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$N\Delta$ interaction in πd breakup

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The effects of the residual nucleon-delta interaction proposed by Ferreira *et al.* to eliminate the discrepancies between theory and experiment in πd elastic scattering are studied for the breakup reaction at 228 and 294 MeV. It is found that in first order the residual interaction lowers the cross section particularly in the region of the neutron-proton final-state interaction. However, the effects of higher-order corrections generated by the $N\Delta$ interaction are found to be non-negligible, which implies that the parameters extracted by Ferreira *et al.* probably must be modified.

I. INTRODUCTION

The effects of a residual nucleon-delta interaction which is not contained in the standard three-body description obtained from the relativistic Faddeev theory, have been studied for πd elastic scattering by Ferreira *et al.*[1-6] Contributions to this residual interaction can arise, for example, from the exchange of a ρ meson between the nucleon and the delta or from a direct $\pi\Delta\Delta$ vertex which would give rise to a t channel one-pion exchange (OPE) interaction. The method used by Ferreira *et al.* was to calculate a correction to the pion-deuteron elastic scattering amplitude coming from the scattering of a nucleon and a delta in intermediate states with the partial-wave nucleon-delta T matrix parametrized in terms of the two parameters δ and η, where δ is the phase shift and η the inelasticity. They then added this correction to the pion-deuteron elastic scattering amplitudes of Ref. 7 and determined, in this way, the parameters δ and η of the $N\Delta$ channels ${}^5S_2$ and ${}^5P_3$ such that they would eliminate almost completely the discrepancies for the total cross section, the differential cross section, and the vector analyzing power iT_{11} throughout the energy region $125 \leq T_\pi \leq 325$ MeV.

The main effect of the residual $N\Delta$ interaction in the case of the differential cross section was to remove a long-standing discrepancy of a factor of 2 in the angular region around 100°. Thus, since also in the case of the πd breakup reaction there exist in several regions of phase space discrepancies of roughly a factor of 2, it is tempting to see if the residual $N\Delta$ interaction would also help to remove these discrepancies here. Some estimates for the effects of this interaction in the closely related $\pi d \to N\Delta$ channel have been carried out already by Dosch and Ferreira.[5] Similarly, the effects of the residual interaction in the reactions $\pi d \to NN$ and $NN \to NN$ have been studied recently by Alexandrou and Blankleider.[8]

An important point raised by Alexandrou and Blankleider[8] is the question of the convergence of the residual $N\Delta$ interaction, that is, whether or not it is justified to use just the lowest-order term as did Ferreira *et al.* Thus, we have also studied the effects of higher-order $N\Delta$ terms. Since the equivalent to the πd elastic reaction in the case of the breakup channel is the region of the neutron-proton final-state interaction, one expects that the effects of the residual interaction will be strong here, and, therefore, we have performed our calculations at $T_\pi = 228$ and 294 MeV in order to compare with the data of Mathie *et al.*[9] and List *et al.*[10] which cover this region of phase space.

II. FIRST-ORDER $N\Delta$ INTERACTION

A. Relativistic three-body model

The relativistic three-body theory of Ref. 7 was obtained by applying the isobar ansatz for the two-body subsystems and the condition that all the spectator particles be on the mass shell. It leads to the integral equations

$$F_{nl,J}^{j_n m_n \nu_n, j_l m_l \nu_l}(k_n,k_l) = V_{nl,J}^{j_n m_n \nu_n, j_l m_l \nu_l}(k_n,k_l) + \sum_{i \neq n} \sum_{j_i m_i \nu_i} \int_0^\infty \frac{k_i^2 dk_i}{2\omega_i} V_{ni,J}^{j_n m_n \nu_n, j_i m_i \nu_i}(k_n,k_i) \tau_{j_i}(k_i) F_{il,J}^{j_i m_i \nu_i, j_l m_l \nu_l}(k_i,k_l) , \quad (1)$$

where $\omega_i = (k_i^2 + m_i^2)^{1/2}$, J is the total angular momentum of the system, and k_i and ν_i are the magnitude of the three-momentum and the helicity of particle i, respectively, while j_i, and m_i are the spin and helicity of the pair jk, which in the isobar ansatz correspond, respectively, to the total angular momentum of the pair and its magnetic projection along the direction $\mathbf{k}_j + \mathbf{k}_k$ [for simplicity we have left out in Eq. (1) the isospin and orbital momentum quantum numbers of a pair]. The transition potentials $V_{ji,J}^{j_j m_j \nu_j, j_i m_i \nu_i}(k_j,k_i)$ in Eq. (1) are the partial-wave projections of the diagram where one exchanges particle k between a state consisting of particle i and the isobar i, and a state consisting of particle j and the isobar j. The coupling of the isobars to the pion and nucleons is given by the standard relativistic vertices that couple particles of spin 0, $\frac{1}{2}$, 1, and $\frac{3}{2}$ of either positive or negative parity. The isobar propagators $\tau_{j_i}(k_i)$ are related to the on-shell amplitudes of the pair jk, as

$$\tau_{j_i}(k_i)=\frac{-4\sqrt{s_i}}{\pi g_{j_i}^2(p_i^2)p_i^{2l_i+1}}\sin\delta(s_i)e^{i\delta(s_i)}\ , \tag{2}$$

where

$$s_i=(\sqrt{S}-\sqrt{k_i^2+m_i^2})^2-k_i^2 \tag{3}$$

and

$$p_i^2=\frac{[s_i-(m_j+m_k)^2][s_i-(m_j-m_k)^2]}{4s_i}\ , \tag{4}$$

with S the total invariant energy squared of the system and $g_{j_i}(p_i^2)=1/(1+p_i^2/\Lambda^2)$, with $\Lambda=1$ GeV/c for the pion-nucleon channels, while for the nucleon-nucleon channels they are constructed from the bound and antibound state solutions of the Paris potential. The input of these equations are the six S- and P-wave pion-nucleon channels (S_{11}, S_{31}, P_{11}, P_{13}, P_{31}, and P_{33}) and the nucleon-nucleon $^1S_0$ and $^3S_1$-$^3D_1$ channels as described in Ref. 7. In the nucleon-exchange diagram for the transition potential from a NN isobar, with a pion as spectator, to a πN isobar, with a nucleon as spectator [see Eq. (77) of Ref. 7], we have included only the positive-energy components in the propagator of the exchanged nucleon since it is our experience that the inclusion of the negative-energy components always leads to a poor description of the elastic and breakup reactions. This is similar to the case of nucleon-nucleon elastic scattering where the negative-energy components lead, in general, to a poor description of the data so that they are usually removed.[11]

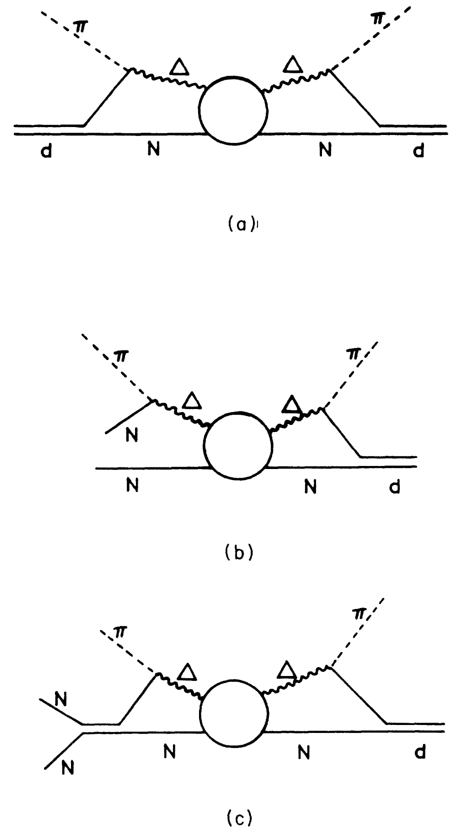

FIG. 1. The lowest-order contributions of the residual $N\Delta$ interaction to: (a) πd elastic scattering, (b) πd breakup, and (c) πd breakup in the region of the neutron-proton final-state interaction.

B. $N\Delta$ interaction in $\pi d\to\pi d$

The possibility of a residual nucleon-data interaction not contained in the integral equations (1) can be understood by considering, for example, contributions to the $N\Delta-N\Delta$ transition potential arising from the exchange of a ρ meson or from a direct $\pi\Delta\Delta$ vertex. In principle, this residual interaction could be simply added to the standard transition potential defined by the three-body theory, and its effect calculated by solving the integral equations (1) with the modified $N\Delta-N\Delta$ transition potential. This is the approach followed by Alexandrou and Blankleider.[8] However, one may also consider using first-order perturbation theory and simply add the $N\Delta$ T matrix generated by the residual interaction alone to the full amplitude generated by the three-body model. This has been the approach followed by Ferreira *et al.* The process considered by Ferreira *et al.* in their study of the effects of the $N\Delta$ interaction in πd elastic scattering is depicted in Fig. 1(a). The expression for this process within the theory of Eq. (1) is

$$\Delta F^{m_f,m_0}_{\pi d\to\pi d,J}(k_f,k_0)=\sum_{m_i\nu_i}\sum_{m_i'\nu_i'}\int_0^\infty\frac{k_i^2dk_i}{2\omega_i}\int_0^\infty\frac{k_i'^2dk_i'}{2\omega_i'}V^{1m_f,m_i\nu_i}_{\pi d\to N\Delta,J}(k_f,k_i)\tau_\Delta(k_i)f^{m_i\nu_i,m_i'\nu_i'}_{N\Delta\to N\Delta,J}(k_i,k_i')\tau_\Delta(k_i')V^{m_i'\nu_i',1m_0}_{N\Delta\to\pi d,J}(k_i',k_0)\ , \tag{5}$$

where m_0 and m_f are the initial and final helicities of the deuteron and implicit in Eq. (5) is the fact that the pion has spin 0 and the delta has spin $\frac{3}{2}$.

The $N\Delta\to N\Delta T$ matrix $f^{m_i\nu_i,m_i'\nu_i'}_{N\Delta\to N\Delta,J}(k_i,k_i')$ in Eq. (5) has been written in the helicity basis, which is related to the corresponding T matrix in the LSJ basis, as

$$f^{m_i\nu_i,m_i'\nu_i'}_{N\Delta\to N\Delta,J}(k_i,k_i')=\sum_{LS}\sum_{L'S'}b^{m_i\nu_i}_{LSJ}f^{LS,L'S'}_{N\Delta\to N\Delta,J}(k_i,k_i')b^{m_i'\nu_i'}_{L'S'J}\ , \tag{6}$$

where

$$b^{m_i\nu_i}_{LSJ}=\sqrt{(2L+1)/(2J+1)}C^{LSJ}_{0,m_i-\nu_i}C^{3/2\,1/2\,S}_{m_i,-\nu_i}\ . \tag{7}$$

In the study of Ferreira *et al.*[4] only the contribution of the $N\Delta$ states with lowest orbital angular momentum was taken into account in the case of $J=2$ and $J=3$, that is, only the $^5S_2$ and $^5P_3$ $N\Delta$ channels, so that with this simplification the corresponding equations (6) become

$$f^{m_i\nu_i,m_i'\nu_i'}_{N\Delta\to N\Delta,2}(k_i,k_i')=b^{m_i\nu_i}_{022}f^{02,02}_{N\Delta\to N\Delta,2}(k_i,k_i')b^{m_i'\nu_i'}_{022} , \quad (8)$$

$$f^{m_i\nu_i,m_i'\nu_i'}_{N\Delta\to N\Delta,3}(k_i,k_i')=b^{m_i\nu_i}_{123}f^{12,12}_{N\Delta\to N\Delta,3}(k_i,k_i')b^{m_i'\nu_i'}_{123} . \quad (9)$$

The residual $N\Delta$ T matrices $f^{LS,LS}_{N\Delta\to N\Delta,J}(k_i,k_i')$ in Eqs. (8) and (9) for $J=2$ and 3 and energies $125\le T_\pi\le 325$ MeV, were parametrized by Ferreira *et al.* as

$$f^{LS,LS}_{N\Delta\to N\Delta,J}(k_i,k_i')=Bt^L_{N\Delta} , \quad (10)$$

where

$$t^L_{N\Delta}=\frac{1}{2i}(\eta_L e^{2i\delta_L}-1) \quad (11)$$

and B is a constant that, in our normalization and in the zero-width approximation (see Appendix A) is given by

$$B=-\frac{p_r^3g_\Delta^2(p_r^2)\sqrt{S}}{s_r\Gamma_\Delta q_r} , \quad (12)$$

where p_r, s_r, q_r, and Γ_Δ are the πN relative momentum, πN invariant mass squared, $N\Delta$ relative momentum, and width of the delta, respectively, all evaluated at the resonance position. The $N\Delta$ phase shifts and inelasticities δ_L and η_L were chosen such that the theoretical description of πd elastic scattering improves dramatically after adding the correction (5) to the standard πd amplitude. We should mention that Ferreira *et al.* took into account, approximately, the finite width of the delta by evaluating the $N\Delta$ on-shell momentum q_r from Eq. (A6) by using the complex Δ mass and then replacing q_r in Eq. (12) by $\mathrm{Re}(q_r)$. We have also used this prescription as well as their values of $\mathrm{Re}(q_r)$ in the applications involving Eqs. (10)–(12) at $T_\pi=228$ and 294 MeV.

C. $N\Delta$ interaction in $\pi d\to\pi NN$

Following the same technique as in Eq. (5), one can evaluate the contribution of the residual $N\Delta$ interaction to the πd breakup reaction depicted by Fig. 1(b) as

$$\Delta F^{\mu_i\mu_j,m_0}_{\pi d\to\pi NN,J}(p_ik_i,k_0)=\sum_{m_i}\sum_{m_i'\nu_i'}\langle\mu_j0|\tfrac{3}{2}m_i\rangle g_\Delta(p_i)\tau_\Delta(k_i)\int_0^\infty\frac{k_i'^2dk_i'}{2\omega_i'}f^{m_i\mu_i,m_i'\nu_i'}_{N\Delta\to N\Delta,J}(k_i,k_i')\tau_\Delta(k_i')V^{m_i'\nu_i',1m_0}_{N\Delta\to\pi d,J}(k_i',k_0) , \quad (13)$$

and that of Fig. 1(c) as

$$\begin{aligned}\Delta F^{\mu_i\mu_j,m_0}_{\pi d\to\pi NN,J}(p_kk_k,k_0)=&\sum_{j_km_k}\sum_{m_i\nu_i}\sum_{m_i'\nu_i'}\langle\mu_i\mu_j|j_km_k\rangle g_{j_k}(p_k)\tau_{j_k}(k_k)\\&\times\int_0^\infty\frac{k_i^2dk_i}{2\omega_i}\int_0^\infty\frac{k_i'^2dk_i'}{2\omega_i'}V^{j_km_k,m_i\nu_i}_{\pi d\to N\Delta,J}(k_k,k_i)\tau_\Delta(k_i)\\&\times f^{m_i\nu_im_i'\nu_i'}_{N\Delta\to N\Delta,J}(k_i,k_i')\tau_\Delta(k_i')V^{m_i'\nu_i',1m_0}_{N\Delta\to\pi d,J}(k_i',k_0) ,\end{aligned} \quad (14)$$

where μ_i and μ_j are the final helicities of the two nucleons and $\langle\mu_i\mu_j|j_km_k\rangle$ is the angular part of the vertex of an isobar of spin j_k and helicity m_k that decays into particles i and j with helicities μ_i and μ_j. Figure 1(b) has been previously estimated by Dosch and Ferreira,[5] while Fig. 1(c), which is the relevant one in the region of the neutron-proton final-state interaction, has never been evaluated before.

III. HIGHER-ORDER $N\Delta$ EFFECTS

There are three points which are questionable within the approach of Ferreira *et al.* that we have described in Sec. II. Firstly, they have considered the effects of the residual $N\Delta$ interaction by means of perturbation theory keeping only the lowest-order term and give no information on the validity of this approximation. It is not obvious that perturbation theory should be valid in this case since the residual $N\Delta$ interaction is not weak. Secondly, they have parametrized the on-shell residual $N\Delta$ amplitude in a way corresponding to a stable Δ particle (zero-width approximation), while one knows that the Δ is not a stable particle but a rather wide pion-nucleon resonance. Thirdly, they have used a parametrization of the off-shell residual $N\Delta$ amplitudes [see Eqs. (10) and (11)] which is identical to the on-shell one, that is, their off-shell amplitudes do not go to zero as k_i or $k_i'\to\infty$. In addition, for a given orbital angular momentum L they do not have the proper threshold behavior due to the angular momentum barrier that it should behave as k_i^L or $k_i'^L$ as k_i or $k_i'\to 0$. Consequently, in this section we will reformulate the contribution of the residual $N\Delta$ interaction in such a way that these problems can be overcome. First of all, we will rewrite the integral equations (1) with the residual $N\Delta$ interaction included to all orders. Secondly, we will introduce a separable model for the residual nucleon-data amplitude which is unitary also in the case of an unstable delta so that no zero-width approximation is needed. This will also allow us to solve the problem of the proper threshold and asymptotic behaviors of the residual amplitude.

A. Higher-order corrections

If one includes a residual nucleon-delta interaction v^R in the integral equations (1), one must make the replacement $V_{\Delta\Delta}\to V_{\Delta\Delta}+v^R$, so that the equation for an initial πd state and a final $N\Delta$ state becomes

$$F_{\Delta d}=V_{\Delta d}+(V_{\Delta\Delta}+v^R)\tau_\Delta F_{\Delta d}+\sum_{j\neq\Delta}V_{\Delta j}\tau_j F_{jd}\ . \quad (15)$$

This equation can be rewritten as

$$F_{\Delta d}=(1+f^R\tau_\Delta)V_{\Delta d}+(1+f^R\tau_\Delta)\sum_j V_{\Delta j}\tau_j F_{jd}\ , \quad (16)$$

where f^R is the residual $N\Delta$ T matrix that satisfies the Lippmann-Schwinger equation

$$f^R=v^R+v^R\tau_\Delta f^R\ . \quad (17)$$

The approach of Ferreira *et al.* would correspond then to solving Eq. (16) with $f^R=0$ and simply adding afterwards just the lowest-order contribution given by the second term in the inhomogeneous part of Eq. (16).

B. Separable model of the residual $N\Delta$ interaction

The Lippmann-Schwinger equation (17) for a given orbital angular momentum L, can be written explicitly as

$$f_L^R(k_i,k_i')=v_L^R(k_i,k_i') + \int_0^\infty \frac{q_i^2dq_i}{2\omega_i}v_L^R(k_i,q_i)\tau_\Delta(q_i)f_L^R(q_i,k_i')\ . \quad (18)$$

If the residual $N\Delta$ interaction is taken to be of the separable form

$$v_L^R(k_i,k_i')=h_L(k_i)\gamma_L h_L(k_i')\ , \quad (19)$$

where γ_L is the strength of the interaction and $h_L(k_i)$ the form factor, then the solution of Eq. (18) can be written in closed form as

$$f_L^R(k_i,k_i')=h_L(k_i)\frac{\sin\delta_L^{N\Delta}e^{i\delta_L^{N\Delta}}}{\alpha_L}h_L(k_i')\ , \quad (20)$$

where

$$\alpha_L=\int_0^\infty \frac{q_i^2dq_i}{2\omega_i}h_L^2(q_i)\mathrm{Im}\tau_\Delta(q_i)\ , \quad (21)$$

and the phase shift $\delta_L^{N\Delta}$ is given by

$$\tan\delta_L^{N\Delta}=\frac{\alpha_L}{\gamma_L^{-1}-\int_0^\infty(q_i^2dq_i/2\omega_i)h_L^2(q_i)\mathrm{Re}\tau_\Delta(q_i)}\ . \quad (22)$$

The Ferreira *et al.* T matrix given by Eqs. (10) and (11) corresponds to the special case of Eq. (20), $h_L(k_i)=$const., if the inelasticity $\eta_L^{N\Delta}$ in Eq. (11) is equal to 1. At the two energies that we are considering both inelasticities $\eta_0^{N\Delta}$ and $\eta_1^{N\Delta}$ of Ref. 4 are indeed equal to 1. Equations (20) and (21) imply that

$$\int_0^\infty \frac{k_i^2dk_i}{2\omega_i}f_L^R(k_i,k_i)\mathrm{Im}\tau_\Delta(k_i)=\sin\delta_L^{N\Delta}e^{i\delta_L^{N\Delta}}\ , \quad (23)$$

which is a straightforward generalization of the unitarity relation to the case of an unstable delta, since in the case of a stable delta $\mathrm{Im}\tau_\Delta(k_i)\propto\delta(k_i-k_0)$ and Eqs. (21) and (23) become the standard expressions for stable particles.

TABLE I. Parameters of the $N\Delta$ separable potentials defined by Eqs. (19) and (24).

L	β_L (fm^{-1})	γ_L (fm^{-4-2L})
0	1.90	−9.86
1	1.68	−98.2

The advantage of Eqs. (20)–(23) is that they allow us to define a nucleon-delta phase shift without having to introduce the zero-width approximation. As Eqs. (21) and (22) indicate, the phase shift $\delta_L^{N\Delta}$ has in this case the meaning of an average phase shift, since in Eqs. (21)–(23) one is integrating over the available width of the P_{33} resonance. We will use for the form factors in Eqs. (20)–(23) the simple forms

$$h_L(k_i)=\frac{k_i^L}{(\beta_L^2+k_i^2)^{L+1}},\ \ L=0,1 \quad (24)$$

which have the correct threshold and asymptotic behaviors as $k_i\to 0$ or $k_i\to\infty$. The parameters γ_L and β_L for $L=0$ and 1 are given in Table I, and they have been obtained by fitting the $N\Delta$ phase shifts of Ferreira *et al.*[4] at the two energies that we are considering.

There is no unambiguous way to relate the parameters of the residual interaction of Ferreira *et al.* which were obtained assuming the zero-width approximation, with those of three-body models where the delta propagator always contains the width of the P_{33} resonance. Thus, Alexandrou and Blankleider, for example, identified the Ferreira *et al.* phase shifts with those between a nucleon and a bare delta in their model. We, on the other hand, have chosen to compare the phase shifts of the unstable delta of our model directly to the zero-width phase shifts of Ferreira *et al.* This choice can be justified by noticing that at $T_\pi=228$ and 294 MeV, one is far above the P_{33} resonance so that when doing the integration over $\mathrm{Im}\tau_\Delta(q_i)$ in Eqs. (21) and (23), one is taking into account most of the width of the delta, and, consequently, the result of these integrations will be similar to those of the zero-width approximation. This of course will not be true at energies below the P_{33} resonance.

IV. RESULTS

A. πd elastic scattering

We show in Figs. 2 and 3 our results for the πd elastic differential cross section and vector analyzing power at $T_\pi=228$ and 294 MeV. We present in these figures the results of the calculation without residual $N\Delta$ interaction (solid lines), with the $N\Delta$ interaction included in first order within the Ferreira *et al.* model as given by Eqs. (10)–(12) (short-dashed lines), with the $N\Delta$ interaction included in first order as given by our model of Eqs. (20)–(22) (long-dashed lines), and with the $N\Delta$ interaction as given by our model included to all orders (dotted lines). The first-order results of the Ferreira *et al.* model are qualitatively similar to those of Ref. 4, although small differences appear due to our use of a different parametrization of the delta propagator [Eq. (2)] and of the deuteron wave function. In particular, Ferreira *et al.* neglected the D-state component of the deuteron in their evaluation of Figs. 1(a) and 1(b), while we have performed all our calculations including the D-state component of the deuteron.

The first-order results of our model follow the same trend as the Ferreira *et al.* model, although they are somewhat stronger as a consequence of our use of form factors in Eqs. (20)–(22). The biggest difference, however, is observed in the results when the $N\Delta$ interaction is included to all orders which have different shape and make the theoretical predictions to move further away from the data. This clearly shows that the first-order approach of Ferreira *et al.*[4] is not adequate as has already been pointed out by Alexandrou ad Blankleider.[8]

It is important to mention that our results when we include the $N\Delta$ interaction to all orders show a much stronger effect than the ones reported by Alexandrou and Blankleider[8] (as seen, for example, in our Fig. 2 for $T_\pi = 228$ MeV). The reason for this difference is that we have chosen the cutoff form factor of the six pion-nucleon channels as $g_{j_i}(p_i^2) = 1/(1+p_i^2/\Lambda^2)$ with $\Lambda = 1$ GeV/c, while the corresponding form factor of Ref. 8 for the dominant P_{33} channel has a cutoff parameter $\Lambda = 200$ MeV/c. We have checked that changing our Λ to 200 MeV/c indeed gives results very similar to those of Alexandrou and Blankleider. One can understand this strong dependence of the higher-order $N\Delta$ effects on the πN form factor by noticing that Eq. (16) can be rewritten as

$$F_{\Delta d} = (1+f^R\tau_\Delta)V_{\Delta d} + (1+f^R\tau_\Delta) \times \sum_{\Delta'} (V_{\Delta\Delta'} + V_{\Delta d}\tau_d V_{d\Delta'})\tau_{\Delta'} F_{\Delta' d} \,, \tag{25}$$

where the kernel $V_{\Delta\Delta'} + V_{\Delta d}\tau_d V_{d\Delta'}$ is stronger when the cutoff parameter Λ is large. Thus, since the higher-order effects of the residual interaction are given by $f^R\tau_\Delta(V_{\Delta\Delta'} + V_{\Delta d}\tau_d V_{d\Delta'})$, they will also be stronger when the cutoff parameter Λ increases.

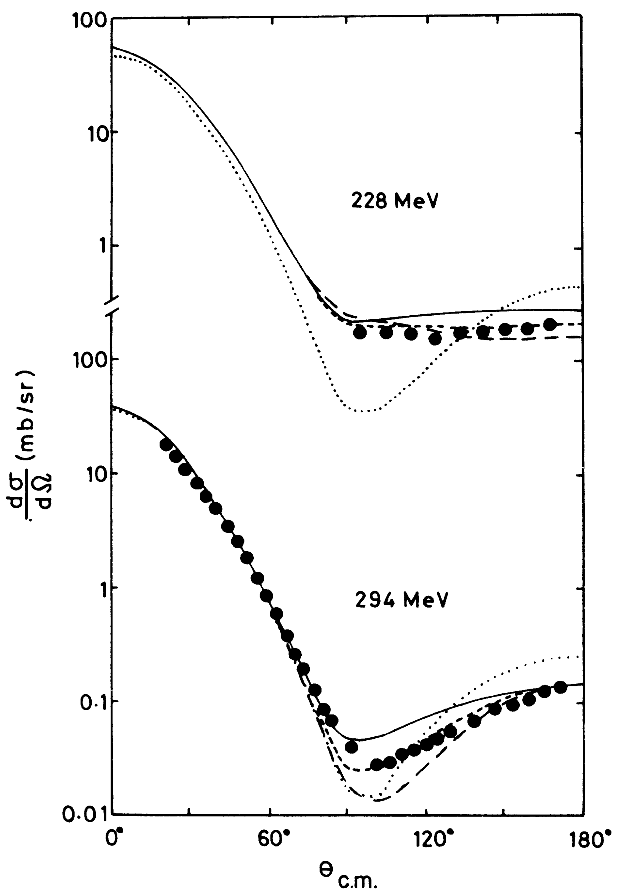

FIG. 2. The effects of the residual $N\Delta$ interaction in the πd elastic differential cross section. The solid lines are the results without $N\Delta$ interaction, the short-dashed lines are the first-order results within the Ferreira *et al.* model given by Eqs. (10)–(12), the long-dashed lines are the first-order results within the separable model given by Eqs. (20)–(22), and the dotted lines are the results of including the $N\Delta$ interaction to all orders within the separable model of Eqs. (20)–(22). The experimental data are from Ottermann *et al.* (Ref. 12).

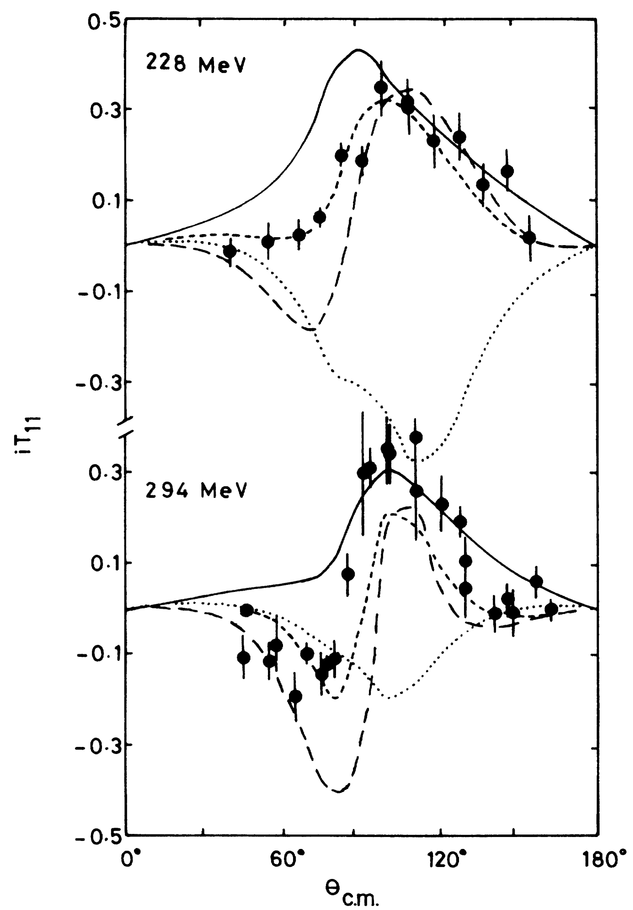

FIG. 3. The effects of the residual $N\Delta$ interaction in the πd elastic vector analyzing power iT_{11}. The curves are labeled as in Fig. 2. The data are from Ottermann *et al.* (Ref. 13) and from Smith *et al.* (Ref. 14). At 228 MeV the data used corresponds to 238 MeV.

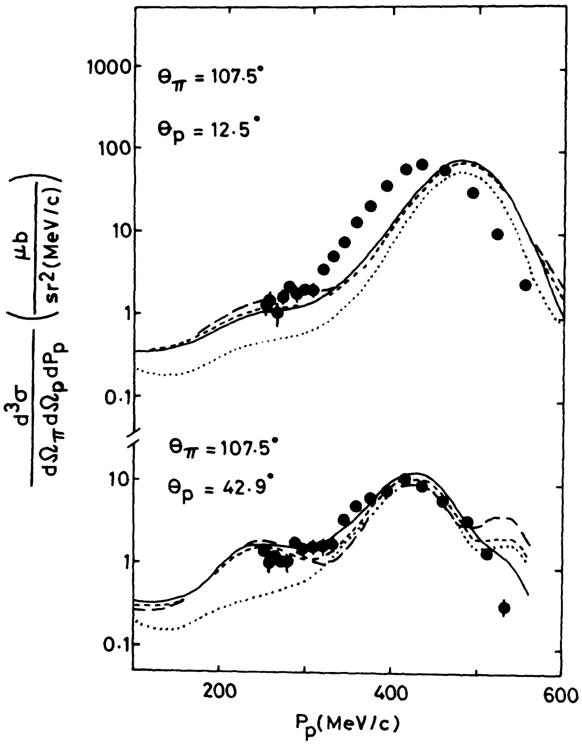

FIG. 4. The effects of the residual $N\Delta$ interaction in the πd breakup differential cross section at $T_\pi = 228$ MeV and two pion-proton angle pairs as a function of the proton momentum. The labeling of the figures is the same as in Fig. 2. The data are from Mathie *et al.* (Ref. 9).

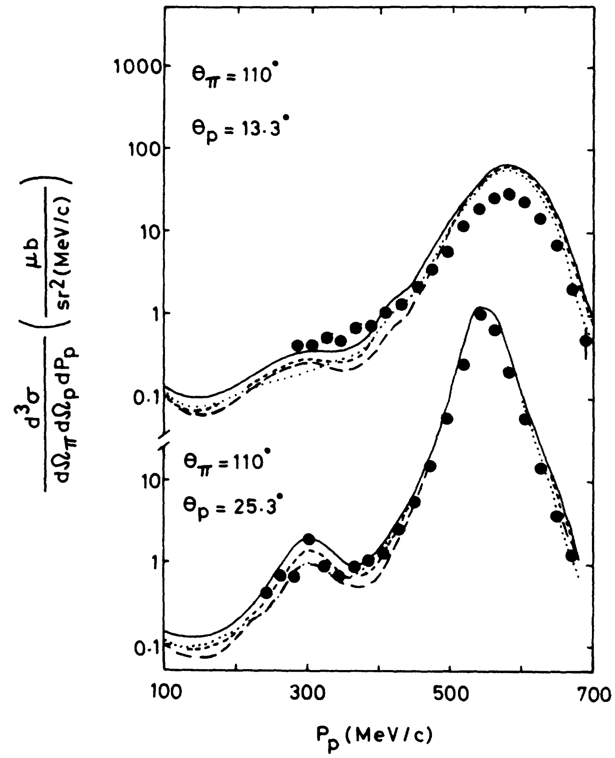

FIG. 6. The effects of the residual $N\Delta$ interaction in the πd breakup differential cross section at $T_\pi = 294$ MeV and two pion-proton angle pairs as a function of the proton momentum. The labeling of the figures is the same as in Fig. 2. The data are from List *et al.* (Ref. 10).

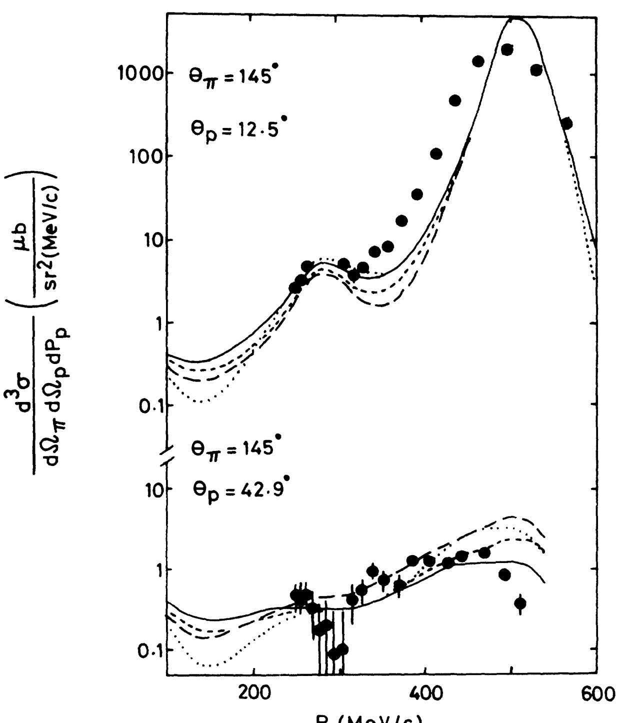

FIG. 5. Same as in Fig. 4.

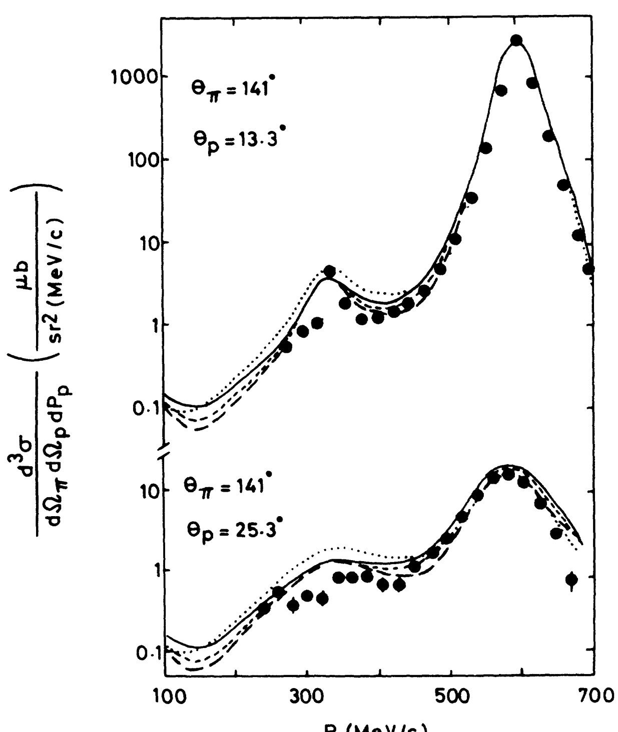

FIG. 7. Same as in Fig. 6.

B. πd breakup

We show in Figs. 4 and 5 the results for the differential cross section of the πd breakup reaction at 228 MeV measured recently by Mathie *et al.*[9] where we have considered only four pion-proton angle pairs. As one sees, the effects of the residual interaction are stronger in the region of low proton momentum which is dominated by the neutron-proton final-state interaction, that is, by Fig. 1(c) in the lowest-order case. There is no clear improvement in the description of the data as a result of including the residual interaction whether in lowest order or to all orders. One also sees large effects from the residual interaction in the high-momentum part for $\theta_p=42.9°$ which corresponds to a Δ^{++} formation in the final state and, therefore, are produced in lowest order by Fig. 1(b). Again, the effects are in the opposite direction of what the data requires. At the two cross sections with $\theta_p=12.5°$, the theoretical predictions are shifted from the data and this situation does not change by including the residual interaction. The origin of this shift is very hard to understand theoretically since the position of the quasifree peak is basically determined by the deuteron wave function and the momentum of the outgoing neutron which is completely determined by the kinematics of this experiment. This shift is not observed at 294 MeV at very similar angles as shown in Figs. 6 and 7.

We show in Figs. 6 and 7 the corresponding results for the differential cross sections at $T_\pi=294$ MeV measured by List *et al.*,[10] where we consider again only four pion-proton angle pairs. As one sees, the effects of the residual interaction are somewhat smaller in this case and again no obvious improvement in the description of the data is obtained. In the region of the neutron-proton final-state interaction, at low proton momentum the ordering of the curves is approximately the same as in the elastic case for similar pion angles, which is of course a consequence of the similarity between Figs. 1(a) and 1(c).

V. CONCLUSIONS

We have studied the effects of a residual interaction of the type proposed by Ferreira *et al.* in the πd elastic and breakup reactions. We have found that this interaction can produce large effects in both reactions, but its inclusion through the use of first-order perturbation theory is not justified. In the case of the breakup channel, the effects of the residual interaction are stronger in the region of the neutron-proton final-state interaction but no obvious improvement in the description of the data is obtained.

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APPENDIX A: THE ZERO-WIDTH APPROXIMATION

The delta propagator $\tau_\Delta(k_i)$ which is given in terms of the P_{33} phase shift by Eqs. (2)–(4) can be rewritten in the form

$$\tau_\Delta(k_i)=\frac{4\sqrt{s_i}}{\pi g_\Delta^2(p_i^2)p_i^3}\times\frac{\sqrt{s_r}\Gamma_\Delta(s_i)}{s_i-s_r+i\sqrt{s_r}\Gamma_\Delta(s_i)}\,, \tag{A1}$$

where s_i and p_i are defined by Eqs. (3) and (4), and

$$\Gamma_\Delta(s_i)=\frac{s_r-s_i}{\sqrt{s_r}}tg\,\delta(s_i) \tag{A2}$$

is the (energy-dependent) delta width. The zero-width approximation is obtained if, in the denominator of the propagator (A1), the width of the delta is neglected. In this approximation the delta propagator becomes

$$\begin{aligned}\tau_\Delta(k_i)&=\frac{4\sqrt{s_i}}{\pi g_\Delta^2(p_i^2)p_i^3}\frac{\sqrt{s_r}\Gamma_\Delta(s_i)}{s_i-s_r+i\epsilon}\\&=\frac{4\sqrt{s_i}\sqrt{s_r}\Gamma_\Delta(s_i)}{\pi g_\Delta^2(p_i^2)p_i^3(\sqrt{S}-\sqrt{m_i^2+k_i^2}+\sqrt{s_r+k_i^2})}\frac{1}{\sqrt{S}-\sqrt{m_i^2+k_i^2}-\sqrt{s_r+k_i^2}+i\epsilon}\,,\end{aligned} \tag{A3}$$

where we have used Eq. (3) in the last step and m_i is the mass of the nucleon. The propagator (A3) is that of a two-body problem with two stable particles of masses m_i and $\sqrt{s_r}$, respectively. Let us now consider the one-channel Lippmann-Schwinger equation for the $N\Delta$ system

$$f_L^R(k_i,k_i')=V_L^R(k_i,k_i')+\int_0^\infty\frac{q_i^2dq_i}{2\omega_i}V_L^R(k_i,q_i)\tau_\Delta(q_i)f_L^R(q_i,k_i')\,, \tag{A4}$$

where the potential is in general complex (as a result, for example, of reducing a set of coupled channel equations into a one-channel problem). Using the propagator (A3) into Eq. (A4) implies that the on-shell $N\Delta$ amplitude is of the form

$$f_L^R(q_r,q_r)=-\frac{p_r^3g_\Delta^2(p_r^2)\sqrt{S}}{s_r\Gamma(s_r)q_r}\frac{\eta e^{2i\delta}-1}{2i}\,, \tag{A5}$$

where

$$q_r^2=\frac{[S-(m_i+\sqrt{s_r})^2][S-(m_i-\sqrt{s_r})^2]}{4S} \tag{A6}$$

and p_r is given by Eq. (4) with $s_i=s_r$.

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