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Shell model calculations for levels and transition rates in ^{204}Pb and ^{206}Pb

D. Wang and M. T. McEllistrem
Department of Physics and Astronomy, University of Kentucky, Lexington, Kentucky 40506

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Level energies and decay rates of both negative and positive parity levels of ^{206,204}Pb have been calculated through mixed-configuration shell model calculations using the modified surface delta interaction (MSDI), the Schiffer-True central interaction, and another two-body interaction. These calculations were all carried out with a full six-orbit neutron hole space. The predicted low-lying levels with the MSDI are in excellent agreement with experiments, accounting for the energies, spins, and parities of essentially all levels below 3 MeV excitation energy except known particle-hole collective excitations in both nuclei. Almost all calculated $E2$ and $M1$ transition rates are consistent with measured branching ratios for γ-ray decay of excited levels. The comparison of the observed and calculated levels demonstrates the important role played by the neutron-hole $i_{13/2}$ configuration in the levels of ^{204}Pb and ^{206}Pb, and interprets an apparent discrepancy over the character and energy spacings of 0^+ levels in ^{204}Pb.

I. INTRODUCTION

The low-lying 0^+ levels of ^{206}Pb and ^{204}Pb are particularly important clues to valence particle structure, because their number reflects directly the number of significant valence configurations near ^{208}Pb. The ^{206}Pb spectrum is well known to have only two excited 0^+ levels below 3 MeV excitation energy, which, together with the ground state, belong primarily to the $p_{1/2}$, $p_{3/2}$, and $f_{5/2}$ valence neutron-hole configurations. The 0^+ levels at 0, 1165, and 2314 keV have energy separations reflecting the single hole energies, which are separated by an average of about 0.6 MeV;[1] exciting pairs to make excited 0^+ levels would lead to about 1 MeV separations for the 0^+ levels, as is observed. One could expect the neutron-hole $i_{13/2}$ configuration to play a role also, but previous[1,2] shell model calculations did not find it to be important for low-lying levels.

The first 2^+ and 4^+ excited levels of ^{204}Pb were shown in careful shell model studies[1,2] to be very similar to pairs of the 2^+ and 4^+ excitations of ^{206}Pb, respectively, with quite similar energy spacings in the two nuclei. Experimental studies[3-5] of ^{204}Pb, in fact, showed a natural parity level spectrum very similar to that of ^{206}Pb. Shell model calculations[1,2] led to the conclusion that the low-lying, natural parity levels of ^{204}Pb could be well described by using the levels of ^{206}Pb as a pair-excitation basis from which ^{204}Pb levels were built. The 0^+ excited levels, on the other hand, are now seen to be difficult to associate between the two nuclei. They are also hard to observe because they do not show the usual γ decays to 2_1^+ levels.

A recent experimental study[6] of the structure of ^{204}Pb suggested the association of three low-lying 0^+ levels in ^{204}Pb, with excitation energies of 0, 1728, and 2433 keV, with apparently the same neutron-hole configurations dominant as those cited for ^{206}Pb. This association was based on similar energy differences and the fact that no γ-ray decays to 2_1^+ levels were observed from these 0^+ levels in ^{204}Pb, just as in ^{206}Pb. However, that proposed association is shown here to be wrong. Kantele *et al.*[7] had proposed that the 2433 keV level was an intruder state, but its excitation energy is now known[8] to be too low to fit intruder state systematics. That systematics would place the intruder 0^+ level above 3 MeV excitation energy.[8] Further, its recently reported γ-ray decay[6] suggests that it is a valence excitation.

Another low-lying excited 0^+ level in ^{204}Pb had been reported near 1584 keV excitation energy in a conversion electron study,[9] and was more recently found in a high resolution study[7] at 1582.7 keV. The next higher 0^+ level was firmly established at 1728 keV in both the newer conversion study[7] and also in careful studies of the (p,t) reactions from ^{206}Pb.[10,11] There seemed to be one more low-lying 0^+ level belonging to valence neutron-hole configurations in ^{204}Pb than in ^{206}Pb, with a spacing between two of these levels of only 147 keV. No remotely similar spacing occurs in ^{206}Pb, in contrast to the findings that there is a strong correlation between the other natural parity excitations of the two nuclei.[2] Thus the configurations and energies expected for 0^+ levels needs examination.

The earlier experiments for levels[3-5] of ^{204}Pb did not emphasize unnatural parity levels, nor provide much information about odd parity levels. This lack of information and the question of the 0^+ levels led to the recent and detailed experimental study of the level and decay schemes of this nucleus[6] which provided several new levels, spin assignments, branching ratios, and $E2/M1$ mixing ratios. This experiment also discovered a previously missed 2^+ level at 1582.8 keV, within 0.5 keV of the excitation energy previously reported for the lowest excited 0^+ level. A question arose in the $(n,n'\gamma)$ study[6] as to whether there were really two distinct levels within 0.5

keV of each other, particularly as the high resolution conversion study[7] had attributed a 684-keV cascade transition as an $E2$ decay of the 0^+ level. The $(n, n'\gamma)$ study of the levels showed in contrast that this transition was in fact dominated by an anisotropic decay from the 2^+ level.[6]

The question of whether there were really two levels of different spin within 0.5 keV of each other has now been resolved[12] in the affirmative by experiments exciting both levels in two different reactions; the "684" keV cascade transition intensity differed by a factor of 25 in the two reactions. The resolution of this problem, and the fact that there are four low-lying 0^+ levels in ^{204}Pb, but only three in ^{206}Pb, encouraged us to make new shell model calculations for these nuclei. Also the large amount of new experimental information about odd-parity and unnatural parity levels, and about electromagnetic decay rates,[6] suggested new calculations.

II. SHELL MODEL CALCULATION

Shell model calculations were carried out on a μ-VAX II computer using the well known code OXBASH82.[13] The advent of new experimental information since 1980 suggested that we make as complete a comparison of valence-space excitations and levels and decay properties of ^{206,204}Pb as possible. A full six-orbit calculation, with up to two holes in ^{206}Pb and four holes in ^{204}Pb, and with no other truncations or occupancy restrictions, has been carried out. Thus the mixed configuration probabilities to be presented for calculated levels are of the form: va_n^2, where the a_n is a probability-amplitude for configuration n of a unit normalized wavefunction and v is the number of valence holes. These probabilities are also spectroscopic factors for single nucleon transfer experiments.

The single hole orbits and their energies relative to the ^{208}Pb core shown in Table I are taken from the experimental ^{207}Pb spectrum[14] and used as the inputs to the various calculations without modification. Several different residual interactions were used in the calculations to generate the two body matrix elements (TBME's). One is the modified surface delta interaction[15] (MSDI) which has often been very successful in characterizing levels and electromagnetic properties. The others were the Schiffer-True[16](ST) and Gogny's[17] D1 central interactions. The ST and D1 interactions were used in the calculations without parameter adjustment, except that the calculated TBME's were normalized to obtain the best level spacing in ^{206}Pb. That TBME normalization was then used unaltered for ^{204}Pb. The MSDI has two parameters each for $T = 0$ and $T = 1$ couplings, respectively. Since the proton orbits of the lead isotopes are closed, the TBME's needed to describe the active neutron holes depend only on the two $T = 1$ parameters, denoted as A_1 and B_1. The diagonal matrix element can then be written as

$$\langle j^2|V^{\text{MSDI}}|j^2\rangle_{J=\text{even},T=1} = -A_1\frac{(2j+1)^2}{2(2J+1)}\langle j-\tfrac{1}{2}j\tfrac{1}{2}|J0\rangle^2 + B_1$$

where j, J, and T are the angular momenta of the single particle state, and angular momentum and isospin of the nuclear state, respectively. The B_1 is a monopole term which only gives a fixed energy shift for the whole set of levels. The ground state and first excited state energies of ^{206}Pb fix A_1 and B_1. The two parameters determined in this way have the values A_1=0.1445 and B_1=0.1090. All levels of both Pb nuclei except special core excitations, as well as electromagnetic properties, are obtained without any additional parameters except an $E2$ effective charge and $M1$ quenching factor. All parameters, including the effective charge and $M1$ quenching factor, are taken to be the same for both nuclei. The effective charge was determined to be $e_n = 0.85e$ and the magnetic dipole quenching factor was set to be 0.5 to optimize fits to branching ratios and $E2/M1$ mixing ratios in the two Pb nuclei.

III. RESULTS AND DISCUSSION

A. ^{206}Pb configurations and level scheme

The principal reasons for reexamining the levels of ^{206}Pb are to test the completeness of the six orbit valence space for few valence-particle Pb nuclei, to test several residual interactions, and to set parameters for each one. The parameters could then be used in the ^{204}Pb calculations, to have a consistent model for both nuclei.

The ground state spectroscopic factors, as defined above, are calculated for ^{206}Pb and shown in Table II, together with experimentally determined spectroscopic factors from (d,p) reaction studies.[18,19] Complete tabulations of spectroscopic factors for all levels are available from the authors. The present model-calculated results appear in rows (c) through (f), different rows representing different interactions. The earlier shell model calculations of Takahashi *et al.*[20] and of Harvey and Clement[21] show very small $i_{13/2}$ probabilities in the ground state wave functions. Row (c) is for the Shiffer-True interaction (ST1), with an effective range of r_2=2.0 fm; it does

TABLE I. The single neutron hole orbits and energies relative to the ^{208}Pb core.

	$p_{1/2}$	$f_{5/2}$	$p_{3/2}$	$i_{13/2}$	$f_{7/2}$	$h_{9/2}$
Energy (MeV)	−7.38	−7.95	−8.27	−9.01	−9.72	−10.85

TABLE II. The squared amplitudes of various of ^{206}Pb ground state configurations.

	$p_{1/2}$	$f_{5/2}$	$p_{3/2}$	$i_{13/2}$	$f_{7/2}$	$h_{9/2}$
(a) Expt.[1]	1.08	0.40	0.24	0.28	0.08	0.0
(b) Expt.[2]	1.30	0.50	0.4			
(c) ST1[3]	1.948	0.072	0.058	0.008	0.006	0.002
(d) ST2[3]	1.448	0.394	0.152	0.060	0.030	0.014
(e) MSDI[4]	1.012	0.464	0.168	0.236	0.074	0.048
(f) D1[4]	1.464	0.232	0.202	0.054	0.036	0.012
(g) TAK[5]	1.264	0.304	0.308	0.054	0.056	0.012
(h) HAR[6]	1.186	0.30	0.332	0.098	0.064	0.020

[1] Reference 18.
[2] Reference 19.
[3] Reference 16.
[4] This work.
[5] Reference 20.
[6] Reference 21.

not correspond well to the experiments. Row (d) shows a modified Schiffer-True interaction (ST2) with the effective range reduced to r_2=0.1 fm. That works much better, but still does not show spectroscopic factors which correspond well to the experimental results, particularly for the $i_{13/2}$ configuration. Row (f) shows results obtained from the use of Gogny's D1 interaction which has been very successful in describing the ground-state properties of a wide range of nuclei. This calculation used only the central, non-density-dependent parts of Gogny's D1 interaction, and that produces good results, nearly identical to the ST2 interaction.

The best results are obtained with the MSDI, which is parametrized for these calculations. The most interesting part of the MSDI results is the fine way they reproduce the $i_{13/2}$ probability, as well as the very good match with other configurations, as is evident in Table II. The excellent agreement with measured spectroscopic factors shows that including the full six orbit valence space, with full inclusion of the $i_{13/2}$ configuration, together with a surface-peaked effective interaction is essential to obtaining the best possible representation of these nuclei.

The importance of including the full, untruncated valence space is also illustrated in the comparisons of experimental and calculated level schemes of Figs. 1 and 2, but most especially that of Fig. 2. The middle column of these level diagrams is always the experimental level scheme. The left-most column is from our MSDI calculations, and the column on the right comes always from the recent shell model calculation of Takahashi *et al.*[20]

The ground state and first excited level energies of ^{206}Pb were used in our calculations to fix the two MSDI parameters: their match is thus guaranteed. The rest of the spectrum is in beautiful agreement with the experimental spectrum shown in the two figures. All the way up to 3.8 MeV excitation energy, and including all levels, the average deviation is only about ± 30 keV. The Takahashi *et al.* results are also good, but with an average deviation about three times ours. Our improvement correlates with the improved calculations of configuration strengths, as given in Table II; only the MSDI results reproduce the experimental configuration strengths well.

The level scheme in Fig. 2 shows the odd and unnatural parity states, the latter being calculated for the first time. The match with the experimental results using the MSDI is again especially good. The model shows many more states than have been observed experimentally, which is to be expected.

We expect and find the three 0^+ levels calculated at 0, 1114, and 2287 keV to correspond to the experimental ones at 0, 1165, and 2314 keV. The calculation shows a fourth 0^+ state calculated at 3243 keV whose configuration is dominated by $(i_{13/2})^2$. Interestingly enough, the cylcotron group at Jyväskyla has observed[22] an $E0$ conversion line from a level above 3 MeV excitation energy. This is probably the fourth valence-space 0^+ level.

Known core excitations of ^{208}Pb are of course not included in these valence space calculations; any level observed in ^{208}Pb is a particle-hole core excitation, and hence not in the valence space of this model. These core excitations would be expected to appear in ^{206}Pb at approximately the same excitation energies as in ^{208}Pb. The collective 3^- in ^{206}Pb is well known at 2.648 MeV; no corresponding level occurs in the calculated spectrum. Another strong collective level in ^{208}Pb, the 5^- at 3.197 MeV, has a corresponding level at 3.54 MeV in ^{206}Pb; that also is missing from our calculated level spectrum.

B. ^{204}Pb configurations and level scheme

The spectroscopic factors of the ground state of ^{204}Pb as measured[23] and as found in several shell model calculations are given in Table III. Again, only the MSDI interaction treats the valence configurations adequately; it is the only one which reflects the fact that the $i_{13/2}$ configuration is quite important in this nucleus, comparable to the $p_{1/2}$, $p_{3/2}$, and $f_{5/2}$ configurations. Thus we conclude that both the form of the residual interaction and the model space seem to be important in getting the $i_{13/2}$ spectroscopic factor as large as that required by experiments; our calculations with interactions other than the MSDI provided results much less satisfactory than those calculated with the MSDI. We conclude that all four valence orbits are equally important for the low-

TABLE III. The squared amplitudes of various of ^{204}Pb ground state configurations.

	$p_{1/2}$	$f_{5/2}$	$p_{3/2}$	$i_{13/2}$	$f_{7/2}$	$h_{9/2}$
(a) Expt.[1]	1.4	2.2	0.6	1.3	0.1	0.0
(b) HAR[2]	1.24	1.11	1.06	0.3	0.23	0.06
(c) MSDI[3]	1.255	1.369	0.473	0.609	0.182	0.112
(d) D1[3]	1.474	1.285	0.777	0.251	0.163	0.049

[1] Reference 23.
[2] Reference 21.
[3] This work.

lying levels of ^{204}Pb. Note that the quoted[23] experimental spectroscopic factors in Table III must contain errors; they should not sum to more than the number of valence holes, or four.

We had originally been drawn to the present calculations in part by the apparent difference in 0^+ level spacings, and number of low-lying 0^+ levels, in the two nuclei as clarified in the experiments of the last few years.[6,7,12] Liotta and Pomar had explicitly identified the appropriate ^{206}Pb pair excitations[2] for each ^{204}Pb level. But the extra 0^+ level of ^{204}Pb had not then been noted, nor was the 1582.8 keV 2^+ level known. The fourth 0^+ level and small 0^+ spacings in ^{204}Pb might be difficult to comprehend. This is now realized to be the result of the sharply increased importance of the $(i_{13/2})^2$ configuration in ^{204}Pb. The calculated 0^+ excitation energies

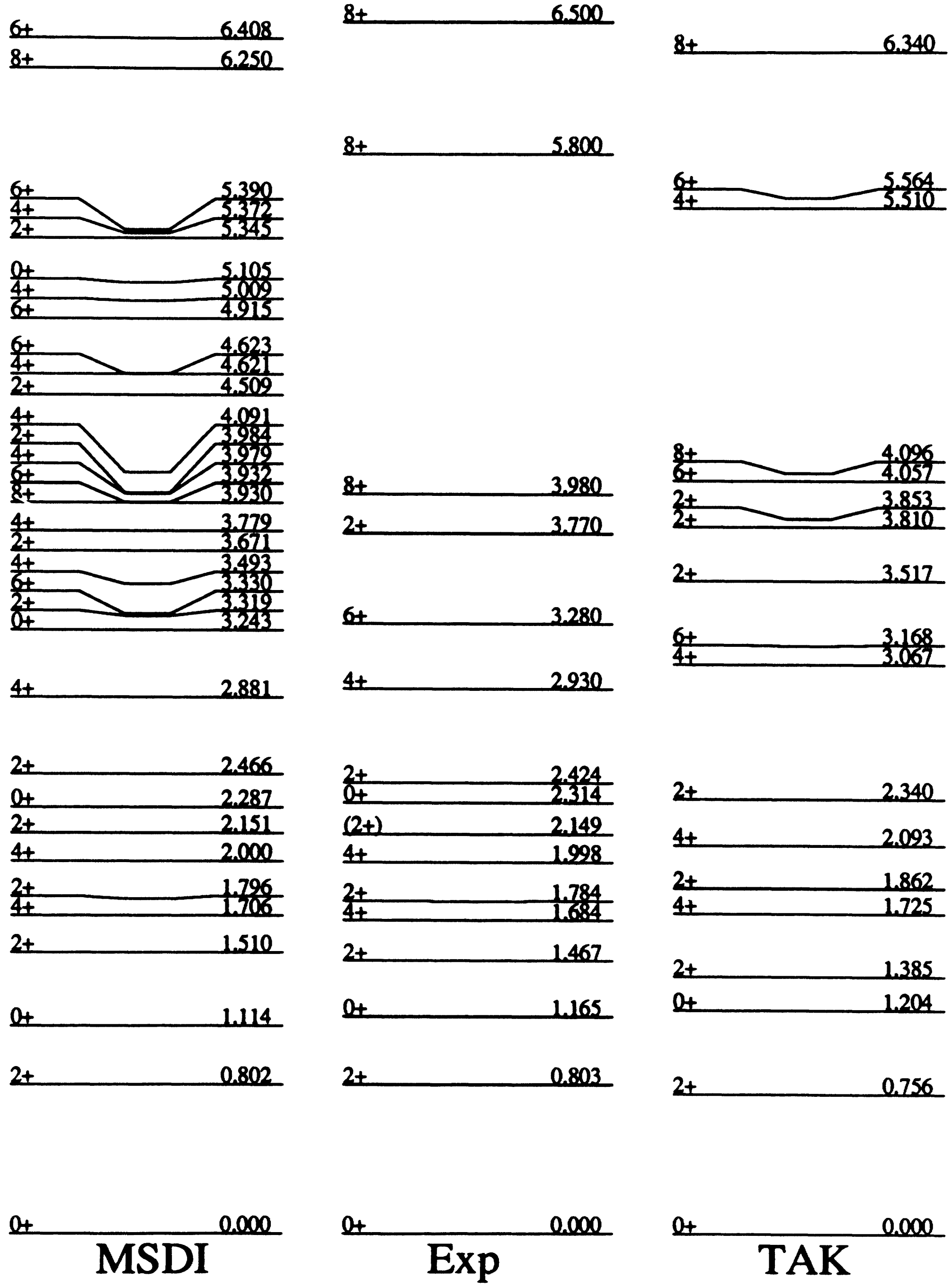

FIG. 1. The level scheme of natural, even parity states in ^{206}Pb.

with the MSDI are 0, 1537, 1809, and 2346 keV, which agree quite well with the measured energies of 0, 1583, 1728, and 2433 keV.

The relationship proposed by McGrory and Kuo[1] and confirmed in the calculations of Liotta and Pomar[2] between levels of the two nuclei still applies very well even to our calculated 0^+ levels, in spite of the very different 0^+ energy spacings in the two Pb isotopes. We can test that picture by examining the calculated configurations of the levels of the two nuclei.

The two particle parentage amplitudes of the ground state and first excited 2^+ levels of ^{204}Pb, based on the ground state of ^{206}Pb, are explicitly given in Table IV. The parentage amplitudes of other levels are not tabulated here, but our calculated configuration probabilities show that the ^{204}Pb ground state is only weakly coupled to any other 0^+ or 2^+ level of ^{206}Pb. Similarly, the calculated 0_2^+ level of ^{204}Pb, at 1537 keV, is strongly coupled to the calculated 1114 keV 0^+ level of ^{206}Pb, and very weakly to all others. A similar coupling occurs between

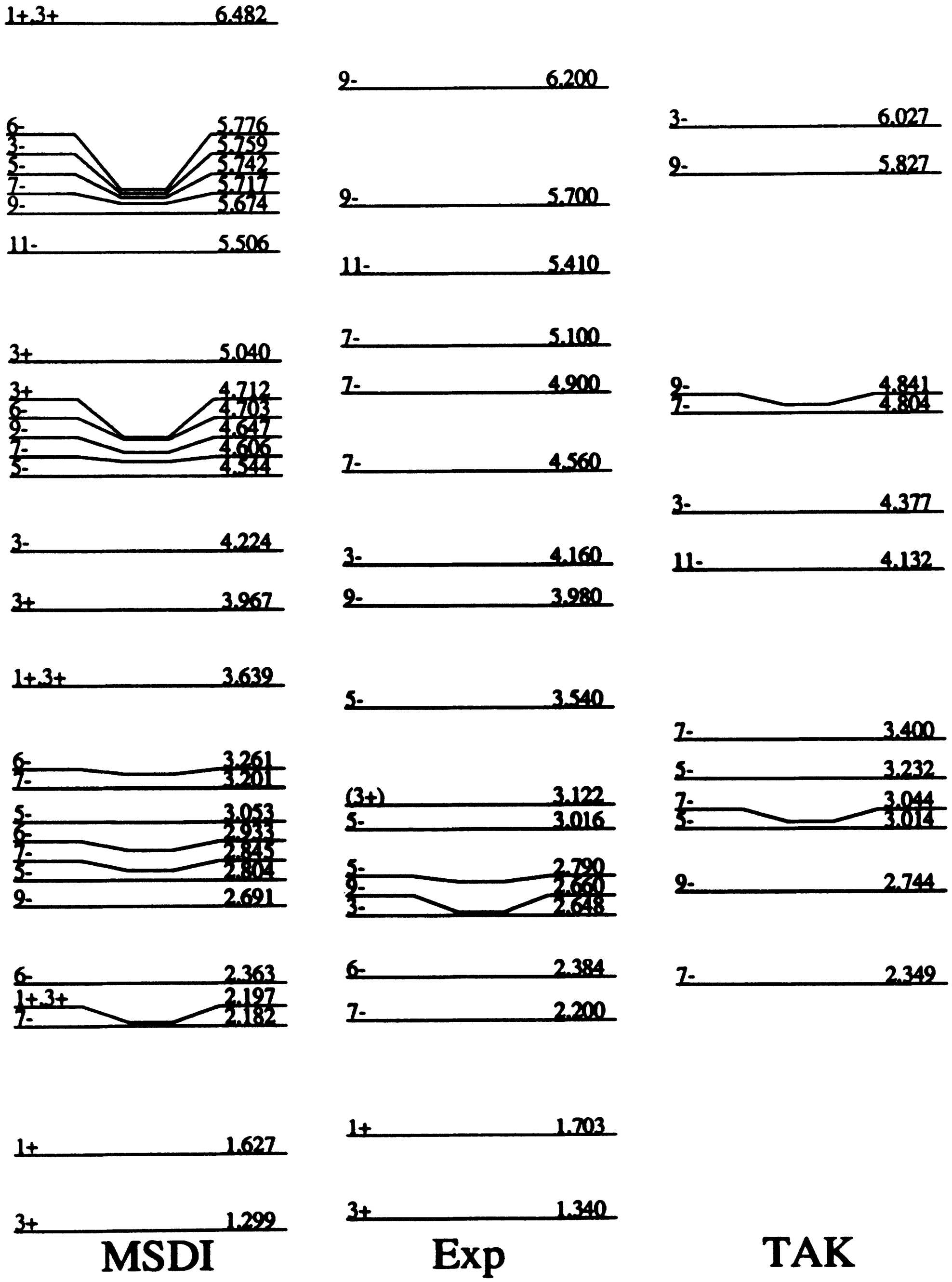

FIG. 2. The level scheme of unnatural and odd parity states in ^{206}Pb.

TABLE IV. The two-particle parentage amplitudes of the ground state (g.s.) and first 2^+ state in ^{204}Pb based on the g.s. configuration of ^{206}Pb

	g.s.		2^+	
	TAK[1]	MSDI[2]	TAK[1]	MSDI[2]
$p^2_{1/2}$	0.59	0.554		
$f^2_{5/2}$	0.805	0.793	−0.552	−0.472
$p^2_{3/2}$	0.672	0.474	−0.242	−0.180
$i^2_{13/2}$	0.281	−0.543	−0.084	0.166
$f^2_{7/2}$	0.287	0.296	−0.097	−0.079
$h^2_{9/2}$	0.125	0.233		−0.059
$p_{1/2}f_{5/2}$			−0.259	−0.388
$p_{1/2}p_{3/2}$			0.248	−0.232
$f_{5/2}p_{3/2}$			0.242	0.223
$f_{5/2}f_{7/2}$			0.062	−0.065
$p_{3/2}f_{7/2}$			−0.188	−0.174
$f_{5/2}g_{9/2}$				−0.158
$f_{7/2}g_{9/2}$				0.027

[1] Reference 20.

[2] This work.

the calculated 0^+_3 level at 1809 keV and the calculated 2287 keV 0^+ level of ^{206}Pb. Finally, the calculated level at 2346 keV in ^{204}Pb would correspond to a (calculated) 0^+ level at 3243 keV in ^{206}Pb. This level has not yet been clearly fixed experimentally, although it has probably been observed.[22] The present model calculations are then quite consistent with the very recent experimental confirmation[12] of the presence of both 0^+ and 2^+ levels within 0.5 keV of each other, near 1583 keV in ^{204}Pb.

The natural, even parity levels of ^{204}Pb are shown in Fig. 3. Here one finds again quite good agreement between the MSDI calculations and the experimental level scheme. Note that no MSDI parameters are adjusted for these calculations; the A_1 and B_1 fixed for ^{206}Pb are used without modification. The good agreement of our results with measurements follows partly from the stronger role played by $(i_{13/2})^2$ components.

It is interesting to note, nonetheless, that *single particle* $i_{13/2}$ amplitudes do not appear at low excitation energies, but only above 2.25 MeV. That is why no odd parity levels appear until that excitation energy in either the observed or MSDI spectra, as is evident in Fig. 4. The lowest observed odd-parity level is at 2.258 MeV, and the lowest calculated one is at 2.27 MeV.

The calculated 2^+ levels at 1673 and 1719 keV both have very strong configuration couplings to the ground state of ^{206}Pb which would lead to strong excitation in (p,t) transfer reactions. Only one state, at 1665 keV, was observed[11] in the (p,t) experiment. It is quite possible that the 2^+ level found[6] at 1583 keV which corresponds to the calculated 1673 keV level was also excited, but obscured in the (p,t) reaction study by the unknown presence of the 0^+, also at 1583 keV, and the 4^+ at 1564 keV, as suggested[12] recently.

C. Lifetimes and $E2$ decay rates

A strong test of the adequacy of the model space used in these calculations comes from examination of electromagnetic transition rates. As noted in the Introduction, the effective charge of $0.85e$ was used for all $E2$ decays of both nuclei. A quenching factor of 0.5 was adopted uniformly for the magnetic dipole operator in both nuclei.

1. Isomer lifetimes

The 4^+_1 state of ^{204}Pb decays mainly to the 2^+_1 by an $E2$ transition, and also with a much lower rate directly to the ground state by an $E4$ transition. The level is known to have a rather long half-life of 0.27 μs. The shell model calculation yields an $E4$ ground state transition branch of about 1%. The measured branching ratio is about 3%, which is rather good agreement. However, the calculation fails to reproduce the half-life of the 4^+_1 state, the calculated value being about two orders of magnitude too long. Thus even with a model which reproduces so many detailed properties well, we see evidence of core polarization components which are outside our model space.

Core polarization components would be expected to be much less important for high spin, odd parity levels, since few particle-hole excitations would couple to high spin. Fortunately for present tests, the 9^-_1 state at 2186 keV is a well known metastable state in ^{204}Pb with a half-life equal to 66.9 minutes. This level decays mainly to the 4^+_1 with a 97% branch and to the 4^+_2 with a 3% branch. The MSDI shell model calculation produces a half-life of 43.4 minutes which is in excellent agreement with the empirical result. The calculation shows that the transition rate to the 4^+_2 is about 1% of the transition rate to the 4^+_1, also in agreement with the experiment. The agreement between experiment and calculation for the 9^-_1 state, both for decay branching ratio and for lifetime, argues that the valence space and interaction are well chosen.

2. 0^+ excited level decays

All of the 0^+ excited levels of both nuclei have been discovered and assigned[7,9] through the intense $E0$ conversion electron decays; none have been reported in γ-ray experiments, although several γ-ray detection experiments have been done in both nuclei.[5,6,24] This is rather unusual. The MSDI calculations help interpret this; the $E2$ decay intensities of all 0^+ excited levels to 2^+_1 levels are calculated to be one or two orders of magnitude smaller than single particle speed, as can be seen in Table V. These would have been the most obvious transitions, but they are too weak to be observable.

The calculated $B(E2)$'s for decay of excited 0^+ levels in both nuclei are presented in Table V, together with the single particle rate. For ^{206}Pb, a rate comparable to single particle speed is calculated only for the 0^+_3 decay to the second and third 2^+ excited levels, corresponding to experimental transition energies of 847 and 530 keV, respectively. Although the most recent γ-ray study[6] was of ^{204}Pb, a radiolead sample enriched to 87 % in ^{206}Pb was also run at all energies and angles used for the ^{204}Pb measurements. Reexamining these spectra in the light of these calculations enables us to identify both 0_3^+ decays, although they are almost obscured by very strong neighboring lines at 856 and 537 keV, respectively.

This is shown in Fig. 5, which shows partial spectra for transitions in ^{206}Pb. These data are from the experiment[6] of Hanly *et al.* The top panel of Fig. 5 shows the E_γ region near 500 to 800 keV. One sees there the clean 657 and 664 keV lines just above channel 1000. The strong 537 keV transition from the 1340 keV 3^+ level partially obscures the 530 keV 0^+ decay, but the broadened region near the base of the line shows clearly the presence of a line near 530 keV. The lower panel of Fig. 5 shows the γ-ray region from 750 to 1150 keV. One

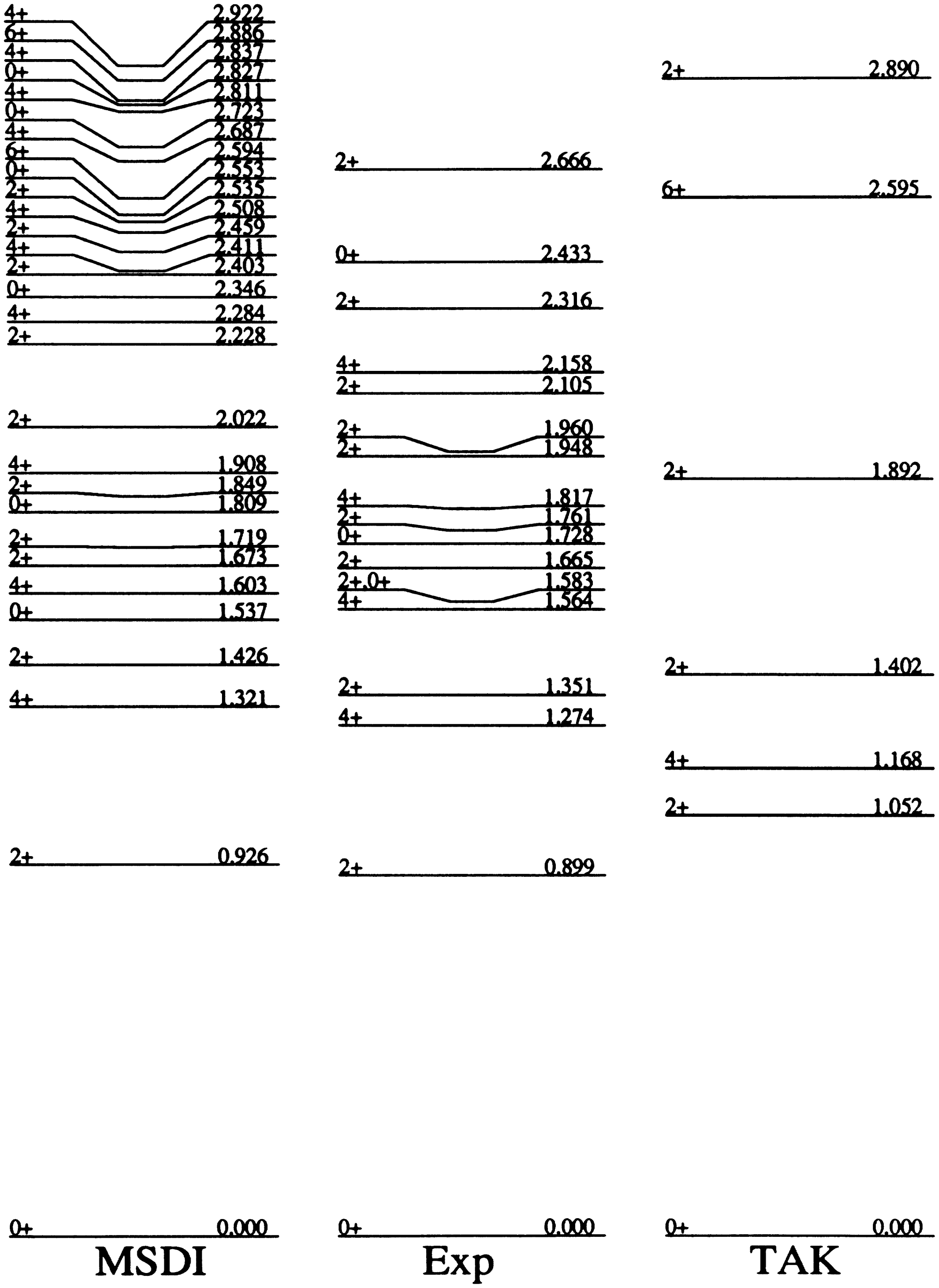

FIG. 3. The level scheme of natural, even parity states in ^{204}Pb.

sees the moderate intensity 856 keV decay of the 2196 keV level,[24] and just below that the very weak 847 keV line which is the other expected decay of the 0_3^+ level of ^{206}Pb. Both of these lines, expected on the basis of our calculations, are marked with small arrows in the spectra. The 0^+ levels are quite weakly excited in $(n, n'\gamma)$ experiments, which accounts for the weakness of their appearance in these spectra.

The only excited 0^+ decays of ^{204}Pb with $B(E2)$ values comparable to single particle speed have transition energies coincident with those of strong decays from other levels. Thus γ-ray decays of 0^+ excited levels could not be expected to be observed for either nucleus; they were not identified in experiments.

A comparison between measured branching and $E2/M1$ mixing ratios and those from these shell model

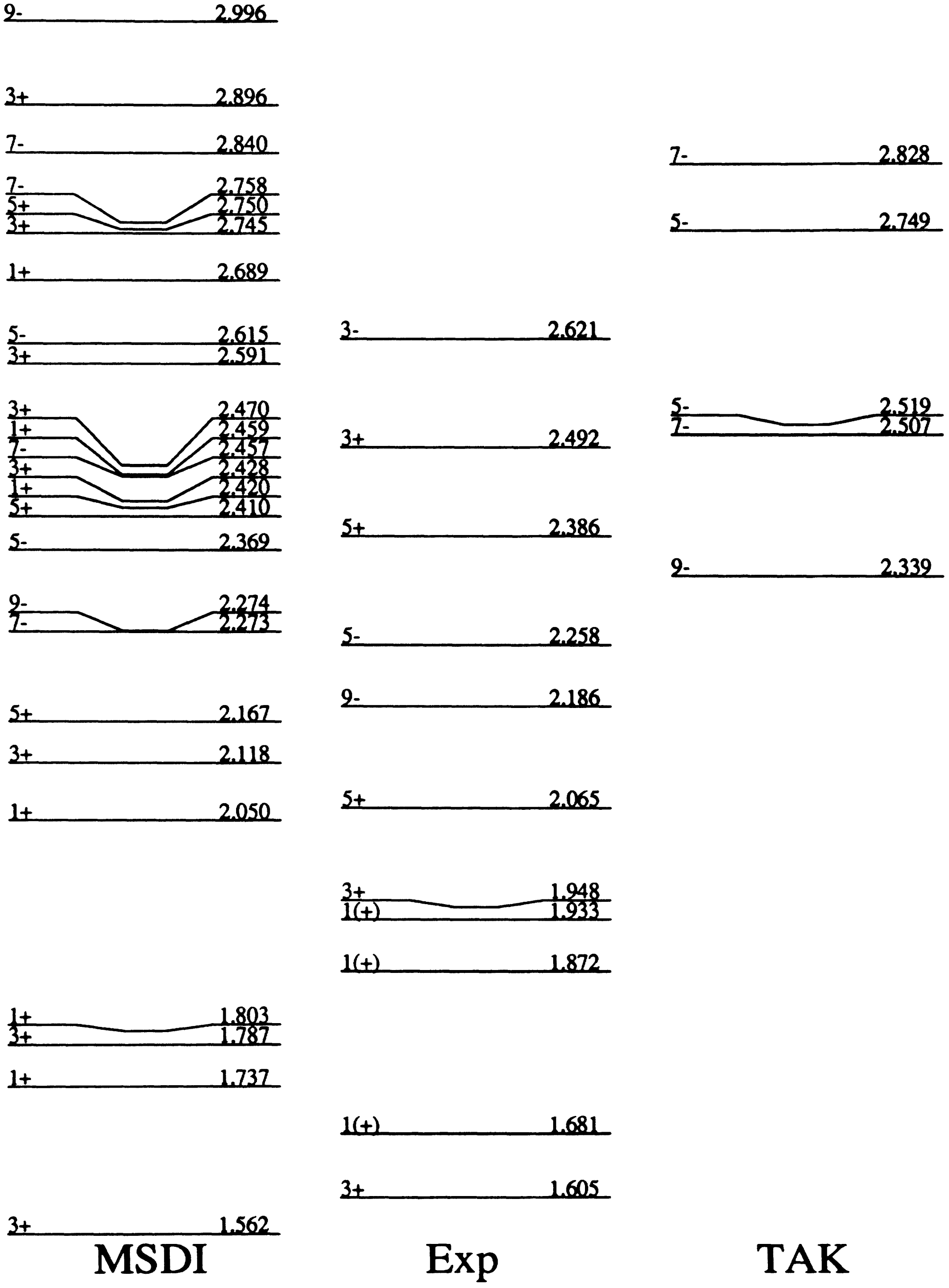

FIG. 4. The level scheme of unnatural and odd parity states in ^{204}Pb.

TABLE V. The reduced electromagnetic transition probabilities of the $0^+ \rightarrow 2^+$ states in ^{206}Pb and ^{204}Pb.

	^{206}Pb	
E_γ (MeV)	$J_i \rightarrow J_f$	$B(E2)\downarrow$ (e^2 fm^4)
0.362	$0_2^+ \rightarrow 2_1^+$	0.069
1.511	$0_3^+ \rightarrow 2_1^+$	2.78
0.847	$0_3^+ \rightarrow 2_2^+$	117.8
0.530	$0_3^+ \rightarrow 2_3^+$	92.92
	^{204}Pb	
E_γ (MeV)	$J_i \rightarrow J_f$	$B(E2)\downarrow$ (e^2 fm^4)
0.684	$0_2^+ \rightarrow 2_1^+$	10.10
0.831	$0_3^+ \rightarrow 2_1^+$	1.521
0.232	$0_2^+ \rightarrow 2_2^+$	103.0
0.379	$0_3^+ \rightarrow 2_2^+$	85.48
0.147	$0_3^+ \rightarrow 2_3^+$	38.53
0.065	$0_3^+ \rightarrow 2_4^+$	43.26

$B_{sp}(E2)\downarrow$ (^{206}Pb)=72.27 e^2 fm^4.
$B_{sp}(E2)\downarrow$ (^{204}Pb)=71.33 e^2fm^4.

calculations is presented for several other levels in Table VI. The levels and decays indicated in Table VI are those for which good measurements exist.[6] The agreement between measurements and calculations is quite good except for the 2_3^+ and 1_3^+ decays.

IV. CONCLUSION

Several shell model calculations with simple two-body effective interactions have been made. These have all

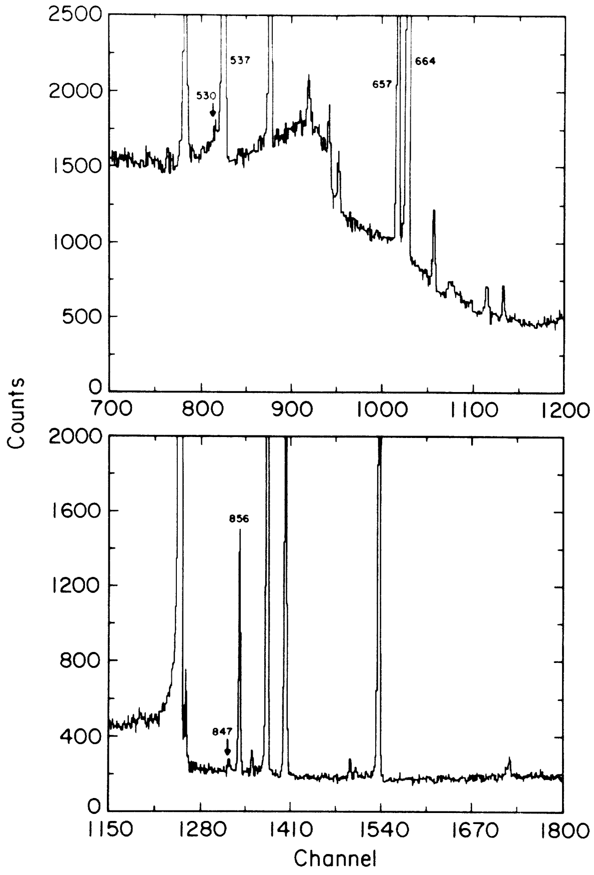

FIG. 5. The energy spectra of ^{206}Pb$(n,n'\gamma)^{206}$Pb, at $E_n = 2.6$ MeV. The top and lower panel show γ-ray energy regions of 500–800 keV and 750–1150 keV, respectively.

TABLE VI. Reduced electromagnetic decay properties of the levels in ^{204}Pb.

$J_i \rightarrow J_f$	$B(E2)\downarrow^1$ (e^2 fm^4)	$B(M1)\downarrow^1$ (e^2 fm^2)	δ Calc.[1]	δ Expt.[2]	Decay prob. Calc.[1]	Decay prob. Expt.[2]
$2_2^+ \rightarrow 0_1^+$	9.5				17	22
$2_2^+ \rightarrow 2_1^+$	175	0.39×10^{-4}	0.93	0.8	83	78
$2_3^+ \rightarrow 0_1^+$	9.2				77	10
$2_3^+ \rightarrow 2_1^+$	117	0.12×10^{-4}	2.0	−1.7	22	90
$3_1^+ \rightarrow 2_1^+$	2.5	0.76×10^{-5}	0.32	0.2	33	41
$3_1^+ \rightarrow 4_1^+$	11.9	0.30×10^{-3}	0.04	0.1	66	59
$2_4^+ \rightarrow 2_1^+$	6.32	0.19×10^{-3}	0.13	0.1	56	51
$2_4^+ \rightarrow 0_1^+$	6.37				44	49
$1_1^+ \rightarrow 0_1^+$		0.61×10^{-4}			60	52
$1_1^+ \rightarrow 2_1^+$	0.47	0.35×10^{-3}	0.03	0.1	35	48
$1_1^+ \rightarrow 2_2^+$	32	0.78×10^{-3}	0.05		4.4	
$1_3^+ \rightarrow 0_1^+$		0.31×10^{-7}			1	89
$1_3^+ \rightarrow 2_1^+$	0.36	0.36×10^{-4}	0.1	0	99	11
$3_2^+ \rightarrow 2_1^+$	0.13	0.11×10^{-6}	0.84		0.5	12
$3_2^+ \rightarrow 4_1^+$	0.03	0.16×10^{-3}	0.006		52	15
$3_2^+ \rightarrow 2_2^+$	5.6	0.32×10^{-3}	0.04		48	72
$2_5^+ \rightarrow 0_1^+$	0.34				4.6	41
$2_5^+ \rightarrow 2_1^+$	159	0.38×10^{-4}	1.6	1.4	90	52
$2_5^+ \rightarrow 2_2^+$	0.91	0.86×10^{-4}	0.04		5.3	7

[1] This work.

[2] Reference 6.

provided realistic level schemes for two even-A Pb nuclei. Calculations of ^{204,206}Pb with the MSDI interaction, however, provide remarkably detailed agreement with level energies and electromagnetic decay properties. They explain fully, for example, that none of the excited 0^+ levels should have observable γ-ray decays to 2_1^+ levels, that the 9_1^- level of ^{204}Pb should have a half-life of approximately 66 minutes, and that the 4_1^+ level of ^{204}Pb should be an isomeric state. We have calculated $B(M1)$ and $B(E2)$ transition rates, $E2/M1$ mixing ratios, and branching ratios which agree favorably with measured decay rates for all but a couple of levels.

The comparisons between model and experiments with the MSDI interaction, with only two parameters for the two nuclei, and two additional parameters for all electromagnetic transition rates, are considerably improved from those reported earlier without altering a principal physical conclusion of earlier studies. The excitations of ^{206}Pb can be considered as a boson basis for the excitations of ^{204}Pb. The strong association between 0^+ and 2^+ levels of the two Pb nuclei is again confirmed, up to an excitation energy of about 2 MeV, just as found by Liotta and Pomar[2] in their detailed examination of that correspondence. These new calculations do show that there should be four 0^+ levels in ^{204}Pb within the excitation energy span in which only three are found in ^{206}Pb, in agreement with recent experiments. The decreased energy spacing between 0^+ levels results from the increased importance of the $i_{13/2}$ configuration in ^{204}Pb as opposed to ^{206}Pb. The results show, in contrast to earlier analyses, that the $(i_{13/2})^2$ configuration is as important as any other in the structure of ^{204}Pb. Finally, the calculations account for all measured levels of both nuclei below an excitation energy of about 2.4 MeV except for the ones clearly identified as particle-hole core excitations by their correspondence to excitations of ^{208}Pb.

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