deviations rapidly grow, reaching many orders of magnitude at energies above 200 MeV. One calculation has also been made for ^{90}Zr. These results are similar, with the point of breakdown of the Fermi-gas form raised to about 200 MeV. The present results should have significant consequences for heavy-ion reaction studies of equilibration and for astrophysics.

The conventional Fermi-gas form for the nuclear level density was first derived by Bethe.[1] It requires the assumption of a constant single-particle level density to obtain the Bethe form. While many low-energy level-density measurements have been fit with the Fermi-gas expression, relatively few tests have been made to see how appropriate it is at energies of 100 MeV and above. We present exact calculations of the level density for ^{40}Ca and show that the Fermi-gas form becomes completely inappropriate above 100 MeV for ^{40}Ca.

Jacquemin and Kataria[2] have described an exact iterative method for calculating the nuclear level density. The only limitation on the method is the restriction to noninteracting particles. While this restriction clearly limits the reliability of the calculation in describing the level density at low energies, the average behavior at high energies ought not to be significantly affected. An approximate calculation of the effect of two-body forces indicates no fundamental change to the conclusion reported here.

The origins of the deviation from the Fermi-gas form are easy to identify. A Fermi gas is characterized by a confining potential well, which for the nucleus is about 50 MeV deep. Thus, the hole density cannot be taken to be constant, if excitation energies high enough to excite holes by 50 MeV are considered. A more subtle problem is that a Fermi gas actually has a density of single-particle states which increases as $E^{1/2}$. This effect, as the first, is not important for small excitation energies, but can become quite significant as the energy increases. For the case of ^{40}Ca, we assume a level-density parameter of $a=40/8=5$. The corresponding single-particle density would be $(6/\pi^2)(5)=3.04$, or 1.52 for protons and 1.52 for neutrons. To fill such a well with 20 nucleons requires 20/1.52 or 13.16 MeV. This is far less than the actual Fermi energy of about 40 MeV, illustrating the inconsistency.

A third contributing factor is more difficult to deal with quantitatively. This is the need to cut the single-particle spectrum off at the top as well. Two arguments can be advanced for such a cutoff. First, if the cutoff is not invoked, excited levels will be generated to infinite excitation energy. This is physically unreasonable,[3] given that any nucleus has a finite binding energy. At energies above 350 MeV, the ^{40}Ca nucleus has enough energy to completely dissociate. At energies beyond this, free-particle phase-space densities (which are much smaller because of the binding energies of the nucleus) would be more appropriate. An alternative justification of the cutoff is that highly excited single-particle states are not strongly coupled to compound nuclear states. Proton-induced reactions have recently been studied at 80, 120, and 160 MeV by Scobel *et al.*[4] on targets of ^{90}Zr and ^{208}Pb. Integration of the outgoing neutron spectrum above 30 MeV appears to give a cross section which is 35–40% of the absorption cross section. Allowing a similar fraction for proton decay and adding an additional 10% to account for nonequilibrium neutrons and protons emitted between 10 and 30 MeV, it is clear that at energies above 80 MeV equilibrium reactions occur almost exclusively as the final stage of a reaction whose first stage is direct or preequilibrium. Viewed in the time-reversed sense, the probability of emission of a high-energy particle from a compound state is very small. While these arguments make an upper cutoff plausible, they do not give a definitive prescription for where it should be.

Calculations of the level density of ^{40}Ca were made with four different single-particle sets for comparison with the Fermi-gas model. In each case, results with different numbers of orbits are presented in order to show the sensitivity of the results to this parameter. Those presented were selected so as to span the range of choices which are physically reasonable, i.e., show a decrease at the point where the nucleus becomes completely unbound. The first calculation was done with the single-particle density set equal to 1.5 MeV. As previously indicated, this forces the Fermi energy to be at 13 MeV rather than 40 MeV. Table I shows the ratio between the actual level density and that predicted for a Fermi gas based on the fit to the lowest 20 MeV of excitation. Figure 1 shows the actual level densities. In both Fig. 1 and Table I, calculations including 7 orbits, 9 orbits, and 11 orbits are presented. All orbits are summed to be spin $\frac{3}{2}$ for this calculation, so the largest space has a capacity of

TABLE I. Ratio between Fermi-gas prediction and actual level density.

Excitation energy E (MeV)	Constant single-particle spacing 7 orbit	9 orbit	11 orbit	Harmonic oscillator 8 orbit	9 orbit	11 orbit	Experimental levels 7 orbit	8 orbit	9 orbit
50	2	1.15	1.1	24	17.8	7.2	5.5	1.26	1.0
100	6×10^3	10	2.5	724	260	10.5	340	4.34	1.05
150	10^8	3×10^2	10	8.3×10^4	1.3×10^4	33	1.1×10^5	160	18
200	10^{19}	4×10^5	10^3	2.4×10^7	1.54×10^6	190	1.4×10^8	1.8×10^4	930
250	a	10^9	10^5	1.7×10^{10}	4×10^8	1.6×10^3	7.4×10^{11}	5.9×10^6	1.3×10^5
300	a	10^{13}	10^7	3×10^{13}	2.2×10^{11}	2.3×10^4	4.1×10^{16}	6.3×10^9	5×10^7

[a]Highest excitation energy was 217 MeV.

44 protons and 44 neutrons. An alternative calculation with all levels assumed to be spin $\frac{1}{2}$ but with twice as many levels gave virtually identical results, indicating the precise grouping for the constant density of single-particle states is not important. Note that discrepancies between the Bethe form and the calculations are modest for 50 MeV, but become quite substantial at 100 MeV for the 7- and 9-orbit calculations. Beyond this point, the divergence is exponential. Based on the actual binding energy, the 9-orbit calculation is probably the most realistic.

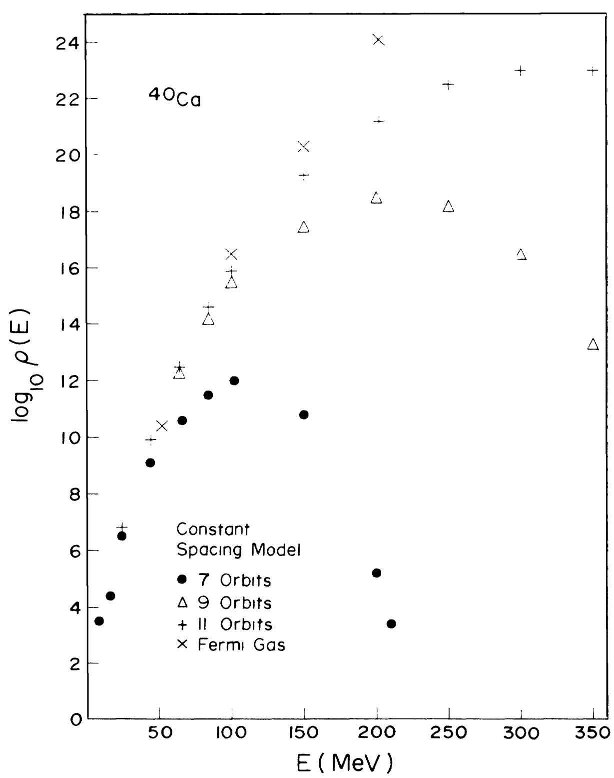

FIG. 1. Shown are the calculations for the level density for ^{40}Ca, with 7 orbits (●), 9 orbits (△), and 11 orbits (+). The density of single-particle states is independent of energy and equal to 1.52/MeV. The ×'s show the energy dependence of a Fermi-gas fit to the first 20 MeV of excitation. All four level densities agree at energies below 20 MeV, but diverge exponentially at high energies.

A second calculation was done with harmonic-oscillator level spacings. A harmonic-oscillator constant of $\hbar\omega=41/A^{1/3}$ MeV was used and the spin-orbit strength was determined from the $d_{5/2}$-$d_{3/2}$ splitting in the $A=40$ system. In this case, the level order as a function of energy gives the physically observed order of the states as determined by the J value. The best-fit Fermi-gas parameters were $a=4.11$, $\delta=1.2$, giving a lower level density than the constant spacing model. Even with the lower Fermi-gas prediction, deviations between the actual and the Fermi-gas values occur at even lower energies than for the constant spacing model. For the harmonic-oscillator levels, the deviation is about an order of magnitude at 50 MeV and grows rapidly beyond this point. The more rapid deviation from the Fermi-gas form

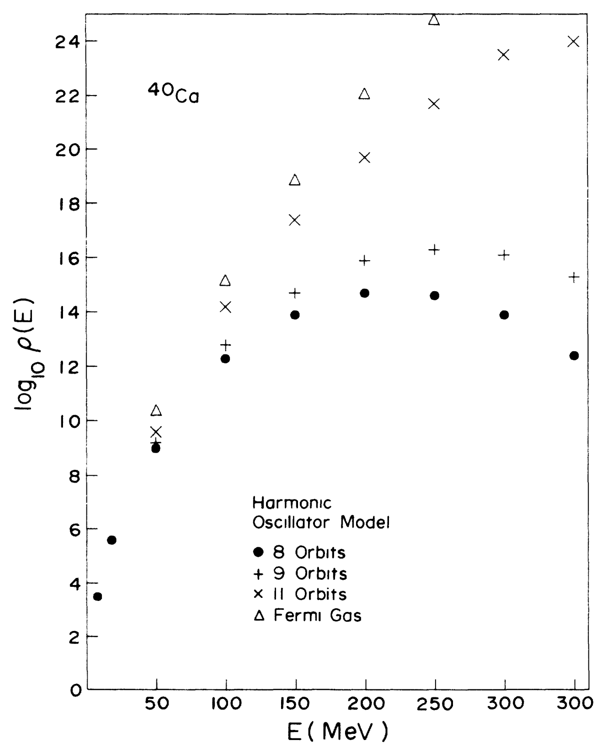

FIG. 2. Same as Fig. 1, except that the single-particle levels are from a harmonic-oscillator Hamiltonian and the bases are for 8, 9, and 11 orbits.

presumably follows from the fact that a harmonic-oscillator single-particle spectrum has a density which increases as E^2, that is, it increases more rapidly relative to the Bethe assumption than does the Fermi-gas single-particle spectrum. These results are shown in Table I and Fig. 2. Six orbits comprise the first three major shells. Thus, the 8-, 9-, and 11-orbit calculations include 2 and 3 orbits from the *f*-*p* shell and, in the last case, the $g_{9/2}$ orbit as well as 4 orbits from the *f*-*p* shell.

A further comparison was made with a "realistic" set of single-particle levels. This set was based on hole energies determined from levels in $A=39$ nuclei, while particle state energies were obtained from the corresponding energy levels in $A=41$. Levels from the first two major shells were taken from the levels proposed by Seeger and Howard.[5] Calculations were made for 7 orbits, 8 orbits, and 9 orbits and a Fermi-gas fit made to the first 20 MeV of the level density. Over this range, the three calculations with different bases all agree. At 50 MeV, the 7-orbit calculation is already a factor of 5 below the Fermi-gas extrapolation. It appears that either the 7- or 8-orbit calculation is probably the most realistic, based on their values in the neighborhood of 350 MeV. For the 8-orbit basis, the Fermi-gas prediction is fairly close at 50 MeV, but begins deviating significantly from the 8-orbit result at 100 MeV, as can be seen in Fig. 3. Thus, it appears as if the Fermi-gas result breaks down somewhere between 50 and 100 MeV for a realistic set of levels for ^{40}Ca.

A final comparison was made between the calculation utilizing Seeger-Howard[5] values for each level and the Fermi-gas model prediction. This gave results which were very close to those of the calculation with experimental levels and is not shown separately. As in the results discussed previously, fitting the level density values with a Fermi-gas function between 0 and 20 MeV gives a good representation up to about 70 MeV but then begins to substantially overestimate the level density.

One calculation using Seeger-Howard levels was done for ^{90}Zr. In this case, the Fermi-gas fit was satisfactory to energies beyond 100 MeV. At about 160 MeV, significant discrepancies began to appear which then grew exponentially.

A simple model for the effects of the two-body interaction indicates that a more complete calculation incorporating these effects would not change the conclusions reported here. Addition of a two-body force to the calculation would produce an additional spreading. This could be approximated in the present calculations by folding them together with an additional Gaussian of appropriate width. The unfolded Gaussians which fit the single-particle Hamiltonian calculations have widths $\sigma \gtrsim 50$ MeV; folding this together with a Gaussian of width 5–10 MeV has a negligible effect except in the tail regions. The peak height is reduced by 10–15 %, but the largest effects are in the tail regions, where the states highest (lowest) in energy are shifted up (down) by 10–15 MeV. Retention of a lower and upper energy cutoff still leaves us with a finite basis, and this guarantees increasing discrepancies with energy between the exact results and the Fermi-gas prediction.

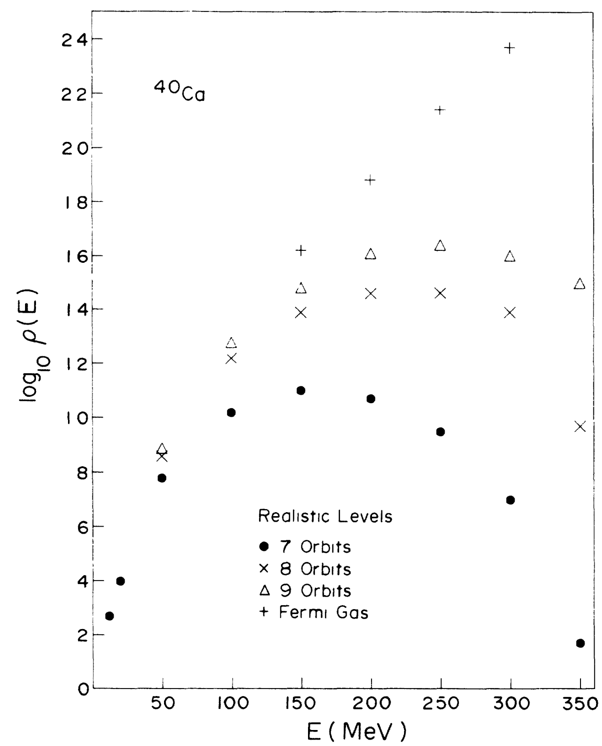

FIG. 3. Same as Fig. 1, except that the single-particle levels are from experimental levels in the $A=39$ and 41 systems and the bases are for 7, 8, and 9 orbits.

It appears that nuclear level densities at energies above approximately $E \sim 2A$ MeV deviate substantially from the Fermi-gas model. While quantitative studies of these effects will require more investigation of criteria for cutting off the high-energy single-particle spectrum, it is possible to say that the discrepancies are always in the direction of an overestimate of the actual level density and that they grow exponentially with energy. A recent paper by Pochodzalla *et al.*[6] discusses the interpolation of the production mechanism of intermediate mass fragments observed in ^{16}O+Ag$^{\rm nat}$ and ^{16}O+^{197}Au reactions at $E/A=84$ MeV. The results concern intermediate mass fragments with $A=100$ and E/A about 6 MeV. This energy region is one in which the Fermi-gas prediction is exponentially incorrect. Consistent with this, the omission of intermediate mass fragments is found to be much suppressed for energies $E/A>4$. The present analysis suggests that, both in heavy-ion physics and in astrophysics, the Fermi-gas level-density formula be used only up to energies about $2A$ MeV. Beyond this point, a Gaussian form which decreases to 0 near $8A$ MeV would be more appropriate. It has been shown[7] that a truncated basis Fermi gas is fit quite well by a Gaussian form.

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