Microscopic calculations of low-energy reaction cross sections

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Microscopic calculations of nuclear reaction cross sections and total reaction probabilities are compared with measurements for the $d+\mathrm{Si}$, $^{3,4}\mathrm{He}+\mathrm{Si}$, and $d+\mathrm{Ge}$ systems at energies ranging from 2 to 53 MeV/nucleon. Good agreement is obtained, except for $^{3}\mathrm{He}+\mathrm{Si}$ at the very lowest energies, when zero-range nucleon-nucleon forces are assumed and realistic nuclear density distributions are used in the tail regions, where the models are most sensitive. The agreement is less good for finite-range forces. A strong absorption model gives much poorer agreement with the recent $^{4}\mathrm{He}+\mathrm{Si}$ measurements than do the microscopic models.

Nuclear reaction cross sections have for years been used to test nuclear reaction models, such as the optical model, and to obtain information about nuclear radii. Moreover, the recent growth of interest in exotic nuclei has stimulated many new reaction cross-section measurements[1] aimed toward identifying anomalously large radii, and possibly neutron halos, in the light neutron-rich nuclei.

Several microscopic theoretical treatments[2–5] based on the Glauber model[6] explain nucleus-nucleus reaction cross sections as resulting from individual nucleon-nucleon interactions. Their common ingredients are the averaged nucleon-nucleon reaction cross section, σ_{NN}, and the nuclear matter densities in the projectile and target nuclei, ρ_P and ρ_T. Most treatments[2–4] begin by considering a projectile with impact parameter b (Fig. 1), which contains $\rho_P dV_P$ nucleons in a volume element dV_P whose displacements from the centers of the two nuclei are $\mathbf{r}_P$ and $\mathbf{r}_T$. The probability of interaction in this volume element, while the projectile travels a distance dz, is just $\sigma_{NN}\rho_T\rho_P dV_P dz$; ρ_T is the target nucleon density in this same volume element. The transparency, or probability that the projectile will *not* interact at this impact parameter, is then

$$T(b)=\exp\left[-\sigma_{NN}\int\int\int\int\rho_T(\mathbf{r}_T)\rho_P(\mathbf{r}_P)dV_P dz\right] , \qquad (1)$$

where the integrations extend over the projectile's trajectory (assumed to be a straight line) and internal coordinates. The reaction cross section is obtained by integration over impact parameters:

$$\sigma_R=2\pi\int[1-T(b)]b\,db \ . \qquad (2)$$

The complete Glauber theory involves expansions in powers of the nucleon-nucleon scattering amplitude.[7] Calculations based on Eqs. (1) and (2) are therefore approximate, but nevertheless have often successfully fitted the experimental data. This method was first used by Karol,[2] who calculated σ_R's at 2.1 GeV/nucleon laboratory energy for many projectile-target combinations, using density functions for which Eqs. (1) and (2) could be evaluated analytically. Later, DeVries and Peng[3] applied the model at somewhat lower energies, using more realistic density functions which required numerical integration. To make the model useful at these energies, they modified Eq. (2) to allow for the deflection of the projectile's trajectory by the Coulomb field:

$$\sigma_R=2\pi\int[1-T(b')]b\,db \ . \qquad (3)$$

The trajectory, as in the Karol model, was again assumed to be straight but at distance b' from the beam axis, where b' is the classical distance of closest approach for a particle having impact parameter b.

Although Glauber theory was developed for application at high energies, these models have successfully predicted σ_R's for heavy-ion collisions at bombarding energies down to 10 MeV/nucleon.[3,4] Therefore we decided to apply it to the recent data[8–10] for light projectiles on Si and Ge, some of which extend to still lower energies.

We first concentrated on fitting the recent measurements[10] of the reaction probability for 27 to 92 MeV α particles on Si (i.e., the probability that such α particles will initiate a nuclear reaction in a Si detector before the end of their range). These measurements determine an average reaction cross section of 1170±55 mb in this energy range; they are compared in Fig. 2 with our microscopic predictions. The solid and dashed curves are obtained when ρ of Si is taken to be a three-parameter Fermi (3PF) distribution,[11] and the Gaussian function recommended by Karol,[2] respectively; both give an rms radius of 3.08 fm. For the α-particle density, both 3PF (as used by DeVries and Peng[3]) and Gaussian forms were used, with indistinguishable results. Nucleon-nucleon cross sections down to the lowest available energies were taken from the literature;[12] there, σ_{pp} is taken to be $4\pi(d\sigma/d\Omega)$ at 90° c.m., where the Coulomb scattering cross section is negligible compared with those observed. At lower energies σ_{np} was computed from the effective range parameters,[13] and σ_{nn} and σ_{pp} were obtained by extrapolating the existing data assuming a $1/E$ dependence. The latter procedure appeared reliable to about ±10%. This uncertainty has negligible effect on the com-

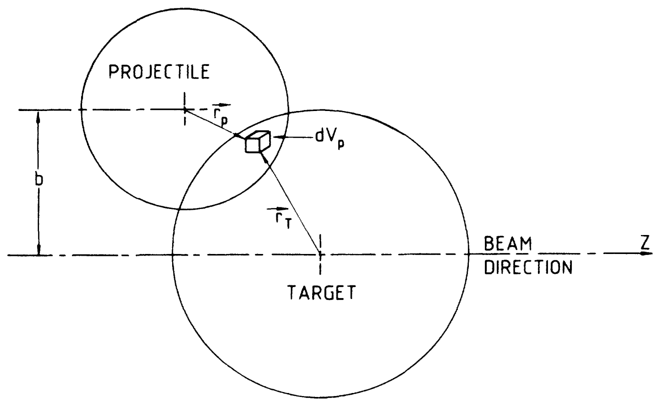

FIG. 1. Overlap region, where nucleon-nucleon collisions between projectile and target nuclei take place.

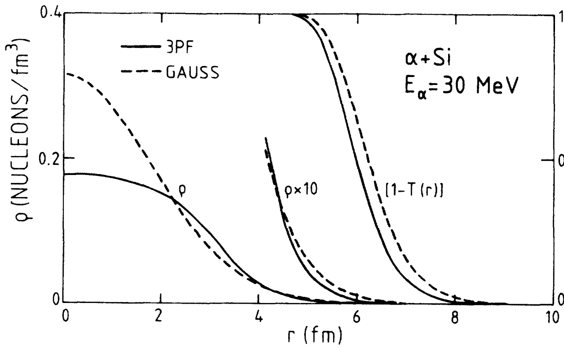

FIG. 3. Nucleon density ρ in Si (left vertical scale), and opacity $[1-T(r)]$ for the $\alpha+$Si system at $E_\alpha=30$ MeV (right vertical scale), plotted vs. distance for two functional forms of ρ.

puted reaction probabilities.

The 3PF Si density function gives better results for $\alpha+$Si ($\chi^2/N=2.4$ vs. 9.4 for the Gaussian function) since the Gaussian cross section is too large, especially at higher energies. For example, at $E_{\alpha 0}=90$ MeV, the 3PF and Gaussian σ_R's are 1201 and 1286 mb, respectively. The explanation is contained in Fig. 3, which shows that the opacity $1-T$ extends to larger impact parameters for the Gauss form. The density functions have two crossing points, at 2.2 and 4.3 fm. Our predictions using the 3PF density inside 4.3 fm, and the Gaussian density otherwise, were indistinguishable from those using the pure Gaussian density. Therefore the excess cross section from the Gaussian function is attributable to its dominance over the 3PF function outside the second crossing point. The Gauss and 3PF functions place 12% and 9%, respectively, of the Si matter distribution in this outer region. This 3% difference in surface matter content leads to a 10% difference in σ_R at 90 MeV. These calculations illustrate the great sensitivity of reaction cross sections, as given by this model, to the matter distribution in the nuclear surface, and their insensitivity to the matter in the interior

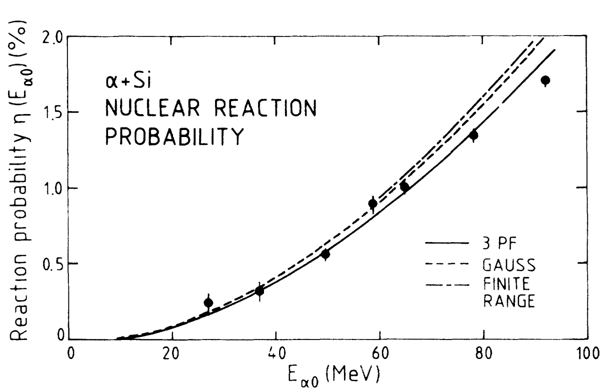

FIG. 2. Total nuclear reaction probability data (Ref. 10) for 27 to 92 MeV α-particles incident upon a thick Si detector, compared with microscopic predictions using the model of DeVries and Peng. Two different functional forms of the nucleon density in Si are used (see text) when the nucleon-nucleon force is assumed to have zero range, while the 3PF density is used in the finite-range calculation.

where both nuclei are more than sufficiently dense to be completely opaque.

Correction for Coulomb effects, as represented by Eq. (3), is essential at these energies. Neglecting this correction raised the 3PF cross section from 1201 to 1305 mb at 90 MeV and had larger effects at lower energies.

Bertsch, Brown, and Sagawa[5] obtained a simpler microscopic model, also based on Glauber theory, by assuming that nucleon-nucleon interactions take place between independent tubes of matter, aligned with the beam direction, in the projectile and target. With this simplification, calculations using both finite-range and zero-range nucleon-nucleon forces are straightforward. When this model was used to calculate $\alpha+$Si reaction cross sections (assuming zero-range forces and 3PF densities, and making Coulomb corrections), the results were indistinguishable (differences $\leq 0.1\%$) from those obtained with Eqs. (1) and (3). A finite-range force with an interaction range of 1 fm increased the average σ_R by about 10%. This increase is comparable to that found by Bertsch, Brown and Sagawa[5] for the Li+^{12}C and Be+^{12}C systems and, as they explain, is expected since the effect of the finite-range force is to increase the surface density.

It seems disappointing that the calculations with finite-range forces overpredict the data. Two possible explanations may be considered. First, the density distributions we use are charge distributions obtained from electron scattering[11] and are not always corrected for proton size. If they are not, the use of finite-range forces may in effect count the nucleon size twice. Secondly, the microscopic models neglect Pauli blocking and the excitation of collective surface modes; these processes have large, opposing effects on the cross section.[14] Thus the discrepancy between these calculations, which should be our most realistic ones, and the experimental data may indicate incomplete cancellation of these effects.

Several strong absorption models exist for calculating σ_R. All such models contain an interaction radius $R_{\rm int}$ which is parametrized in different ways. Gupta and Kailas[15] review the arguments for energy-dependent interaction radii. Their model II, for instance, uses

$$R_{\rm int}=Q_1+Q_2+f(A_1^{1/3}+A_2^{1/3})/E_{\rm c.m.}^{1/3}\ , \qquad (4)$$

where the equivalent uniform radii Q_1 and Q_2 are defined in Ref. 15 and f is the only free parameter in their model. They fit many heavy-ion σ_R data, taking $f = 1.17$ for best fit to their data set. With this f we find $\alpha+$Si σ_R's about 50% larger than those given by the microscopic models, and $\chi^2/N \cong 100$. However, when much smaller f's are used (0.21 and 0.29 for their models I and II, respectively) we obtain fits indistinguishable from the 3PF prediction. Their model thus lacks universality in that it predicts too strong an energy dependence for the $\alpha+$Si interaction radius. Similarly, Kox *et al.*[4] found it inadequate to describe the reaction cross sections for $^{12}C + ^{12}C$.

We found the reaction probabilities for 2 to 6 MeV/nucleon d and ^{3}He on Si with the microscopic theory of DeVries and Peng, using deuteron densities from Humberston and Wallace (Ref. 16), ^{3}He densities from McCarthy *et al.*,[17] and the 3PF silicon density. The results are compared with measurements[9] in Fig. 4. For the $d+$Si system, both measurements and calculations give average cross sections of about 1200 mb in this region, and the fit is excellent. Replacement of the deuteron density with a Gaussian function having the same rms radius leads to serious underprediction of the data, since the Gauss function is very weak in the tail region in comparison with the realistic deuteron wave function.[16] This again illustrates the high sensitivity of the cross sections to the matter distribution in this region.

The predicted ^{3}He$+$Si cross section is negligible up to 2 MeV/nucleon since, even for head-on collisions, there is essentially no overlap of target and projectile. This effect makes the theory generally underpredict the data. However, at higher energies the predictions and data have equal slopes, signifying equal average cross sections (about 900 mb).

Finally, we consider the 15 to 53 MeV/nucleon $d+$Ge measurements,[8] noting first that the data given are ratios of deuterons reacting in a Ge detector telescope to those which do not react. We present in Table I the corresponding reaction probabilities. The microscopic calculations with zero-range NN forces give good predictions at the highest energies but are too low at lower energies. Finite-range calculations provide a still better fit at high energies but again underpredict η below 75 MeV. The problem may lie with the measurements, since at the lowest energies the deuterons went only a short distance into the stopping detector. However, microscopic calculations[3] for 10 to 30 MeV/nucleon deuterons on the nearby nucleus ^{90}Zr also are lower than the data points. New, more accurate measurements would therefore be of interest.

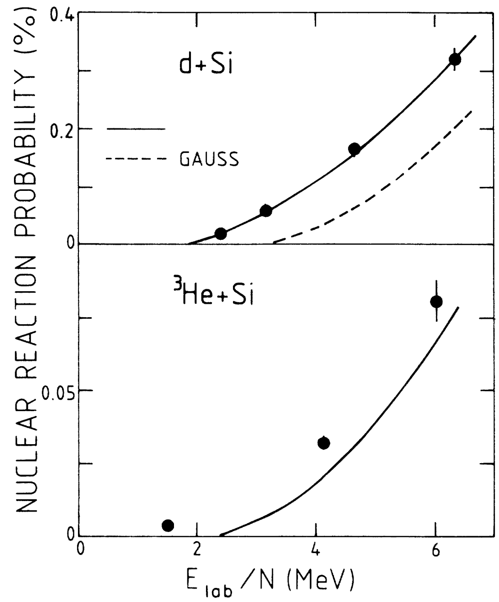

FIG. 4. Predicted and measured (Ref. 9) nuclear reaction probabilities for 2 to 6 MeV/nucleon d and ^{3}He on Si. The microscopic calculations employ zero-range nucleon-nucleon forces; nuclear density functions are specified in the text.

We conclude that the microscopic models, developed from the Glauber theory, predict reaction cross sections in good agreement with those measured for the $d+$Si, ^{3,4}He$+$Si, and $d+$Ge systems at energies as low as 2 MeV/nucleon provided realistic density distributions are used in the tail region, where the models are most sensitive. The prescription of DeVries and Peng[3] and of Kox *et al.*[4] for including Coulomb-force effects appears adequate. The one serious failure occurs for 1.5 MeV/nucleon ^{3}He on Si, where the closest-approach distance, even for head-on collisions, is so large that the density distributions have negligible overlap. The inclusion of finite-range nucleon-nucleon somewhat spoils the agreement with the most accurate data set we consider (27 to 92 MeV α- particles on Si). This suggests that

TABLE I. Total nuclear reaction probabilities $\eta(E_d)$ for deuterons of laboratory energy E_d incident upon germanium. Measurements are from (Ref. 8). Predictions are for zero- and finite-range nucleon-nucleon forces, using the method of Bertsch, Brown, and Sagawa (Ref. 5).

E_d (MeV)	Measured η	Predicted η (zero range)	Predicted η (finite range)
34.9	0.030±0.005	0.015	0.016
44.4	0.040±0.002	0.024	0.026
60.8	0.059±0.003	0.042	0.046
75.3	0.068±0.004	0.062	0.066
90.0	0.093±0.004	0.084	0.089
105.2	0.109±0.005	0.109	0.116

omission from the models of certain physical effects—such as Pauli blocking, eclipsing, and collective surface excitations—causes errors which only partially cancel one another in determining the reaction cross sections.

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