Structure and decay modes of antisymmetric β vibrations in the O(6) limit of the neutron-proton interacting boson model

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We investigate the structure of the intrinsic states for the symmetric and antisymmetric β vibrations in the O(6) limit of the neutron-proton interacting boson model and of the low-lying configurations projected from these intrinsic states. We study the decay modes of these low-lying states, particularly the antisymmetric 0^+ state. This level appears to decay only to the 1^+ and 2^+ mixed-symmetry states. The signature for these decays is that the $M1$ and $E2$ strengths go to zero for $N_\pi = N_\nu$.

I. INTRODUCTION

Ever since the theoretical papers of Iachello[1] suggesting the algebraic properties of mixed-symmetry states within the proton-neutron interacting boson model[2] (also referred to as the IBM-2) and the subsequent experimental observation[3] of the predicted $M1$ scissors mode in ^{156}Gd, there has been considerable interest, both theoretically[4] and experimentally,[5] in describing and locating such excitations in a wide variety of medium-to-heavy-mass nuclei. Besides the small amplitude, scissorlike counter oscillation[1,6,7] between the proton and neutron distributions in well-deformed nuclei [the SU(3) limit of the IBM-2] leading to an excitation of $J^\pi = 1^+$, there are predictions of a mixed-symmetry 2^+ state in the U(5) limit of the IBM-2,[1,6,-9] corresponding to small-amplitude β vibrations of the protons and neutrons which are 180° out of phase, with some experimental indication of their existence.[5]

While the properties of SU(3)- and U(5)-like nuclei have been described theoretically in several approaches,[4,6-11] including the IBM, the structure of O(6)-like nuclei is less well understood, with many predictions regarding the properties of such nuclei coming from the IBM.[1,4,6,7] Detailed studies have been made of the low-lying properties of O(6) nuclei within the IBM-2 for the totally symmetric states $[N]$ (i.e., the IBM-1 configurations or equivalently the states of maximal F spin[2] $F_{\max} = \frac{1}{2}N$, where $N = N_\pi + N_\nu$ = number of proton bosons + number of neutron bosons) and the first set of mixed-symmetry states $[N-1,1]$ (i.e., those with an F-spin value of $F_{\max} - 1$). The label $[N-f,f]$ denotes the U(6) representation of the IBM-2 configurations. In previous studies[6,7] one has concentrated on the states with the maximum O(6) symmetry or, equivalently, maximum sigma quantum numbers for each group of states, namely, $\langle N,0\rangle$ for $[N,0]$ and $\langle N-1,1\rangle$ for $[N-1,1]$, where $\langle \sigma_1,\sigma_2\rangle$ denotes the O(6) representation of each IBM-2 state.[6] In particular, the lowest-lying configurations of $[N-1,1]\langle N-1,1\rangle$ are a 2^+ state and a 1^+ state, with the 1^+ state being the O(6) analog of the $M1$ scissors mode in the SU(3) limit.[12]

In a recent paper, Balantekin, Barrett, and Halse[13] have described and discussed in the IBM-2 the O(6) analogy of the 2^+ excitations in the U(5) limit, representing symmetric and antisymmetric vibrations of the β degree of freedom of the protons and the neutrons. Contrary to the U(5) case,[8,9] in the O(6) limit these β vibrations occur in the next higher sigma representation, being $\langle N-2,0\rangle$ for both $[N,0]$ and $[N-1,1]$; in each case these excitations are built on an intrinsic state whose lowest wave function has $J^\pi = 0^+$. As pointed out in Ref. 13, the antisymmetric β vibrational 0^+ state may occur as low as 3.1 MeV in ^{134}Ba and 2.1 MeV in ^{196}Pt, and, hence, be available to experimental detection; the problem being one of determining an appropriate "signature" for identifying this particular 0^+ excitation. In this paper, we will discuss the structure of this 0^+ state and its decay modes, pointing out features which may make it stand out in the experimental data.

II. STRUCTURE OF THE O(6) IBM-2 MIXED-SYMMETRY β VIBRATIONS

We will consider the O(6) limit of the IBM-2 corresponding to the group chain

$$\begin{array}{llllll} \mathrm{U}(6)_\pi \times \mathrm{U}(6) & & & & & \\ \supset \mathrm{U}_{\pi+\nu}(6) & \supset \mathrm{O}_{\pi+\nu}(6) & \supset \mathrm{O}_{\pi+\nu}(5) & \supset \mathrm{O}_{\pi+\nu}(3) & & (1) \\ [N-f,f] & \langle \sigma_1,\sigma_2\rangle & (\tau_1,\tau_2) & L\ , & & \end{array}$$

where the quantum numbers for each step in the subgroup chain are listed below each group classification.[6] A possible spectrum of a "typical" nucleus with this dynamical symmetry chain is illustrated in Fig. 1, where only the first few states of each representation are shown.

The states of interest to us are those of the $[N-1,1]\langle N-2\rangle$ representation, particularly the 0^+

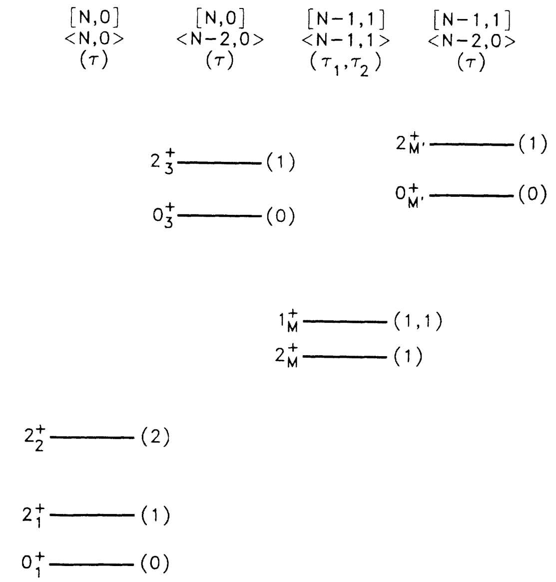

FIG. 1. "Typical" spectrum of a nucleus with the dynamical symmetry chain (1), including the $\langle N-2,0\rangle$ O(6) configurations.

band head, denoted by

$$|[N_\pi][N_\nu][N-1,1]\langle N-2\rangle(0)0^+\rangle \equiv |0^+_{M'}\rangle \ . \qquad (2)$$

For convenience and conciseness, we hereafter omit second quantum numbers (e.g., f, σ_2, or τ_2) which are zero. We use M' to distinguish the $\langle N-2\rangle$ mixed-symmetry states from the $\langle N-1,1\rangle$ mixed-symmetry states which are usually denoted by a subscript M.

Using the notation of Van Isacker *et al.*,[7] we refer to the symmetric β vibrational band-head state as

$$|[N_\pi][N_\nu][N]\langle N-2\rangle(0)0^+\rangle \equiv |0^+_3\rangle \ . \qquad (3)$$

According to the study of O(6)-like nuclei by Casten and von Brentano,[14] states of this possible structure exist in the spectra of the Xe and Ba isotopes.

Before describing a specific form for the states $|0^+_{M'}\rangle$ and $|0^+_3\rangle$, we will discuss the structure of the intrinsic states, from which the states $|0^+_{M'}\rangle$ and $|0^+_3\rangle$ and the other states in their corresponding bands can be projected.

The IBM-2 intrinsic wave function, including the γ degrees of freedom, is generally defined as[15]

$$|N_\pi,\beta_\pi,\gamma_\pi;N_\nu,\beta_\nu,\gamma_\nu\rangle = \frac{1}{\sqrt{N_\pi!N_\nu!}}[\lambda^\dagger_\pi(\beta_\pi,\gamma_\pi)]^{N_\pi} \times[\lambda^\dagger_\nu(\beta_\nu,\gamma_\nu)]^{N_\nu}|0\rangle \ , \qquad (4)$$

where

$$\lambda^\dagger_\rho(\beta_\rho,\gamma_\rho) = \frac{1}{\sqrt{1+\beta_\rho^2}}\left[s^\dagger_\rho+\beta_\rho\cos\gamma_\rho d^\dagger_{0_\rho} +\beta_\rho\sin\gamma_\rho\frac{(d^\dagger_2+d^\dagger_{-2})_\rho}{\sqrt{2}}\right] \quad \text{for } \rho=\pi \text{ or } \nu \ . \qquad (5)$$

Otsuka and Sugita[16] and Leviatan[17] have shown that for $\beta_\pi=\beta_\nu=1$ and $\gamma_\pi=\gamma_\nu$ the above intrinsic state generates all the O(6) state with $\langle\sigma\rangle=\langle N\rangle$.

The mixed-symmetry intrinsic state, from which $|0^+_{M'}\rangle$ can be projected, can be written in the form[17] (assuming $\beta_\rho=1$)

$$|\Phi^{\rm in}(0^+_{M'})\rangle = [\sqrt{N_\pi/N}\,\eta_\nu(S^\dagger_\nu)(\lambda^\dagger_\nu)^{N_\nu-2}(\lambda^\dagger_\pi)^{N_\pi} -\sqrt{N_\nu/N}\,\eta_\pi(S^\dagger_\pi)(\lambda^\dagger_\pi)^{N_\pi-2}(\lambda^\dagger_\nu)^{N_\nu}]|0\rangle \ , \qquad (6)$$

where

$$S^\dagger_\rho=\lambda^\dagger_\rho\sigma^\dagger_\rho-(d^\dagger_1d^\dagger_{-1})_\rho+(d^\dagger_2d^\dagger_{-2})_\rho \ , \qquad (7a)$$

$$\sigma^\dagger_\rho=\frac{1}{\sqrt{2}}\left[\cos\gamma_\rho d^\dagger_{0_\rho}+\sin\gamma_\rho\frac{(d^\dagger_2+d^\dagger_{-2})_\rho}{\sqrt{2}}-s^\dagger_\rho\right] \ , \qquad (7b)$$

$$\eta_\rho=\left[\frac{N_\rho(N_\rho-1)}{N_{\rho'}!(N_\rho+1)!}\right]^{1/2} \qquad (7c)$$

and $\rho=\pi$ or ν while $\rho'=\nu$ or π. Since $S^\dagger_\rho$ is an O(6) scalar,[17] it is seen that the intrinsic state (6) has $\langle\sigma\rangle=\langle N-2\rangle$.

In the limit of large N (i.e., $N_\pi\to\infty$, $N_\nu\to\infty$), Eq. (6) takes the form

$$|\Phi^{\rm in}(0^+_{M'})\rangle_{N\to\infty} \propto \underbrace{[\sigma^\dagger_\nu\lambda^\dagger_\pi-\sigma^\dagger_\pi\lambda^\dagger_\nu]}_{\rm I}\ \underbrace{(\lambda^\dagger_\pi)^{N_\pi-1}(\lambda^\dagger_\nu)^{N_\nu-1}}_{\rm II}|0\rangle, \qquad (8)$$

where the second and third terms on the right-hand side of Eq. (7a) are of $O(1/N_\rho)$ compared with the first term. In Eq. (8), Part I is seen to be antisymmetric in the π and ν degrees of freedom and is, in fact, an $F=0$ pair of a proton boson and a neutron boson. Because of the structure of Part I, only the ξ_2 term in the Majorana interaction[2] has a nonvanishing contribution. By a straight forward calculation one can obtain

$$\langle\Phi^{\rm in}(0^+_{M'})|M_{\pi\nu}|\Phi^{\rm in}(0^+_{M'})\rangle=N\xi_2 \ , \qquad (9)$$

where

$$M_{\pi\nu}=\xi_2(s^\dagger_\nu d^\dagger_\pi-d^\dagger_\nu s^\dagger_\pi)^{(2)}(s_\nu\tilde{d}_\pi-\tilde{d}_\nu s_\pi)^{(2)} +\sum_{k=1,3}\xi_k(d^\dagger_\nu d^\dagger_\pi)^{(k)}(\tilde{d}_\nu\tilde{d}_\pi)^{(k)} \ . \qquad (10)$$

Note that Eq. (9) holds for the intrinsic state with finite N's in Eq. (6).[17] The intrinsic state (8) is no longer a state of good $\langle\sigma\rangle$, because of the neglecting of terms of

$O(1/\sqrt{N_\rho})$.

The intrinsic state, from which $|0_3^+\rangle$ [see Eq. (3)] can be projected, is similar to (6), except that it has a plus sign, instead of a minus sign, between the two terms and the factors of $\sqrt{N_\rho}$ are interchanged.

Returning to Eqs. (2) and (3), we first observe that except for the decomposition $U_{\pi+\nu}(6)\supset O_{\pi+\nu}(6)$, the two states $|0_3^+\rangle$ and $|0_{M'}^+\rangle$ have identical structure, where this difference represents coefficients depending only on N which make the states orthogonal. Using the isoscalar factors of Van Isacker, Frank, and Sun[18] one can construct the corresponding states for $N_\nu=N-1$ and $N_\pi=1$ in a straightforward manner to obtain

$$|0_3^+\rangle=|[N_\nu=N-1][N_\pi=1];[N]\langle N-2\rangle(0)0^+\rangle = \frac{\sqrt{N+2}}{N}\{\langle N-1\rangle\} + \frac{\sqrt{(N+1)(N-2)}}{N}\{\langle N-3\rangle\} \tag{11a}$$

$$|0_{M'}^+\rangle=|[N_\nu=N-1][N_\pi=1];[N-1,1]\langle N-2\rangle(0)0^+\rangle=\frac{\sqrt{(N+1)(N-2)}}{N}\{\langle N-1\rangle\}-\frac{\sqrt{N+2}}{N}\{\langle N-3\rangle\}\ , \tag{11b}$$

where

$$\{\langle N-1\rangle\}=\sum_{\tau,l}\left\langle\begin{matrix}\langle N-1\rangle & \langle 1\rangle \\ (\tau) & (\tau)\end{matrix}\middle|\begin{matrix}\langle N-2\rangle \\ (0)\end{matrix}\right\rangle\left\langle\begin{matrix}(\tau) & (\tau) \\ l & l\end{matrix}\middle|\begin{matrix}(0) \\ 0\end{matrix}\right\rangle(|[N_\nu]\langle N-1\rangle(\tau)l\rangle_\nu(b_l^\dagger)_\pi)^0 \tag{12a}$$

and

$$\{\langle N-3\rangle\}=\sum_{\tau,l}\left\langle\begin{matrix}\langle N-3\rangle & \langle 1\rangle \\ (\tau) & (\tau)\end{matrix}\middle|\begin{matrix}\langle N-2\rangle \\ (0)\end{matrix}\right\rangle\left\langle\begin{matrix}(\tau) & (\tau) \\ l & l\end{matrix}\middle|\begin{matrix}(0) \\ 0\end{matrix}\right\rangle(|[N_\nu]\langle N-3\rangle(\tau)l\rangle_\nu(b_l^\dagger)_\pi)^0\ . \tag{12b}$$

The brackets $\langle\ |\ \rangle$ represent the O(6) and O(5) isoscalar factors, respectively. A similar type of structure exists for $|2_3^+\rangle$ and $|2_{M'}^+\rangle$, the configurations corresponding to the 2^+ states in each band.

III. RESULTS

The similarity of the structure of the states $|0_{M'}^+\rangle$ and $|0_3^+\rangle$ and their associated 2^+ states leads to some immediate conclusions:

(1) The energy of the 0^+ band-head states is the same except for the ξ_2 term in the Majorana interaction, which pushes the mixed-symmetry state up in energy. This conclusion follows directly from the structure of the intrinsic states described in Sec. II and is the same as the result in Balantekin, Barrett, and Halse,[13] for the classical limit. For $\xi_2=0$ the two states are degenerate, a result borne out by explicit IBM-2 calculations.

(2) The $E2$ reduced transition matrix elements between $|2_3^+\rangle$ and $|0_{M'}^+\rangle$ and between $|2_{M'}^+\rangle$ and $|0_3^+\rangle$ are identical, namely,

$$\langle 2_{M'}^+||Q_\rho||0_3^+\rangle^2=\langle 2_3^+||Q_\rho||0_{M'}^+\rangle^2 = \frac{4(N+1)N_\nu N_\pi}{(N-1)N^2}\quad \text{for } \rho=\nu \text{ or } \pi \tag{13a}$$

and

$$Q_\rho=(s^\dagger\tilde{d}+d^\dagger s)_\rho\ . \tag{13b}$$

As pointed out in Balantekin, Barrett, and Halse,[13] the states in the representation $[N-1,1]\langle N-2\rangle$ cannot decay to the states in the lowest representation $[N]\langle N\rangle$ by $M1$ transitions $[L_\rho=\sqrt{10}(d_\rho^\dagger\tilde{d}_\rho)^{(1)}]$ or $E2$ transitions of the form (13b), because these operators transform according to the O(6) representation $\langle 1,1\rangle$ and so cannot couple $\langle N,0\rangle$ to $\langle N-2,0\rangle$.

If the state $|2_3^+\rangle$ lies higher in energy than the state $|0_{M'}^+\rangle$, as appears to be so from model calculations (see Fig. 1), then the state $|0_{M'}^+\rangle$ has the possibility of decaying only to the states $|1_M^+\rangle$ and $|2_M^+\rangle$, the mixed-symmetry 1^+ and 2^+ states in the representation $[N-1,1]\langle N-1,1\rangle$. The reduced matrix elements for the $M1$ and $E2$ decays, respectively, to these states are

$$\langle 2_M^+||Q_\rho||0_{M'}^+\rangle^2=\frac{2(N+3)}{N(N+2)(N-2)}(N_\pi-N_\nu)^2 \tag{14a}$$

and

$$\langle 1_M^+||L_\rho||0_{M'}^+\rangle^2=\frac{3(N+3)}{N^2(N-2)}(N_\pi-N_\nu)^2 \quad \text{for } \rho=\pi \text{ or } \nu\ . \tag{14b}$$

One immediately observes that these reduced matrix elements are proportional to $(N_\pi-N_\nu)$ and, hence, vanish for $N_\pi=N_\nu$. Since Eqs. (14) hold for proton and neutron bosons separately, the vanishing of these reduced matrix elements occurs irrespectively of the value of the F-scalar and the F-vector boson charges.[19] One also notes the $F_z=\frac{1}{2}(N_\pi-N_\nu)$=the z component of the F spin, so that these reduced matrix elements are proportional to F_z. It now becomes apparent why Eqs. (14a) and (14b) take this form. We are making an isovector transition in F spin between two configurations having the *same* total F spin. The Clebsch-Gordan coefficient for such a transformation is simply

$$(FF_z,10|FF_z)=\frac{F_z}{\sqrt{F(F+1)}} . \qquad (15)$$

An immediate consequence of this result is a new selection rule for the O(6) limit of the IBM-2, which says that the state $|0^+_{M'}\rangle$ cannot decay for $N_\pi=N_\nu$ and, as such, forms an isomeric state.

We would like to encourage experimentalists to look for the decay of the $|0^+_{M'}\rangle$ state in a series of isotopes, such as the Ba isotopes, which are thought to be good O(6) nuclei.[14] In such an investigation the $B(E2,0^+_{M'}\to 2^+_M)$ strength or the $B(M1,0^+_{M'}\to 1^+_M)$ should go through a minimum for $N_\pi=N_\nu$, e.g., for ${}^{132}{}_{56}\mathrm{Ba}_{76}$.

In the two limiting cases, $\xi_2=0$ and $\xi_1=\xi_3=-2\xi_2$, one obtains the following simple relations for $E_{0^+_{M'}}$:

$$E_{0^+_{M'}}=E_{0^+_3} \quad \text{for } \xi_2=0 \qquad (16)$$

and

$$E_{0^+_{M'}}=\frac{(N-1)}{2(N+1)}E_{0^+_3}+E_{1^+_M} \quad \text{for } \xi_1=\xi_3=-2\xi_2 , \qquad (17)$$

where Eq. (17) was derived by Balantekin, Barrett, and Halse.[13] The energies of the states projected from the intrinsic state (6) depend only on the ξ_2 terms in the Majorana interaction (10) and not on the ξ_1 and ξ_3 terms. Consequently, the $0^+_{M'}$ state may lie lower in energy than the 1^+_M state whose energy also depends upon the ξ_1 and ξ_3 terms.[17] Also a low-lying $0^+_{M'}$ state (near the 0^+_3 state) would indicate a small value for ξ_2.

We have performed IBM-2 calculations for the Ba isotopes, and these calculations numerically verify all the results previously stated.

IV. SUMMARY AND CONCLUSIONS

We have described the structure of the intrinsic states for the symmetric and antisymmetric O(6) configurations with $\langle\sigma\rangle=\langle N-2\rangle$. We have also discussed specific representations for the $|0^+\rangle$ and $|2^+\rangle$ states, which can be projected from these intrinsic states. From the structure of the mixed-symmetry 0^+ state, we have seen that it has very limited possibilities for decay, only to the $|1^+_M\rangle$ and $|2^+_M\rangle$ mixed symmetry, $\langle\sigma_1,\sigma_2\rangle=\langle N-1,1\rangle$ states, and have obtained analytic formulas for the reduced matrix elements for these decay modes. The "signature" for these particular $M1$ and $E2$ decays is that they are "forbidden" for $N_\pi=N_\nu$, i.e., for a "good" O(6) nucleus, the $0^+_{M'}$ state is an isomeric state. We encourage experimentalists to study the decay of O(6)-like nuclei for the presence of this signature, particularly for an isotropic chain of nuclei, such as the Ba or Xe isotopes.

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