Contribution of meson exchange currents to pion double charge exchange at low energies

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The magnitude of the effect of meson exchange currents on low-energy pion nucleus double charge exchange is reexamined. In particular, the role of chiral symmetry in limiting the size of the effect is studied, using the method of effective Lagrangians. The equivalence of two commonly used versions of the theory is shown, which resolves some of the disagreements in the literature. The effect is calculated for the case of ^{42}Ca, and found to be smaller than that of ordinary multiple scattering.

I. INTRODUCTION

The subject of this paper is the magnitude of the contribution of meson exchange currents to double charge exchange (DCX) scattering of pions from nuclei at low energies. Recent experiments have produced data on a variety of nuclear targets at pion energies from 20 to 80 MeV.[1,2] It is usually assumed that the reaction is dominated by multiple scattering involving (at least) two charge exchange π-nucleon scatterings in the target. This view is supported by a number of calculations which give cross sections of the right order of magnitude for the transition to the double isobaric analog state (DIAS).[3,4] The idea that meson exchange currents could also contribute to the DCX reaction, by π-π scattering of the projectile on a virtual pion exchanged between two target nucleons, was introduced by Germond and Wilkin.[5] Although Robilotta and Wilkin[6] concluded that the meson exchange current (MEC) effect would be small for analog DCX, the issue has remained somewhat unsettled in the recent literature. It is important to know the relative magnitude of the MEC effects to the multiple-scattering contribution, particularly since most attempts to extract nuclear-structure information from DCX rely on the latter interpretation.

Germond and Wilkin first suggested the role of the MEC contribution to DCX, based on the process illustrated in Fig. 1(a),[5] considering the example of the nonanalog DCX reaction on ^{4}He. The point is that the π-π scattering in the intermediate state gives DCX in one step, with an amplitude large compared to nuclear DCX. (The relevant π-π scattering lengths are of order ~ 0.1 fm, while nuclear DCX amplitudes are ~ 0.01 fm.) This must be multiplied by the transition probability of exchanging a pion in the target which is $\ll 1$, to give the MEC contribution. Shortly after this work, Robilotta and Wilkin[6] recognized that chiral symmetry requires the inclusion of a second process of the same order as the virtual π-π scattering, as shown in Fig. 1(b). The two terms are conventionally referred to as [Fig. 1(a)] the *pole* term, and [Fig. 1(b)] the *contact* term. To calculate both terms in a consistent manner, they used an effective Lagrangian method (which we discuss further below), and found that the two terms of Fig. 1 tend to cancel, producing a much reduced final amplitude. Although the calculation of Ref. 6 was specifically for the π-deuteron scattering length, the authors noted that the same cancellation would also reduce the MEC contribution to DCX. As we show later, we agree with this conclusion.

The issue has been revived in recent years, starting with work by Oset and collaborators,[7] who were interested in the MEC contribution to DCX in the Δ-resonance region. They have pointed out that with a particular choice of effective Lagrangian, the contact term does not contribute, and have therefore dropped it from their calculations. There have been some disagreements with this argument and some discussions of this issue in the recent literature.[8-13] More recently, Auerbach *et al.*[4] calculated the contribution of the *pole* term to DCX on ^{42}Ca at low energies, using the theoretical approach of Ref. 7. They found this part of the MEC amplitude to be comparable to the multiple-scattering contribution and to the experimental DCX amplitude, with considerable sensitivity to the assumed π-nucleon form factor. Following Oset's arguments, they neglect the *contact* term. Their results, if correct, would throw considerable doubt on the possibility of analyzing the DCX cross sections in terms of nuclear-structure effects through multiple-scattering theory.

We find that this method of calculating the MEC contribution is not consistent with the requirements of chiral symmetry. This is an approximate symmetry which constrains the form of low-energy pion scattering processes, e.g., πN and $\pi\pi$, through so called "soft-pion theorems." Although originating from current algebra theory and PCAC (partially conserved axial current) in the 1960s, the results, somewhat modified, have been rederived within the context of QCD in the form of *chiral perturbation theory*.[14]

In this paper we clarify the role of chiral symmetry for the MEC contribution to DCX, and calculate the effect at low energies, for the specific case of ^{42}Ca. We use an effective Lagrangian theory which maintains the approximate chiral symmetry, as is relevant at low energies.

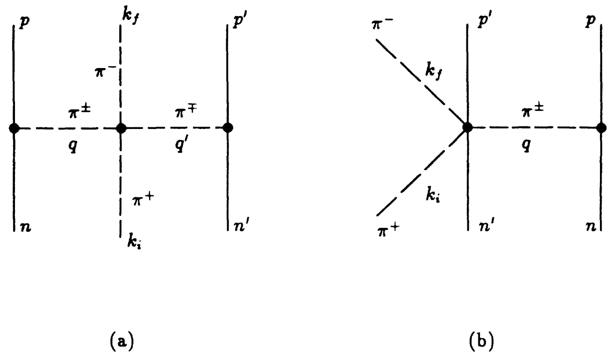

FIG. 1. Diagrams contributing to the (π^+,π^-) reaction in the meson-exchange current model: (a) is the pion-pole diagram and (b) is the pion-contact diagram. Note that the exchanged pions are always charged.

The paper is organized as follows. In Sec. II, we introduce the effective Lagrangian method, and discuss some essential points. We stress the equivalence of two apparently different sets of Lagrangians. In Sec. III, we calculate the transitions amplitudes for DCX at the two-nucleon level, reduced to a nonrelativistic form appropriate to low-energy scattering. In Sec. IV, these amplitudes are then integrated over target states to give the analog DCX scattering amplitude for ^{42}Ca$(\pi^+,\pi^-)^{42}$Ti. Results are shown, and sensitivity to parameters are exhibited. Discussion and conclusions follow in Sec. V.

II. EFFECTIVE LAGRANGIAN FORMALISM

The effective Lagrangian formalism for low-energy pion scattering provides a method of calculation which includes the requirements of approximate chiral invariance, and thus will give the appropriate "soft-pion" limits for a variety of related processes. The effective Lagrangian contains the degrees of freedom being considered: In this case, pions and nucleons only. (We do not consider the explicit contribution of heavier mesons in this paper.) The form is constrained by the requirements of current algebra and PCAC. The calculation of physical amplitudes is to be performed to lowest significant order, at the "tree" level (no closed-diagram loops). These specifications do not lead to a unique form for the Lagrangians, although equivalent Lagrangians of different form will lead to the same physical amplitudes, as we discuss below.

In order to calculate the processes of Fig. 1, we need a Lagrangian with three terms: $\mathcal{L}_{NN\pi}$, for π-N coupling; $\mathcal{L}_{NN\pi\pi\pi}$, which gives $\pi\leftrightarrow 2\pi$ on a nucleon; and $\mathcal{L}_{\pi\pi\pi\pi}$, which give π-π scattering. Lagrangians of this type have been introduced by Weinberg[15] and Schwinger.[16,17] For our purposes we consider two forms used in the literature for parametrization of the $\pi N\to\pi\pi N$ reaction at low energy. One was first introduced by Olsson and Turner,[18] which we write in the following form:

$$\begin{aligned}\mathcal{L}_{NN\pi}&=\frac{g_\pi}{2m_N}\bar{\psi}\gamma^\mu\gamma_5\boldsymbol{\tau}\psi\cdot(\partial_\mu\boldsymbol{\phi})\ ,\\ \mathcal{L}_{NN\pi\pi\pi}&=-\frac{g_\pi}{2m_N}\frac{1}{4f_\pi^2}(\bar{\psi}\gamma^\mu\gamma_5\boldsymbol{\tau}\psi)\\ &\quad\times[\xi(\partial_\mu\boldsymbol{\phi})\boldsymbol{\phi}^2+(\xi-1)\boldsymbol{\phi}(\partial_\mu\boldsymbol{\phi}^2)]\ ,\\ \mathcal{L}_{\pi\pi\pi\pi}&=-\frac{1}{4f_\pi^2}[\tfrac{1}{2}(\xi-1)(\partial_\mu\boldsymbol{\phi}^2)^2+\xi\boldsymbol{\phi}^2(\partial_\mu\boldsymbol{\phi})^2\\ &\qquad-\tfrac{1}{2}(\tfrac{3}{2}\xi-2\eta-1)m_\pi^2\boldsymbol{\phi}^4]\ ,\end{aligned}\tag{1}$$

where ψ and $\boldsymbol{\phi}$ are, respectively, the nucleon and pion fields, g_π is the πN coupling constant, and f_π is the pion decay constant. (In this work, we use $g_\pi=13.5$ and $f_\pi=87$ MeV.) The two parameters ξ and η, can be chosen to reproduce the earlier specific choices. For example, Weinberg's Lagrangian[15] can be obtained by setting $\xi=1$, $\eta=-\frac{1}{4}$ in $\mathcal{L}_{\pi\pi\pi\pi}$ of Eq. (1) while his π-π scattering amplitude derived from current algebra (without Lagrangians) corresponds to the choice of $\xi=0$ and $\eta=0$.[19] There are also two versions of Schwinger's choice; one can be obtained from $\xi=1$, $\eta=0$ (Ref. 16) and the second from $\xi=-2$, $\eta=0$.[17]

Another form of the effective Lagrangians which has frequently been referred to in the literature[8,20–22] has the form

$$\begin{aligned}\mathcal{L}_{NN\pi}&=\frac{g_\pi}{2m_N}\bar{\psi}\gamma^\mu\gamma_5\boldsymbol{\tau}\psi\cdot(\partial_\mu\boldsymbol{\phi})\ ,\\ \mathcal{L}_{NN\pi\pi\pi}&=-\frac{g_\pi}{2m_N}\frac{1}{4f_\pi^2}(\bar{\psi}\gamma^\mu\gamma_5\boldsymbol{\tau}\psi)\cdot[(\partial_\mu\boldsymbol{\phi})\boldsymbol{\phi}^2]\ ,\\ \mathcal{L}_{\pi\pi\pi\pi}&=-\frac{1}{4f_\pi^2}[\boldsymbol{\phi}^2(\partial_\mu\boldsymbol{\phi})^2-\tfrac{1}{2}(1-\tfrac{1}{2}\bar{\xi})m_\pi^2\boldsymbol{\phi}^4]\ .\end{aligned}\tag{2}$$

At first glance, these two sets of Lagrangians appear to be quite different, but they can be shown to be entirely equivalent, in the sense that they produce the same physical (on-shell) amplitudes. It is easy to see that for $\xi=1$, $\eta=\frac{1}{4}(\bar{\xi}-1)$, Eq. (1) becomes identical to Eq. (2). However, Olsson and Turner have also shown[18] that only one linear combination of the two parameters appears in the physical amplitudes for $\pi+\pi\to\pi+\pi$ and $\pi+N\to\pi+\pi+N$, namely, $\xi+4\eta$. Therefore, with the relation $\bar{\xi}=\xi+4\eta$, we have the equivalence of the two Lagrangian forms, Eqs. (1) and (2), for these reactions.

In fact, one can easily prove the equivalence of the two Lagrangians for the DCX reaction, by calculating the amplitudes for Fig. 1, in the nonrelativistic limit. This is seen most directly for the forward amplitude, which is given in the next section in Eq. (10). (The equivalence is also true for all angles for $J=0$ targets, as we show in Appendix B.) Similarly, for the π-d elastic forward amplitude, we find the result

$$T=-\frac{g_\pi^2}{4m_N^2}\frac{1}{f_\pi^2}\frac{(\boldsymbol{\sigma}_1\cdot\mathbf{q})(\boldsymbol{\sigma}_2\cdot\mathbf{q})}{(\mathbf{q}_2+m_\pi^2)^2}(1+\tfrac{5}{2}\bar{\xi})m_\pi^2\tag{3}$$

for either Lagrangian, with $\bar{\xi}=\xi+4\eta$. This result was originally obtained by Robilotta ad Wilkin,[6] using Eq.

(1), with $\eta=0$ (and therefore $\bar{\xi}=\xi$). It must be emphasized that the equivalence of Eqs. (1) and (2) will lead to the same physical amplitudes for DCX only if the contributions of both diagrams of Fig. 1 are included; the values of the separate terms do depend on the choice of Lagrangian.

The on-shell parameter $\bar{\xi}$ can be related to the ratio of $T=0$ and $T=2$ π-π scattering lengths:

$$\frac{a_0}{a_2}=\frac{\frac{5}{2}\bar{\xi}-7}{\bar{\xi}+2} . \tag{4}$$

In the original Weinberg theory of π-π scattering, $\bar{\xi}$ was assumed to be zero, as suggested by the σ model. Experimental data from $\pi N\rightarrow\pi\pi N$ favor small values: $-0.5\leq\bar{\xi}\leq0.3$, within errors.[23,24] [Note that because (ξ,η) are used as two parameters in Eq. (1) of this work, we use $\bar{\xi}$ as the chiral symmetry-breaking parameter which is denoted by ξ in much of the literature.]

When the parameter $\eta\equiv0$, the Lagrangian of Eq. (1) will produce amplitudes which obey the Adler consistency condition, which is a constraint of the off-shell behavior derived from current algebra. With this assumption, $\bar{\xi}=\xi$. The Lagrangian of Eq. (2) will give different off-shell amplitudes, however.

III. TRANSITION AMPLITUDES

We calculate the DCX transition amplitudes corresponding to the two diagrams of Fig. 1, by standard methods described in Appendix A. We use the Lagrangian form of Olsson and Turner, given by Eq. (1). The invariant transition amplitudes are given by

$$\begin{aligned}T^{(p)}=&\frac{g_\pi^2}{4m_N^2}\frac{2}{f_\pi^2}[\bar{u}(p)\gamma^\mu\gamma^5u(n)q_\mu][\bar{u}(p')\gamma^\nu\gamma^5u(n')q'_\nu]\\&\times\frac{1}{(q^2-m_\pi^2+i0^+)(q'^2-m_\pi^2+i0^+)}\\&\times[(\xi-1)(s+u)+\xi t-(3\xi-4\eta-2)m_\pi^2]\end{aligned} \tag{5}$$

for the pion-pole diagram of Fig. 1(a), and

$$\begin{aligned}T^{(c)}=&\frac{g_\pi^2}{4m_N^2}\frac{2}{f_\pi^2}[\bar{u}(p)\gamma^\mu\gamma^5u(n)q_\mu]\frac{1}{q^2-m_\pi^2+i0^+}\\&\times[\bar{u}(p')\gamma^\nu\gamma^5u(n')]\\&\times[\xi(k_f-k_i)_\nu-(\xi-1)(2q+k_i-k_f)_\nu] ,\end{aligned} \tag{6}$$

for the pion-contact diagram of Fig. 1(b). In the above, s, t, and u are Mandelstam variables as given in Appendix A.

With a nonrelativistic reduction

$$\begin{aligned}&[\bar{u}(p')\gamma^\mu\gamma^5u(p)q_\mu]\rightarrow-(\boldsymbol{\sigma}\cdot\mathbf{q}) ,\\&\frac{1}{\mathbf{q}^2-m_\pi^2+i0^+}\rightarrow-\frac{1}{m_\pi^2+\mathbf{q}^2} ,\end{aligned} \tag{7}$$

and

$$\begin{aligned}s&=m_\pi^2-\mathbf{q}^2-2\mathbf{q}\cdot\mathbf{k}_i ,\\u&=m_\pi^2-\mathbf{q}^2+2\mathbf{q}\cdot\mathbf{k}_f ,\\t&=-(\mathbf{k}_f-\mathbf{k}_i)^2 ,\end{aligned} \tag{8}$$

one can derive the following nonrelativistic amplitudes for the process $\pi^+nn\rightarrow\pi^-pp$:

$$\begin{aligned}T^{(p)}=&\frac{g_\pi^2}{4m_N^2}\frac{2}{f_\pi^2}\frac{(\boldsymbol{\sigma}_1\cdot\mathbf{q})(\boldsymbol{\sigma}_2\cdot\mathbf{q}')}{(\mathbf{q}^2+m_\pi^2)(\mathbf{q}'^2+m_\pi^2)}\\&\times[4\eta m_\pi^2+2\mathbf{q}\cdot\mathbf{q}'-\xi(m_\pi^2+\mathbf{q}^2+\mathbf{q}'^2)] ,\\T^{(c)}=&-\frac{g_\pi^2}{4m_N^2}\frac{2}{f_\pi^2}\frac{1}{\mathbf{q}^2+m_\pi^2}(\boldsymbol{\sigma}_1\cdot\mathbf{q})\\&\times[(\boldsymbol{\sigma}_2\cdot\mathbf{q})+(1-2\xi)(\boldsymbol{\sigma}_2\cdot\mathbf{q}')] ,\end{aligned} \tag{9}$$

where $\mathbf{q}'=\mathbf{q}-(\mathbf{k}_f-\mathbf{k}_i)$ with $\mathbf{q}=\mathbf{n}-\mathbf{p}$.

These amplitudes must be integrated over the $\mathbf{q}$ distribution of the nuclear target state overlap. However, it can be seen that there is considerable cancellation between the two terms of Eq. (9), particularly for the limit of forward scattering ($\mathbf{q}=\mathbf{q}'$), for which

$$T^{(p)}+T^{(c)}=\frac{g_\pi^2}{2m_N^2}\frac{m_\pi^2}{f_\pi^2}\frac{(\boldsymbol{\sigma}_1\cdot\mathbf{q})(\boldsymbol{\sigma}_2\cdot\mathbf{q})}{(\mathbf{q}^2+m_\pi^2)^2}(\xi+4\eta-2) . \tag{10}$$

Notice that the q dependence of the term [ ] in $T^{(p)}$ of Eq. (9), which comes from the π-π scattering interaction in Eq. (5), is entirely removed from the sum. The total mesic contribution then exhibits a well-defined soft-pion limit, i.e., vanishing when $m_\pi\rightarrow0$. This behavior is a result of the current algebra that has been built into the effective Lagrangian formalism. (This was observed by Robilotta ad Wilkin[6] in a similar study of the pion-deuteron scattering length.)

One also sees in Eq. (10) that $\bar{\xi}=\xi+4\eta$ is the only relevant parameter. [Equation (10) only shows this property for $\mathbf{q}=\mathbf{q}'$; however, it is also true for any momentum transfer, *providing* the target states are isotropic ($J=0$); see Appendix B.] From this one can show explicitly the equivalence of the Lagrangian forms in Eqs. (1) and (2), for the DCX amplitude, Eq. (10), following the argument given in Sec. II. As we noted, Eq. (2) is a special case of Eq. (1) with $\xi=1$, $\eta=\frac{1}{4}(\bar{\xi}-1)$. Since Eq. (10) depends only on the value of $\bar{\xi}$, the equivalence follows.

The integration of Eq. (10) over momentum $\mathbf{q}$ in the target is quite well behaved, since the integrand goes as $\mathbf{q}^{-2}$ for large q. By contrast, the separate terms $T^{(p)},T^{(c)}$ do not decrease at large q. The integrated sum is therefore considerably smaller, and also less sensitive to assumptions about nuclear wave functions, form factors, etc., than the individual integrals of $T^{(p)}$ and $T^{(c)}$, as we see in the next section. In fact, the size of the two terms can be changed by different assumptions about the parameters ξ and η. But we have seen that the sum depends only on the combination $\bar{\xi}=\xi+4\eta$, which is in turn related to the π-π scattering lengths and $\pi N\rightarrow\pi\pi N$ amplitudes.

The reason for stressing this point is the problem that can arise by treating the processes in $T^{(p)}$ and $T^{(c)}$

differently. For example, Auerbach *et al.*[4] have calculated the contribution of $T^{(p)}$ to low-energy DCX using a model of π-π scattering (Veneziano and Lovelace) as adopted by Oset *et al.*[7] Although this is not done in the effective Lagrangian formalism, it is equivalent for low momenta to our calculation with the Lagrangian of Eq. (1), setting $\xi=\eta=0$ (i.e., Adler consistency and the "σ condition"). The contribution of $T^{(c)}$ is not calculated, for which the argument has been given[9,13] that with the Lagrangian of Eq. (2), there is no contribution of $T^{(c)}$ to DCX. This is correct, but corresponds to a different choice of Lagrangian than used (effectively) to calculate $T^{(p)}$, since (2) corresponds to (1) with $\xi=1$, and $\eta=\frac{1}{4}(\xi-1)=-\frac{1}{4}$. The point is that the Lagrangian in Eq. (2), with $\xi=0$, predicts different off-shell behavior for π-π scattering than given by the Veneziano-Lovelace model, even for very small momenta. Therefore it is inconsistent to calculate the two terms in this way. The requirement of consistent choice of Lagrangian is equivalent to the conditions of current algebra in the low-momentum (soft-pion) limits. (This will be discussed more fully in a separate paper.) In fact, as we will see in the next section, if we use the Lagrangian of Eq. (2), for which the contribution of the pion-contact term vanishes, the corresponding pion-pole term is almost one magnitude smaller than that given in Ref. 4.

IV. RESULTS

As an example, we calculate the scattering amplitude for the analog DCX reaction $\pi^+ + {}^{42}\mathrm{Ca} \to \pi^- + {}^{42}\mathrm{Ti}$ at low pion energies, for which there are calculations of double scattering and of mesic current contributions,[4] as well as recent experimental measurements.[2] Since our main point is to establish the order of magnitude of the mesic current contribution and compare it to the multiple-scattering contribution, we neglect distortion of the pion waves, using plane waves instead. We also neglect contributions of the Ca core to DCX, treating the valence particles as a pure $(f_{7/2})^2$, $J=0$ state, denoted by $\phi(1,2)$. Finally, we ignore small differences in the wave functions for ^{42}Ca and ^{42}Ti.

The scattering amplitudes for the two processes of Fig. 1 are then obtained by taking the expectation values of Eq. (10),

$$F^{(p)}=-\frac{1}{2\pi}\frac{g_\pi^2}{4m_N^2}\frac{1}{f_\pi^2}\int\frac{d^3\mathbf{q}}{(2\pi)^3}\frac{4\eta m_\pi^2+2\mathbf{q}\cdot\mathbf{q}'-\xi(m_\pi^2+\mathbf{q}^2+\mathbf{q}'^2)}{(\mathbf{q}^2+m_\pi^2)(\mathbf{q}'^2+m_\pi^2)}$$

$$\times\int d\mathbf{r}_1 d\mathbf{r}_2 \phi^*_{J=0}(12) v(q)(\sigma_1\cdot\mathbf{q})(\sigma_2\cdot\mathbf{q}')v(q')e^{-i\mathbf{q}\cdot\mathbf{r}_1+i\mathbf{q}'\cdot\mathbf{r}_2}\phi_{J=0}(12) \tag{11}$$

and

$$F^{(c)}=\frac{1}{2\pi}\frac{g_\pi^2}{4m_N^2}\frac{1}{f_\pi^2}\int\frac{d^3\mathbf{q}}{(2\pi)^3}\frac{1}{(\mathbf{q}^2+m_\pi^2)}\int d\mathbf{r}_1 d\mathbf{r}_2\phi^*_{J=0}(12)v(q)(\sigma_1\cdot\mathbf{q})[(\sigma_2\cdot\mathbf{q})+(1-2\xi)(\sigma_2\cdot\mathbf{q}')]$$

$$\times v(q')e^{-i\mathbf{q}\cdot\mathbf{r}_1+i\mathbf{q}'\cdot\mathbf{r}_2}\phi_{J=0}(12)\ . \tag{12}$$

TABLE I. Mesic effect amplitudes of the pion DCX reaction on ^{42}Ca calculated with the chiral-symmetry-breaking parameter $\xi=0$ and cutoff parameter $\alpha=3.0\ \mathrm{fm}^{-1}$. Details are explained in the text. The labels "p term" and "c term" denote the pion-pole and the pion-contact contributions, respectively.

	$F(Q)$ (fm)					
		($\xi=0$, $\eta=0$)			($\xi=1$, $\eta=-0.25$)	
Q (fm^{-1})	p term	c term	$p+c$	p term	c term	$p+c$
0.00	0.9190×10^{-2}	-0.1158×10^{-1}	-0.2390×10^{-2}	-0.2389×10^{-2}	0.0	-0.2389×10^{-2}
0.20	0.7964×10^{-2}	-0.1025×10^{-1}	-0.2286×10^{-2}	-0.2102×10^{-2}	-0.1870×10^{-3}	-0.2289×10^{-2}
0.40	0.4988×10^{-2}	-0.6891×10^{-2}	-0.1903×10^{-2}	-0.1338×10^{-2}	-0.5652×10^{-3}	-0.1903×10^{-2}
0.60	0.1805×10^{-2}	-0.2962×10^{-2}	-0.1157×10^{-2}	-0.3986×10^{-3}	-0.7579×10^{-3}	-0.1157×10^{-2}
0.80	-0.3096×10^{-3}	0.7330×10^{-4}	-0.2363×10^{-3}	0.3041×10^{-3}	-0.5404×10^{-3}	-0.2363×10^{-3}
1.00	-0.1028×10^{-2}	0.1491×10^{-2}	0.4630×10^{-3}	0.4947×10^{-3}	-0.3182×10^{-4}	0.4629×10^{-3}
1.20	-0.8060×10^{-3}	0.1474×10^{-2}	0.6680×10^{-3}	0.2420×10^{-3}	0.4265×10^{-3}	0.6685×10^{-3}
1.40	-0.2986×10^{-3}	0.7428×10^{-3}	0.4442×10^{-3}	-0.1175×10^{-3}	0.5617×10^{-3}	0.4442×10^{-3}

The single-particle wave functions are taken to be bound in a harmonic-oscillator potential with $\hbar\omega = 10.5$ MeV, which has been frequently used in studies of nuclear spectroscopy for $A = 42$.[25] We also include a πN vertex function of the form $v(q) = \alpha^2/(q^2+\alpha^2)$ with a cutoff parameter α. Various values of this parameter have been used in the literature, from $\alpha = 1.5$ to 6 fm^{-1}.[3,4,26] We test the sensitivity to the value of α, although we favor the larger values, $\alpha \geq 3\ \text{fm}^{-1}$. The details of the method of calculation of the nuclear matrix elements are given in Appendix B.

A set of calculated amplitudes is given in Table I for two choices of (ξ, η), and $\alpha = 3.0\ \text{fm}^{-1}$. The amplitudes are independent of incoming pion energy, and are functions of momentum transfer Q only ($\mathbf{Q} = \mathbf{k}_f - \mathbf{k}_i$). The chosen value of (ξ, η) are (0,0) and $(1, -\frac{1}{4})$, both of which give the chiral-symmetry-breaking parameter value $\bar{\xi} = \xi + 4\eta = 0$.

The main result evident in Table I is that the summed amplitudes are *small*: $|F| < 2.4 \times 10^{-3}$ fm ($< \sqrt{0.06\ \mu\text{b}}$). For $\xi = \eta = 0$, this comes from considerable cancellation between the pole and contact terms, as discussed in connection with Eqs. (9) and (10). For $\xi = 1$, $\eta = -\frac{1}{4}$, the contributions of the two terms change; in fact, $F^{(c)} = 0$ for $Q = 0$ in this case. But the summed amplitude is identical to that for $\xi = \eta = 0$, as expected.

It can also be seen from Table I that the summed amplitudes depend on (ξ, η) only through their combination $\bar{\xi} = \xi + 4\eta$, for all values of the momentum transfer Q. This property can easily be shown to come from the integrations in Eqs. (11) and (12) for isotropic ($J = 0$) nuclear wave functions, as shown in Appendix B. Table I shows explicitly the equivalence of the Lagrangian forms given in Eqs. (1) and (2), since the results calculated for $\xi = 1$, $\eta = -\frac{1}{4}$ are identical to the results that would be calculated for $\bar{\xi} = 0$, using Eq. (2).

As remarked earlier, the results for $\xi = 1$, $\eta = -\frac{1}{4}$ give a vanishing contribution for the contact term at $Q = 0$. This property of the Lagrangian of Eq. (2) was noted in Refs. 9 and 23, and used to eliminate the contact term altogether. However, we emphasize that the corresponding contribution for the pion-pole term is reduced significantly from that calculated for $\xi = \eta = 0$, so that the summed amplitudes of the two calculations are identical (and for *all* values of Q). This is a direct consequence of the equivalence of the two Lagrangians.

The sensitivity of the calculated amplitudes to the cutoff parameter α in the vertex function $v(q)$ is shown in Fig. 2 for $\bar{\xi} = 0$. What is plotted is $|F(Q)|^2$ (in μb) as a function of Q. For $Q = 0$, there is an order-of-magnitude difference between the "soft" value of $\alpha = 1.5\ \text{fm}^{-1}$, and the "hard" limit of $\alpha \to \infty$ (point nucleons). A more realistic range is $3.0 < \alpha < 6.0\ \text{fm}^{-1}$, for which the variation in $|F|^2$ is a factor of 2. It should be noted that this range of variation is considerably smaller for the summed amplitude than for $F^{(p)}$ and $F^{(c)}$ separately, since the q integrations of the former are more convergent than for the latter, as can be seen by comparing Eqs. (9) and (10).

The dependence of the calculated amplitudes on the parameter $\bar{\xi}$ is shown in Fig. 3, for the range $-1 \leq \bar{\xi} \leq 1$, here calculated for $\alpha = 3.0\ \text{fm}^{-1}$. Again, the overall variation of $|F|^2$ is about a factor of 10 at $Q = 0$. Experimental data favor $\bar{\xi} \sim 0$ (see Refs. 23 and 24).

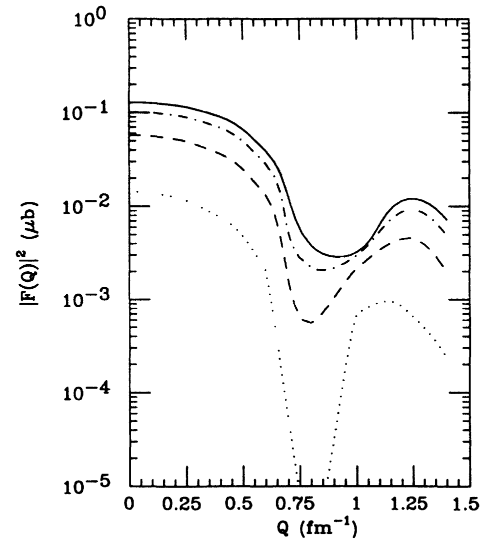

FIG. 2. Sensitivity of the meson exchange current effects to the cutoff parameter α. The results, including both pion-pole and pion-contact diagrams, are all calculated at $\xi = \eta = 0$, with $\alpha = 1.5\ \text{fm}^{-1}$ (dotted line), $\alpha = 3.0\ \text{fm}^{-1}$ (dashed line), $\alpha = 6.0\ \text{fm}^{-1}$ (dash-dotted line), and $\alpha = \infty$ (solid line). $|F(Q|^2$ are plotted in units of μb.

Last, we show the sensitivity to the harmonic-oscillator parameter $\hbar\omega$ (see Fig. 4) of the nuclear wave function, which is a measure of the mean-square radius of the valence nuclear distribution. Although not a negligible variation, this is the least sensitivity (and best known) parameter

The main result is that within considerable range of the parameter $(\alpha, \bar{\xi})$, the amplitudes $|F| < 3 \times 10^{-3}$ fm. This is to be compared to amplitudes for the double scattering

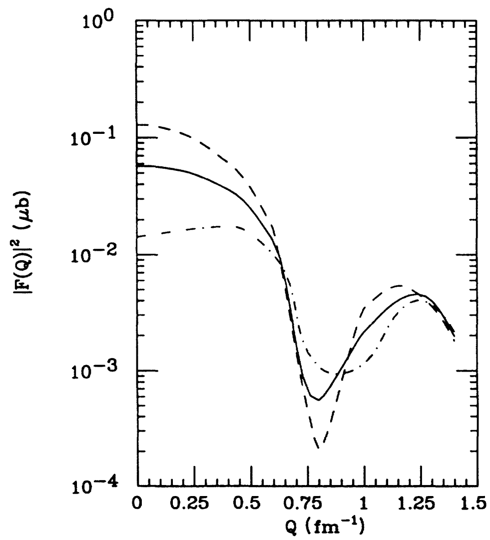

FIG. 3. Shown is the ξ dependence of the meson exchange current effects of Figs. 1(a) and (b). The curves are calculated at $\eta = 0$ and $\alpha = 3.0\ \text{fm}^{-1}$, with $\xi = -1$ (dashed line), $\xi = 0$ (solid line), and $\xi = 1$ (dash-dotted line).

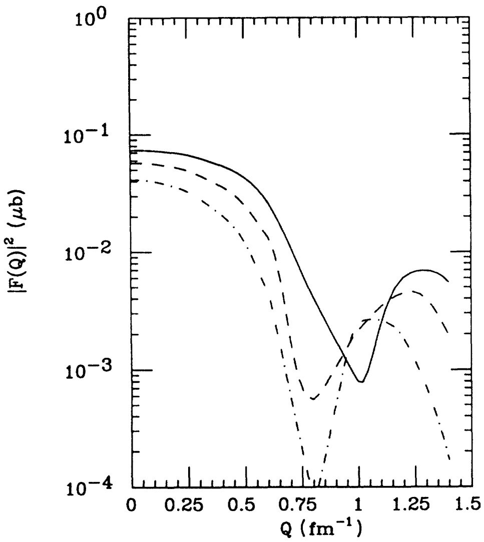

FIG. 4. Shown is the $\hbar\omega$ dependence, calculated at $\xi=\eta=0$, $\alpha=3.0\ \mathrm{fm}^{-1}$, with $\hbar\omega=12.5$ MeV (solid line), 10.5 MeV (dashed line), and 8.5 MeV (dash-dotted line).

contribution to DCX, which is of order $|F|\sim10^{-2}$ fm at low energies for ^{42}Ca (see, e.g., Ref. 4).

V. SUMMARY AND CONCLUSIONS

Our main conclusion is that at low energy the MEC contribution to analog DCX is constrained by chiral symmetry to give amplitudes small compared to the multiple-scattering contribution. We have shown that the result is not dependent on the form of the effective Lagrangian, for fixed values of the symmetry-breaking parameter ξ (and the cutoff parameter). The *relative* contribution of the two amplitudes, Eqs. (11) and (12), does depend on the choice of Lagrangian, as we have seen (e.g., in Table I). For the form given in Eq. (1), there is considerable cancellation between the two terms, while for that of Eq. (2), the contact term vanishes at $Q=0$. The summed results are, of course, the same.

The magnitude of the MEC contribution as calculated is not very sensitive to nuclear structure, but does show some sensitivity to the value of the cutoff parameter. However, the result is well behaved, even in the limit of point nucleons (no cutoff).

The calculation of the MEC effect at low energy in Ref. 4 obtains larger results than ours, and greater sensitivity to the cutoff parameter. We believe this to be a serious overestimate of the MEC effect, because their model does not preserve the soft-pion limits of chiral symmetry, due to the different treatment of the two amplitudes, as we discussed in Sec. III. In particular, their model of π-π scattering calls for a nonvanishing contact term, which will reduce their total amplitudes accordingly.

In a subsequent paper we shall give a direct demonstration of the constraints of chiral symmetry on the MEC terms in DCX, based on Weinberg's theory of pion scattering lengths.[9] We are also investigating the extension of the present approach to include the ρ meson in π-π scattering in a form consistent with chiral symmetry.

Last, we note that there are other physical mechanisms other than multiple scattering and meson exchange which must be accounted for in a complete theory of DCX; i.e., the effect of pion absorption.[27,28]

ACKNOWLEDGMENTS

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APPENDIX A: AMPLITUDES FOR THE $\pi^+nn\to\pi^-pp$ REACTION

As usual, to write the amplitudes for a given set of Lagrangians, one has first to construct the Feynman rules, where an important step is to obtain the relevant vertices. In the following, we start by deriving the vertices which are used for the process $\pi^+nn\to\pi^-pp$. Throughout this work, we follow the conventions of Bjorken and Drell.[29]

Each vertex is defined in terms of an invariant amplitude $\mathcal{M}$, which is in turn given by a normalized plane-wave matrix element of the interaction Lagrangian, omitting any Dirac spinors. For example, the $NN\pi$ vertex $\mathcal{M}_{NN\pi}$ is derived from

$$\frac{\delta^4(p_f+q-p_i)m_N}{\sqrt{2\pi}\sqrt{2\omega_q}\sqrt{E_fE_i}}\mathcal{M}_{NN\pi}=i\int d^4x\,\langle\mathcal{L}_{NN\pi}\rangle\ , \tag{A1}$$

where $\mathcal{L}_{NN\pi}$ is given in the first line of Eq. (1). Similarly, vertices corresponding to $\mathcal{L}_{NN\pi\pi\pi}$ and $\mathcal{L}_{\pi\pi\pi\pi}$ can be obtained from

$$\frac{\delta^4(p+k_f-q-k_i-n)}{(2\pi)^{7/2}\sqrt{8\omega_q\omega_{k_i}\omega_{k_f}}}\frac{m_N}{\sqrt{E_pE_n}}\mathcal{M}_{N\pi\pi\to N\pi}=i\int d^4x\,\langle\mathcal{L}_{NN\pi\pi\pi}\rangle \tag{A2}$$

for the $NN\pi\pi\pi$ vertex and

$$\frac{\delta^4(q'+k_f-k_i-q)}{(2\pi)^2\sqrt{16\omega_{k_i}\omega_{k_f}\omega_q\omega_{q'}}}\mathcal{M}_{\pi\pi\to\pi\pi}=i\int d^4x\,\langle\mathcal{L}_{\pi\pi\pi\pi}\rangle \tag{A3}$$

for the $\pi\pi\pi\pi$ vertex. The results for the charged pions, which correspond to Figs. 5(a)–(c), are given by

$$\mathcal{M}_{NN\pi}=-\frac{g_\pi}{2m_N}\sqrt{2}\gamma^\mu\gamma^5q_\mu\ , \tag{A4}$$

$$\mathcal{M}_{N\pi\pi\to N\pi}=\frac{g_\pi}{2m_N}\frac{1}{4f_\pi^2}2\sqrt{2}\gamma^\mu\gamma^5\times[\xi(k_f-k_i)_\mu-(\xi-1)(2q+k_i-k_f)_\mu]\ , \tag{A5}$$

and

$$\mathcal{M}_{\pi\pi\to\pi\pi}=-\frac{i}{f_\pi^2}[(\xi-1)(s+u)+\xi t-(3\xi-4\eta-2)m_\pi^2]\ , \tag{A6}$$

where we use the Mandelstam variables

$$s=(q+k_i)^2=(q'+k_f)^2\ ,$$
$$u=(q'-k_i)^2=(q-k_f)^2\ , \qquad \text{(A7)}$$
$$t=(q'-q)^2=(k_f-k_i)^2\ .$$

In the above $\omega_q=\sqrt{m_\pi^2+\mathbf{q}^2}$ and $E_p=\sqrt{M_N^2+\mathbf{p}^2}$ denote the pion and nucleon energy, respectively. With the vertices defined, the other necessary diagram rules can be easily found by standard methods of quantum field theory.

For the process $\pi^+ nn \to \pi^- pp$, which is assumed to be described by the interacting Lagrangians given in Eq. (1), the lowest-order scattering amplitude consists of two parts as shown in Figs. 1(a) and (b). By means of the Feynman rules, it is straightforward to write the transition amplitudes for them. However, since it sometimes causes confusion in the literature we would like to discuss first the weighting factors attached to these two diagrams.

The weighting factor for each diagram can be obtained from the analysis of S-matrix expansion,

$$S=\sum_{n=0}^{\infty} S^{(n)}\ , \qquad \text{(A8)}$$

with $S^{(0)}=1$ and

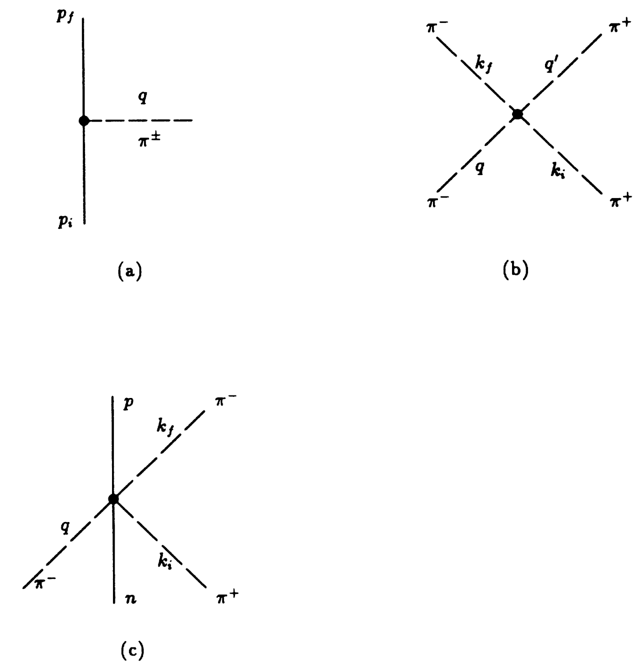

FIG. 5. The vertices relevant to this work: $NN\pi$ [part (a)], $\pi\pi\pi\pi$ [part (b)], and $NN\pi\pi\pi$ [part (c)], representing the expressions given in Eqs. (A4)–(A6).

$$S^{(n)}=\frac{i^n}{n!}\int d^4x_1\int d^4x_2\cdots\int d^4x_n\,\mathcal{T}[\mathcal{L}_I(x_1)\mathcal{L}_I(x_2)\cdots\mathcal{L}_I(x_n)]\ , \qquad \text{(A9)}$$

where $\mathcal{T}$ is a time-ordering operator and $\mathcal{L}_I$ stands for the interaction Lagrangian which is assumed to be in a normal-product order. In the present work, $\mathcal{L}_I$ is simply the sum of three interacting Lagrangians as given in Eq. (1), namely, $\mathcal{L}_I=\mathcal{L}_{NN\pi}+\mathcal{L}_{\pi\pi\pi\pi}+\mathcal{L}_{NN\pi\pi\pi}$. Noting that Figs. 1(a) and (b), respectively, correspond to the third order and the second order in S-matrix expansion, therefore we have

$$S^{[\text{Fig. 1(a)}]}=\frac{i^3}{3!}\int d^4x_1\int d^4x_2\int d^4x_3\,\mathcal{T}[\mathcal{L}_I(x_1)\mathcal{L}_I(x_2)\mathcal{L}_I(x_3)]_{\text{Fig. 1(a)}}$$
$$=\frac{i^3}{2}\int d^4x_1\int d^4x_2\int d^4x_3\,\mathcal{T}[\underbrace{\mathcal{L}_{NN\pi}(x_1)\overbrace{\mathcal{L}_{NN\pi}(x_2)\mathcal{L}_{\pi\pi\pi\pi}(x_3)}}]\qquad \text{(A10)}$$

and

$$S^{[\text{Fig. 1(b)}]}=\frac{i^2}{2!}\int d^4x_1\int d^4x_2\,\mathcal{T}[\mathcal{L}_I(x_1)\mathcal{L}_I(x_2)]_{\text{Fig. 1}(b)}$$
$$=i^2\int d^4x_1\int d^4x_2\,\mathcal{T}[\underbrace{\mathcal{L}_{NN\pi}(x_1)\mathcal{L}_{NN\pi\pi\pi}(x_2)}]\ . \qquad \text{(A11)}$$

In the above, a factor of 3 in Eq. (A10) [a factor of 2 in Eq. (A11)] is removed since we only pick up one characteristic diagram in the second equality among those topologically equivalent diagrams from the expansion of $[\mathcal{L}_I]^3$ ($[\mathcal{L}_I]^2$). Both underbrace and overbrace stand for an operation of pion-field contraction in different Lagrangians, which brings about a free pion propagator. From Eqs. (A10) and (A11), it is rather obvious that these two diagrams, Figs. 1(a) and (b), have different weighting factors. Considering that there is another factor of 2 which comes from the interchange of a pair of nucleons simultaneously in both initial and final states, as seen from the nucleon fields in Eqs. (A10) and (A11), therefore the final invariant transition amplitudes for both pion-pole and pion-contact contributions, using the Feynman rules mentioned above, can be written as

$$T^{(p)}=\frac{g_\pi^2}{4m_N^2}\frac{2}{f_\pi^2}[\bar{u}(p)\gamma^\mu\gamma^5u(n)q_\mu][\bar{u}(p')\gamma^\nu\gamma^5u(n')q'_\nu]\frac{1}{(q^2-m_\pi^2+i0^+)(q'^2-m_\pi^2+i0^+)}$$
$$\times[(\xi-1)(s+u)+\xi t-(3\xi-4\eta-2)m_\pi^2] \qquad \text{(A12)}$$

for the pion-pole diagram contribution of Fig. 1(a), and

$$T^{(c)}=\frac{g_\pi^2}{4m_N^2}\frac{2}{f_\pi^2}[\bar{u}(p)\gamma^\mu\gamma^5u(n)q_\mu]\frac{1}{q^2-m_\pi^2+i0^+}[\bar{u}(p')\gamma^\nu\gamma^5u(n')][\xi(k_f-k_i)_\nu-(\xi-1)(2q+k_i-k_f)_\nu] \qquad \text{(A13)}$$

for the pion-contact diagram contribution of Fig. 1(b). In the above $q=n-p$ and $q'=q+(k_i-k_f)$. (We define $T=i\mathcal{M}$, where $\mathcal{M}$ is an invariant amplitude obtained by means of the Feynman rules.)

APPENDIX B: MULTIPOLE EXPANSION OF PHYSICAL AMPLITUDES

In this appendix, we present some details about the multipole expansion of the physical amplitude, which is frequently used in practical numerical the calculations. The approach and notation are similar to those of Auerbach *et al.*[4]

As seen from the main text, both the pion-pole and the pion-contact amplitudes can be rewritten in a general form

$$F(\mathbf{q})=\tilde{F}(\boldsymbol{\sigma}_1\cdot\mathbf{e})(\boldsymbol{\sigma}_2\cdot\mathbf{e}_2) \ , \qquad \text{(B1)}$$

where $\tilde{F}$ is a spin-independent operator on the unit vectors $\mathbf{e}_1$ and $\mathbf{e}_2$. The physical amplitude is obtained by calculating the matrix element between initial and final nuclear states. In the present study, we choose ^{42}Ca as the target, and restrict ourselves to investigating the double isobaric-analog transition; thus the physical amplitude is defined as

$$F_{fi}=\langle (f_{7/2})^2,J=0|F|(f_{7/2})^2,J=0\rangle \ . \qquad \text{(B2)}$$

Here we neglect any nuclear core excitations.

We can transform rather easily from jj coupling to LS coupling using 9-j symbols. Since we study only the $J=0\rightarrow J=0$ transition, the transformation is in fact much simpler, and only the 6-j symbol is needed in the transformation:

$$|j^2,J=0\rangle=\sum_{\lambda m_\lambda}(-1)^{l+(1/2)+j+m_\lambda}\sqrt{2j+1}\begin{Bmatrix} l & l & \lambda \\ \frac{1}{2} & \frac{1}{2} & j\end{Bmatrix}|(ll)\lambda m_\lambda\rangle|(\tfrac{1}{2}\tfrac{1}{2})\lambda\bar{m}_\lambda\rangle \ . \qquad \text{(B3)}$$

We use the standard Clebsch-Gordan coupling

$$|(ll)\lambda m_\lambda\rangle=\sum_{m_1,m_2}(lm_1,lm_2;\lambda m_\lambda)Y_{lm_1}(\hat{r}_1)Y_{lm_2}(\hat{r}_2) \ , \qquad \text{(B4)}$$

and similarly for $|(\frac{1}{2}\frac{1}{2})\lambda\bar{m}_\lambda\rangle$.

With this transformation and some angular momentum coupling algebra, we can get the following multipole expansion form:

$$F_{fi}(\mathbf{q})=\sum_L F_L \ , \qquad \text{(B5)}$$

with

$$F_L=-\sum_{\lambda\lambda'}D_{\lambda Llj}D_{\lambda' Llj}\int d\mathbf{r}_1\int d\mathbf{q}_2\tilde{F}\rho_{nlj}(r_1,r_2)\mathcal{Y}_{(\lambda 1)L}(\hat{r}_1)\cdot\mathcal{Y}_{(\lambda 1)L}(\hat{r}_2) \ . \qquad \text{(B6)}$$

$D_{\lambda Llj}$ is defined as

$$D_{\lambda Llj}=\sqrt{2(2j+1)}(2l+1)(l0,l0;\lambda 0)\times\begin{Bmatrix} l & \frac{1}{2} & j \\ l & \frac{1}{2} & j \\ \lambda & 1 & L\end{Bmatrix} \qquad \text{(B7)}$$

and

$$\mathcal{Y}_L\cdot\mathcal{Y}_L=\sum_M(-1)^M\mathcal{Y}_{LM}\mathcal{Y}_{L\bar{M}}$$

with the vector spherical harmonic defined in terms of the standard spherical harmonic Y_{lm} by

$$\mathcal{Y}_{(\lambda 1)LM}(\hat{r}_1)=\sum_{m_\lambda m_1}(\lambda m_\lambda,1m_1;LM)Y_{\lambda m_\lambda}(\hat{r}_1)Y_{1m_1}(\hat{e}_1) \ . \qquad \text{(B8)}$$

Here $\rho_{nlj}(r_1,r_2)$ is simply an uncorrelated nuclear density function for a pair of nucleons,

$$\rho_{nlj}(r_1,r_2)=R_{nl}^2(r_1)R_{nl}^2(r_2) \ , \qquad \text{(B9)}$$

where R_{nl} denotes the radial wave function. For simplicity, in this work, we use the harmonic-oscillator wave function for the $0f_{7/2}$ orbit and R_{nl} is normalized as

$$\int R_{nl}^2(r)r^2dr=1 \ .$$

In the pion plane-wave approximation, the above expression can be further simplified. Using the notation given by Auerbach *et al.*,[4] we arrive at the following F_L for the pion-pole contribution:

$$F_L^{(p)} = \frac{3}{\pi}\frac{f^2}{m_\pi^2}\frac{1}{2f_\pi^2}\int \frac{d^3\mathbf{q}}{(2\pi)^3}\tilde{H}_L(q)\tilde{H}_L(q')P_L(\cos\theta_1) \times[4\eta m_\pi^2 + 2\mathbf{q}\cdot\mathbf{q}' - \xi(m_\pi^2 + \mathbf{q}^2 + \mathbf{q}'^2)] , \tag{B10}$$

with

$$\tilde{H}_L(q) = \sum_\lambda D_{\lambda L l j} H_\lambda(q)\sqrt{2\lambda+1}\frac{qv(q)}{q^2+m_\pi^2}(\lambda 0, 10; L0) . \tag{B11}$$

$H_\lambda(q)$ is the multipole nuclear form factor given by

$$H_\lambda(q) = \int r^2 dr\, j_\lambda(qr) R_{nl}^2(r) , \tag{B12}$$

where $j_\lambda(qr)$ is the common spherical Bessel function, θ_1 is the angle between vectors $\mathbf{q}$ and $\mathbf{q}'$ with $\mathbf{q}' = \mathbf{q} + \mathbf{k}_i - \mathbf{k}_f$, and $P_L(t)$ is Legendre polynomial, with $t = \cos\theta_1$.

For the pion-contact contribution, the amplitude as given in Eq. (11) includes two different types of spin-dependent vertices: One is related to $(\boldsymbol{\sigma}_1\cdot\mathbf{q})(\boldsymbol{\sigma}_2\cdot\mathbf{q}')$ while the other has the form $(\boldsymbol{\sigma}_1\cdot\mathbf{q})(\boldsymbol{\sigma}_2\cdot\mathbf{q})$. The former can be managed in the same way as used for the pion-pole term; the latter leads to the second term in the following expression:

$$F_L^{(c)} = \frac{3}{\pi}\frac{f^2}{m_\pi^2}\frac{2\xi-1}{2f_\pi^2}\int \frac{d^3\mathbf{q}}{(2\pi)^3}\tilde{H}_L(q)\tilde{H}_L(q') \times P_L(\cos\theta_1)(m_\pi^2 + \mathbf{q}'^2) - \frac{3}{\pi}\frac{f^2}{m_\pi^2}\frac{1}{2f_\pi^2}\int \frac{d^3\mathbf{q}}{(2\pi)^3}\tilde{H}_L(q)\mathcal{H}_L(\mathbf{q},\mathbf{q}') . \tag{B13}$$

Here we introduce a new function $\mathcal{H}_L(\mathbf{q},\mathbf{q}')$ which is defined by

$$\mathcal{H}_L(\mathbf{q},\mathbf{q}') = \sum_\lambda D_{\lambda L l j} H_\lambda(q')\sqrt{2\lambda+1}\, qv(q') \times(\lambda 0, 10; L0) P_\lambda(\cos\theta_1) . \tag{B14}$$

In practice, it is more convenient to introduce the transformation $\mathbf{y} = \mathbf{q} - \mathbf{Q}/2$, where $\mathbf{Q}$ is the momentum transfer defined as $\mathbf{Q} = (\mathbf{k}_f - \mathbf{k}_i)$. Thus the final expressions used in numerical calculations are actually given by

$$F_L^{(p)} = \frac{3}{m_\pi^2}\frac{f^2}{4\pi}\frac{1}{2f_\pi^2\pi^2}\int y^2 dy\left[4\eta m_\pi^2 + 2(y^2 - Q^2/4) - \xi\left[m_\pi^2 + 2y^2 + \frac{Q^2}{2}\right]\right]\int_{-1}^{1} dx\, \tilde{H}_L(q)\tilde{H}_L(q')P_L(\cos\theta_1) \tag{B15}$$

and

$$F_L^{(c)} = \frac{3}{m_\pi^2}\frac{f^2}{4\pi}\frac{2\xi-1}{2f_\pi^2\pi^2}\int y^2 dy\int_{-1}^{1} dx\, \tilde{H}_L(q)\tilde{H}_L(q')P_L(\cos\theta_1)\left[m_\pi^2 + y^2 + \frac{Q^2}{4} - yQx\right] - \frac{3}{m_\pi^2}\frac{f^2}{4\pi}\frac{1}{2f_\pi^2\pi^2}\int y^2 dy\int_{-1}^{1} dx\, \tilde{H}_L(q)\mathcal{H}_L(\mathbf{q},\mathbf{q}') , \tag{B16}$$

with

$$q = \sqrt{y^2 + Q^2/4 + yQx} , \quad q' = \sqrt{y^2 + Q^2/4 - yQx} , \quad \cos\theta_1 = \frac{y^2 - Q^2/4}{\sqrt{(y^2+Q^2/4)^2 - y^2Q^2x^2}} . \tag{B17}$$

Here, $x = \cos\theta$ with θ being an angle between $\mathbf{y}$ and $\mathbf{Q}$.

In this work, we carry out these integrations over (r,x,y) numerically, using mesh points $N_r = 24$, $N_x = 18$, and $N_y = 45$ accordingly, where a satisfactory convergency in achieved.

As one sees from Eq. (B16) that $\tilde{H}_L(q)\tilde{H}_L(q')P_L(\cos\theta_1)$ is an even function of variable x while its product with Qx is an odd function, therefore, the latter vanishes after integrating x. As a result, by adding two amplitudes of Eqs. (B15) and (B16), one has

$$F_L = F_L^{(p)} + F_L^{(c)} = \frac{f^2}{4\pi}\frac{3}{2f_\pi^2\pi^2}\int y^2 dy\left[(\xi + 4\eta) + \frac{y^2 - \frac{3Q^2}{4} - m_\pi^2}{m_\pi^2}\right]\int_{-1}^{1} dx\, \tilde{H}_L(q)\tilde{H}_L(q')P_L(\cos\theta_1) - \frac{3}{m_\pi^2}\frac{f^2}{4\pi}\frac{1}{2f_\pi^2\pi^2}\int y^2 dy\int_{-1}^{1} dx\, \tilde{H}_L(q)\mathcal{H}_L(\mathbf{q},\mathbf{q}') . \tag{B18}$$

One can see clearly that the summed amplitude depends only on the combination $\bar{\xi}=\xi+4\eta$, which holds for all possible Q values. This is a natural consequence of isotropic ($J=0$) nuclear wave functions, which is also an important condition for the equivalence of two sets of Lagrangians as mentioned in the text.

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