Neutron polarization in polarized ^{3}He targets

J. L. Friar and B. F. Gibson
Theoretical Division, Los Alamos National Laboratory, Los Alamos, New Mexico 87545

G. L. Payne
Department of Physics and Astronomy, University of Iowa, Iowa City, Iowa 52242

A. M. Bernstein
Department of Physics and Laboratory for Nuclear Science, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

T. E. Chupp
The Physics Laboratories, Harvard University, Cambridge, Massachusetts 02138
(Received 12 July 1990)

Simple formulas for the neutron and proton polarizations in polarized ^{3}He targets are derived assuming (1) quasielastic final states; (2) no final-state interactions; (3) no meson-exchange currents; (4) large momentum transfers; (5) factorizability of ^{3}He SU(4) response-function components. Numerical results from a wide variety of bound-state solutions of the Faddeev equations are presented. It is found that this simple model predicts the polarization of neutrons in a fully polarized ^{3}He target to be 87%, while protons should have a slight residual polarization of -2.7%. Numerical studies show that this model works very well for quasielastic electron scattering.

INTRODUCTION

Because of the Pauli principle, ^{3}H and ^{3}He nuclei have magnetic moments nearly equal to those of (free) protons and neutrons, respectively. This is easily understood in the context of the weak binding of these nuclides. The angular momentum barrier suppresses the influence of higher partial waves of the nucleon-nucleon force. If we assume only s waves between each pair of nucleons, the "like" nucleons in ^{3}He and ^{3}H (protons and neutrons, respectively) are restricted by the Pauli principle to be in a spin-singlet state. Thus their magnetic moments cancel, leaving only the magnetic moment of the "unlike" particle. This dominance was indicated long ago in the seminal calculations of Mallet and Tjon[1] and confirmed by the benchmark calculations of Ref. 2 and others:[3] the $^1S_0$ and $^3S_1$-$^3D_1$ partial waves of a nucleon-nucleon force typically generate all but 100–300 keV of the binding energy of ^{3}He.

This result can also be obtained by considering the symmetry properties of various components of the wave function.[4] The SU(4)-orbital decomposition of a typical wave function yields a tiny amount of (positive parity) P-state components (which we will always ignore), roughly 10% D-state components generated by the tensor force, and several s-wave components. The dominant part of the latter is the space-symmetric S state, which consequently has a completely antisymmetric spin-isospin wave function because of the Pauli principle and leads to no net magnetic moment for the "like" nucleons because they are constrained to be spin singlet. Differences between the $T=0$ and 1 forces generate a small (1–2 %) S'-state wave-function component which reflects (spin-isospin)-space correlations. The S' and D states generate small contributions to the total trinucleon magnetic moment from the like particles, which can then exist in relative p waves. In addition to the impulse approximation mechanism considered above, meson-exchange currents (two-body operators) also generate relatively small (primarily isovector) contributions to the magnetic moments.

A synopsis of representative results (in nuclear magnetons) from a recent calculation[5] is listed in Table I, and these numbers are further broken down according to their origin. The S'- and D-wave-function components serve to decrease the magnitude of the impulse approximation result, but this is compensated by the pion-exchange currents. Overall agreement is rather good, although the small disagreement in the ^{3}H case indicates a remaining problem with both isoscalar and isovector parts of the magnetic moment.

Because the magnetic moment of any system is defined as the expectation value of the magnetic moment operator with respect to that state of the system with the highest magnetic quantum number (i.e., a "fully polarized" nucleus), it is clear that a polarized ^{3}He nucleus automatically provides a highly polarized neutron which is rather loosely bound. In order to estimate how large this polarization may be and what mechanisms might lead to depolarization, we generate below a "figure of merit" for this quantity which is simple and quantitative, but dependent upon a number of approximations. After specifying these approximations, we will calculate both the neutron and proton polarizations using a wide variety of nuclear force models, including three-nucleon forces in some cases.

TABLE I. Experimental magnetic moments (in units of nuclear magnetons) of the trinucleons ^{3}He and ^{3}H compared to the free-nucleon magnetic moment (neutron and proton, respectively) and typical impulse approximation, pion-exchange current, and (a synopsis of) total theoretical results from a recent calculation (Ref. 5).

	μ (expt)	μ_N	$\mu_{\rm imp}$	μ_π	$\mu_{\rm Th}$
^{3}He	-2.12	-1.91	-1.76	-0.35	-2.11
^{3}H	2.98	2.79	2.54	0.35	2.89

MODEL CALCULATIONS

The amount of polarization of a nucleon in polarized ^{3}He can be most easily estimated using an extremely simple and graphic model, which is physically motivated. It is clear that the estimate depends on the details of how we pose the problem although hopefully not in a sensitive way. We ask the question: If a nucleon is simply pulled out of the ^{3}He target without disturbing its spin, what is the degree of polarization of the spin of that nucleon? The answer to the posed question is determined by the quantities

$$P_{n,p}^{(\pm)}=\langle {}^3\mathrm{He}\, m=+\tfrac{1}{2}|\hat{P}_{n,p}^{(\pm)}|{}^3\mathrm{He}\, m=+\tfrac{1}{2}\rangle \ , \tag{1}$$

where

$$\hat{P}_n^{(\pm)}=\sum_i \frac{1-\tau_3(i)}{2}\frac{1\pm\sigma_z(i)}{2} \ , \tag{2a}$$

$$\hat{P}_p^{(\pm)}=\sum_i \frac{1+\tau_3(i)}{2}\frac{1\pm\sigma_z(i)}{2} \ . \tag{2b}$$

The isospin factor in $\hat{P}_n$ ($\hat{P}_p$) counts the number of neutrons (protons) which are aligned (+) or antialigned ($-$) with the ^{3}He spin. Clearly, $P_{n,p}^{(+)}+P_{n,p}^{(-)}=N,Z$, where $N=1$ is the number of neutrons and $Z=2$ is the number of protons. This information can be put into the form of separate density matrices for neutrons and protons:

$$\rho_n=\begin{bmatrix} P_n^{(+)} & 0 \\ 0 & P_n^{(-)} \end{bmatrix} \ , \tag{3}$$

and

$$\rho_p=\begin{bmatrix} P_p^{(+)}/2 & 0 \\ 0 & P_p^{(-)}/2 \end{bmatrix} \ , \tag{4}$$

where $\mathrm{Tr}(\rho)=1$ in both cases. What makes the calculations of the P's tractable is that the various pieces of the operators $\hat{P}$ are the generators of SU(4), which is used to classify the basis states of the wave function $(S,S',D,\ldots)$. They are also the magnetic moment operators in the impulse approximation. The results (neglecting the tiny P-state components, as noted above) are well known[5]

$$\langle m=\tfrac{1}{2}|\sigma_z|m=\tfrac{1}{2}\rangle=P(S)+P(S')-P(D) \ , \tag{5a}$$

$$\langle |\tau_z|\rangle=1=P(S)+P(S')+P(D) \ , \tag{5b}$$

$$\left\langle m=\tfrac{1}{2}\left|\sum_{i=1}^{3}\sigma_z(i)\tau_z(i)\right|m=\tfrac{1}{2}\right\rangle =-[P(S)-\tfrac{1}{3}P(S')+\tfrac{1}{3}P(D)] \ , \tag{5c}$$

and thus

$$P_n^{(+)}=1-\Delta \ , \tag{6a}$$

$$P_n^{(-)}=\Delta \ , \tag{6b}$$

where

$$\Delta=[P(S')+2P(D)]/3 \ , \tag{6c}$$

and the normalization condition (5b) has been used to eliminate $P(S)$. In addition,

$$P_p^{(+)}/2=\tfrac{1}{2}-\Delta' \ , \tag{7a}$$

and

$$P_p^{(-)}/2=\tfrac{1}{2}+\Delta' \ , \tag{7b}$$

where

$$\Delta'=[P(D)-P(S')]/6 \ . \tag{7c}$$

These results have a very simple and physical interpretation. If the forces between all pairs of nucleons were identical (spin and isospin independent), we would have $P(S')=P(D)=0=\Delta=\Delta'$. The neutron would then be completely polarized and the protons completely unpolarized, as we discussed above. In a realistic ^{3}He nucleus, however, the neutron is somewhat depolarized and the protons gain a slight downward (negative) polarization. The larger value of $P(D)$ compared to $P(S')$ makes it dominant in both cases. The numerically important negative contribution of $P(D)$ in Eq. (5a) arises because the orbital D wave allows the concomitant $S=\frac{3}{2}$ spin component to point downward while preserving $m=\frac{1}{2}$; this is not possible with S waves.

The quantities Δ and Δ' can be evaluated easily for any model. We note that wave-function probabilities are not experimental observables, because of the problems in defining uniquely the relativistic corrections to the interaction operators.[6] Consequently, this model can only be an approximation (although it might be a very good one). Values of Δ and Δ' for a wide variety of nuclear force models are shown in Figs. 1 and 2. Each point on the plot is a theoretical calculation for a model plotted versus the binding energy of ^{3}He for that model. Many observables plotted in this way show a very strong dependence on binding, although these do not. Most of the models with binding in excess of 7 MeV include a three-body force.[7] The primary exception is the lowest point, corresponding to one of the Bonn potentials,[8] which has a weaker tensor force, and therefore a smaller $P(D)$. Questions about the adequacy of particular nuclear forces

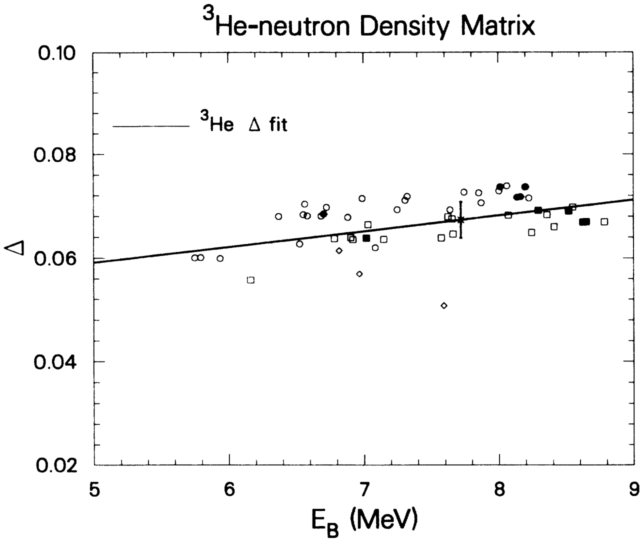

FIG. 1. Neutron density matrix parameter Δ calculated for a wide variety of nuclear models versus the binding energy of ^{3}He for that model. The size of the theoretical error bar at the experimental ^{3}He binding energy is determined by the average spread of the points about the simple fit.

must be settled experimentally. Using the fitted curves, we deduce, at the physical ^{3}He binding energy (7.72 MeV),

$$\Delta \cong 0.07(1) \tag{8a}$$

and

$$\Delta' \cong 0.014(2) , \tag{8b}$$

where *entirely subjective* theoretical error bars have been attached to Δ and Δ' to give a feeling for the spread of the values. We have taken these to be 3 times the average value of the spread of the calculated points about the fit.

This model, although schematic, can be expected to work reasonably well under certain conditions specified by the way we posed the polarization question. We are assuming that the nucleon polarization seen by an external interaction is given only by the distribution of spins in the ground state. Clearly, any spin-dependent final-state interactions can alter the nucleon's spin. Spin-dependent meson-exchange currents will have the same effect and must also be ignored. The nucleon polarization could also differ under different kinematic conditions if the various (radial) wave-function components responded differently to the act of removing the nucleon from the nucleus. This would produce effective values of ρ_n and ρ_p (or Δ and Δ') which depended on the kinematic variables, such as electron energy and momentum transfer in electron scattering. A more detailed "derivation" of Eq. (1), which probably overestimates its flaws, is given in the Appendix.

It should be clear to the reader at this point that the set of conditions which most closely corresponds to "plucking a nucleon out of the nucleus" is quasielastic scattering. Although no detailed study has been made of the effect of spin-dependent final-state interactions and meson-exchange currents in this process for the ^{3}He case,[9] studies on the deuteron clearly indicate that there are only very small corrections at the quasielastic peak.[10]

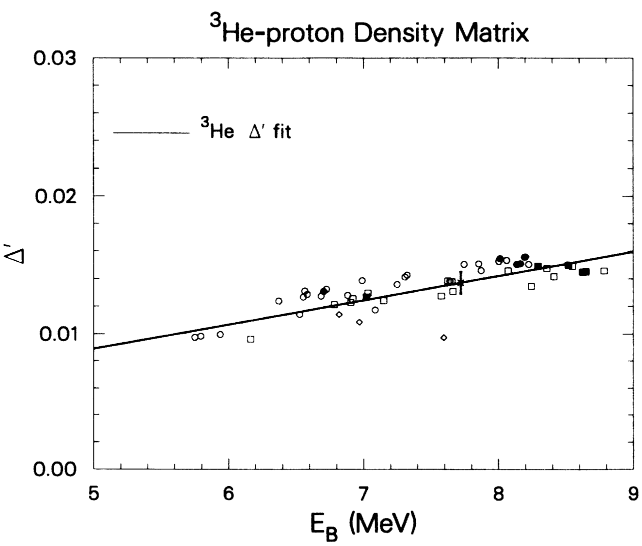

FIG. 2. Proton density matrix parameter Δ' as in Fig. 1.

CROSS SECTIONS AND ASYMMETRIES

In order to use this model in a practical application, we suppose that a process is nucleon-spin dependent in the following schematic way:

$$\frac{d\sigma}{d\Omega}(\pm) \sim A_0 + A_1 \langle \sigma_z \rangle = A_0 \pm A_1 . \tag{9a}$$

This leads to an asymmetry for a free, polarized nucleon:

$$A(N) = \frac{\frac{d\sigma}{d\Omega}(+) - \frac{d\sigma}{d\Omega}(-)}{\frac{d\sigma}{d\Omega}(+) + \frac{d\sigma}{d\Omega}(-)} = \frac{A_1}{A_0} , \tag{9b}$$

where $(\pm)$ refer to the nucleon-spin components in Eq. (9a). For a polarized ^{3}He nucleus the same calculation folded with the density matrices in Eqs. (3) and (4) (for a spin-up ^{3}He nucleus) leads to separate neutron and proton asymmetries:

$$A_n(^3\mathrm{He}) = p_n A(n) f_n , \tag{10a}$$

$$p_n = 1 - 2\Delta , \tag{10b}$$

$$f_n = \sigma_n / (\sigma_n + 2\sigma_p) , \tag{10c}$$

and

$$A_p(^3\mathrm{He}) = p_p A(p) f_p , \tag{10d}$$

$$p_p = -2\Delta' , \tag{10e}$$

$$f_p = 2\sigma_p / (\sigma_n + 2\sigma_p) = 1 - f_n , \tag{10f}$$

where $A(n)$ and $A(p)$ are the asymmetries for elastic $\vec{e} - \vec{n}$ and $\vec{e} - \vec{p}$ scattering, and f_n and f_p are the neutron and proton fractions of the quasifree scattering from ^{3}He. The effective neutron and proton polarizations in ^{3}He are p_n ($=0.865$) and p_p ($=-0.027$) in terms of the fits in Figs. 1 and 2. We can now compare this model to the quasielastic electron scattering model of Blankleider and Woloshyn (BW),[11] which also assumes no final-state interactions and no meson-exchange currents, but does not assume that the density matrix is independent of the external interaction. Indeed, one way of testing the quality of our model is to calculate numerically $A_N(^3\mathrm{He})$ from Eq. (10) and from their model and form the ratio. Deviations from 1.0 indicate the inherent difference due to our factorizability approximation.

The truncated Afnan-Birrell (UPA) wave function[12] used in Ref. 11 had $P_{S'}=1.54\%$ and $P_D=8.37\%$ although the original wave function had $P_{S'}=1.6\%$ and $P_D=9.1\%$. We find, therefore, $\Delta=0.061$ and $\Delta'=0.0114$. Figure 3 shows the ratio of our model (with the values of Δ and Δ' corresponding to Ref. 11) to the Blankleider-Woloshyn neutron asymmetry for electron scattering of 0.88- and 3.0-GeV electrons. The ratio is only weakly dependent on the electron energies and on the momentum transfers, but is slightly larger than 1.0 ($\sim 5\%$). The same quantity for the protons in Fig. 4 is a somewhat stronger function of the electron kinematic variables, and is larger than 1.0 for most of the range of momentum transfers. We note that the deviations from 1.0 are not much larger than the truncation of the Afnan-Birrell D-state wave-function components ($\sim 10\%$) actually used in Ref. 11. These deviations from 1.0 are consistent with those found ($\sim 5\%$) in the calculations used in Ref. 13.

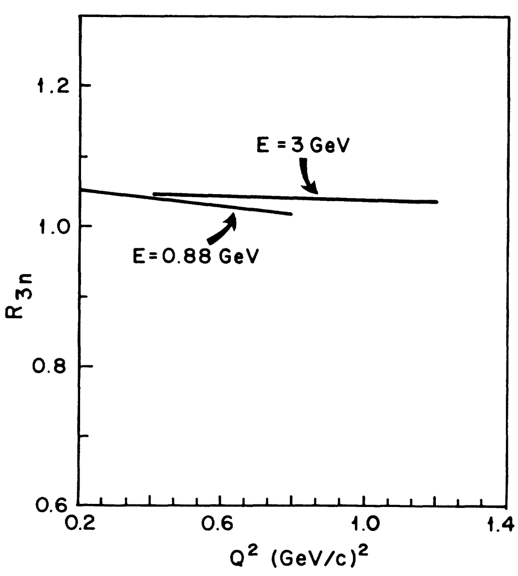

FIG. 3. Ratio of predictions for the neutron asymmetry of our model [Eq. (10)] (with the BW values of Δ and Δ') to that of the complete BW calculation, plotted versus square of momentum transfer for two incident electron energies.

SUMMARY

We have constructed a simple model for the polarization of the neutrons and protons in polarized ^{3}He. The model works well if one assumes that final-state interactions and meson-exchange currents are unimportant at the quasielastic peak.

ACKNOWLEDGMENTS

One of us (J.L.F.) would like to thank Ray Arnold for suggesting this problem, and to Cathleen Woodward, Bob McKeown, and Mike Vetterli for encouragement. Another of us (A.M.B.) would like to thank Boris Blankleider for stimulating discussions and for making his computer code available. The work of G. L. P. and A. M. B. was supported in part by the U. S. Department of Energy, while the work of J. L. F. and B. F. G. was performed under the auspices of the U. S. Department of Energy. The work of T. E. C. was supported in part by the National Science Foundation.

APPENDIX

In order to be reasonably specific about reaction mechanisms, we assume that we are treating inelastic electron scattering, although details are not very important in our approach. We further assume that we are treating quasielastic final-state configurations. This set of kinematics reduces the influence of both final-state interactions and meson-exchange currents, which we assume are sufficiently small to ignore. We also assume that momentum transfers are fairly high, so that the quasielastic energy ($\sim \mathbf{q}^2/2m$) of a nucleon ejected with a momentum transfer $\mathbf{q}$ completely dominates the energies of any important final states of the residual (two-particle) system. In that situation, only the process in Fig. 5(a) contributes significantly, where it is understood that we will take the

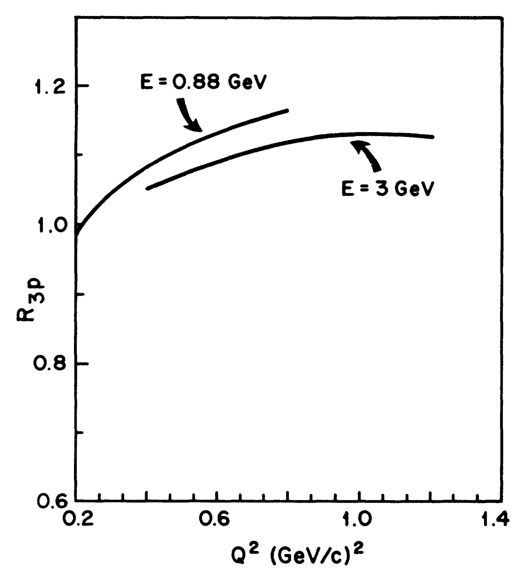

FIG. 4. The proton asymmetry ratio, as in Fig. 3.

imaginary part indicated by the dashed line.

The "cats ears" or correlation-interchange diagram of Fig. 5(a) decreases rapidly for momentum transfers larger than the correlation length (i.e., the average distance between nucleons). For the single neutron in ^{3}He, of course, only Fig. 5(a) exists. Our previously listed approximations prevent mesons from dripping off the struck nucleon to the residual nucleus and altering the nucleon spin states (labeled with Greek letters). The latter situation could occur either at or between the currents J corresponding, respectively, to meson-exchange currents or final-state interactions.

Cutting the graph in Fig. 5(a) along the dotted line, we obtain for the transition probability, assuming that the neutron is struck,

$$T^x \sim \sum_{\substack{ppn \\ \alpha,\beta,\gamma}} \langle 0x^{\gamma}|J^{\gamma\beta^{\dagger}}|ppn^{\beta}\rangle\langle ppn^{\beta}|J^{\beta\alpha}|0x^{\alpha}\rangle\delta(E_N-E_0) , \tag{A1}$$

where we have labeled the ^{3}He spin projection by x, the final neutron spin by β, and the spin-changing ($\mu\rightarrow\lambda$) interaction "current" by $J^{\lambda\mu}$. We further assume that the elementary amplitudes factorize, so that the details of the currents can be removed from the matrix element:

$$T^x \sim \sum_{\alpha,\beta,\gamma} T_{\alpha\beta}T^{\dagger}_{\beta\gamma}\bar{\rho}^{\,x}_{\alpha\gamma}(\mathbf{q}) , \tag{A2}$$

where

$$\bar{\rho}^{x}_{\alpha\gamma}(\mathbf{q}) = \sum_{ppn} \langle 0x|\hat{P}^{\gamma}_{n}|ppn\rangle\langle ppn|\hat{P}^{\alpha}_{n}|0x\rangle\delta(E_N-E_0) , \tag{A3}$$

and the spin projection operators [Eq. (2)] have been inserted to enforce the spin conditions. We further assume that most of the energy is transferred to the neutron so that closure can be performed on the protons, and that the response of the neutron to the momentum injected into the ^{3}He is factorizable; that is, all components of the wave function respond in proportion to their size:

$$\bar{\rho}^{x}_{\alpha\gamma}(\mathbf{q}) = n(\mathbf{q})\rho^{x}_{\alpha\gamma} , \tag{A4}$$

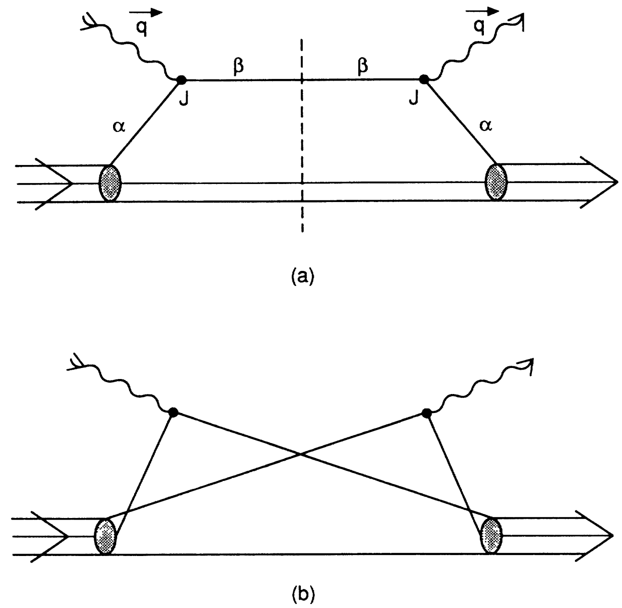

FIG. 5. Processes which contribute to the two-photon amplitude for ^{3}He, whose imaginary part is the transition probability for (virtual) photon absorption.

where

$$\rho^{x}_{\alpha\gamma} = \langle 0x|\hat{P}^{\gamma}_{n}\hat{P}^{\alpha}_{n}|0x\rangle . \tag{A5}$$

Moreover, the projection operators are diagonal:

$$\hat{P}^{\gamma}_{n}\hat{P}^{\alpha}_{n} = \delta^{\alpha\gamma}\hat{P}^{\alpha}_{n} . \tag{A6}$$

Thus

$$T^x \sim \sum_{\alpha\beta} |T_{\alpha\beta}|^2 n(\mathbf{q})\langle 0x|\hat{P}^{\alpha}|0x\rangle , \tag{A7}$$

where the last factor is also Eq. (1) and determines the spin orientation of the neutron α in a ^{3}He nucleus that has an orientation x. It is possible that a derivation of this result can be found which is less restrictive and makes fewer assumptions.

[1]R. A. Malfliet and J. A. Tjon, Nucl. Phys. **A127**, 161 (1969).

[2]C. R. Chen, G. L. Payne, J. L. Friar, and B. F. Gibson, Phys. Rev. C **31**, 2266 (1985).

[3]T. Sasakawa and S. Ishikawa, Few-Body Syst. **1**, 3 (1986); **1**, 143 (1986).

[4]J. L. Friar, E. L. Tomusiak, B. F. Gibson, and G. L. Payne, Phys. Rev. C **24**, 677 (1981).

[5]J. L. Friar, B. F. Gibson, G. L. Payne, E. L. Tomusiak, and M. Kimura, Phys. Rev. C **37**, 2852 (1988); E. L. Tomusiak, M. Kimura, J. L. Friar, B. F. Gibson, G. L. Payne, and J. Dubach, *ibid.* **32**, 2075 (1985). For a somewhat different approach see, R. Schiavilla and D. O. Riska, Phys. Lett B **244**, 373 (1990).

[6]J. L. Friar, Phys. Rev. C **20**, 325 (1979).

[7]C. R. Chen, G. L. Payne, J. L. Friar, and B. F. Gibson, Phys. Rev. C **33**, 1740 (1986).

[8]R. Machleidt, K. Holinde, and C. Elster, Phys. Rep. **149**, 1 (1987).

[9]E. van Meijgaard and J. A. Tjon, Phys. Rev. Lett. **61**, 1461 (1988).

[10]H. Arenhövel, W. Leideman, and E. L. Tomusiak, Z. Phys. **331**, 123 (1988); **334**, 363(E) (1989); E. L. Tomusiak and H. Arenhövel, Phys. Lett. B **206**, 187 (1988).

[11]B. Blankleider and R. M. Woloshyn, Phys. Rev. C **29**, 538 (1984).

[12]I. R. Afnan and N. D. Birrell, Phys. Rev. C **16**, 823 (1977).

[13]C. E. Woodward *et al.*, Phys. Rev. Lett. **65**, 698 (1990).

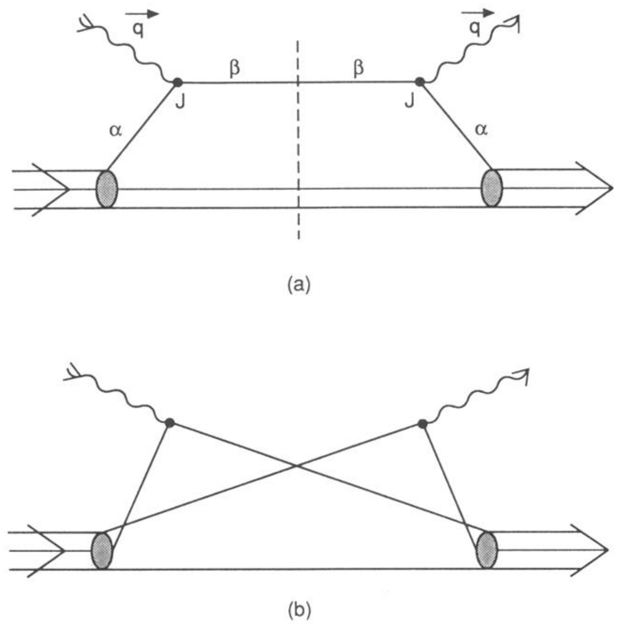

FIG. 5. Processes which contribute to the two-photon amplitude for ^3He, whose imaginary part is the transition probability for (virtual) photon absorption.