PHYSICAL REVIEW C VOLUME 42, NUMBER 5 NOVEMBER 1990

Soft giant dipole mode of ^{11}Li

A. C. Hayes and D. Strottman
Theoretical Division, Los Alamos National Laboratory, Los Alamos, New Mexico 87545
(Received 23 July 1990)

The low-lying electric dipole distribution for ^{11}Li is examined in detail in a large basis nonspurious shell model which includes up to $3\hbar\omega$ of excitation. A very strong enhancement of the $E1$ strength between 1 and 4 MeV of excitation is predicted when Woods-Saxon single-particle wave functions are used. The relativistic Coulomb excitation cross section is underestimated. However, an analysis of the contributions to the $E1$ matrix elements indicates that small changes in the structure of the low-lying states can lead to a significant increase in the predicted $E1$ strengths, so that excitations normally omitted from shell-model calculations are not necessarily needed to explain the anomalous Coulomb cross section.

Electromagnetic interactions in peripheral relativistic heavy-ion (RHI) collision have proven to be a powerful probe of giant resonances in nuclei, and the measured Coulomb fragmentation cross sections are well understood in terms of the corresponding photoabsorption cross sections. Recent experiments have observed[1] the electromagnetic dissociation cross section for ^{11}Li on ^{208}Pb at incident energies of 0.8 GeV/nucleon to be anomalously large, 20 times larger than for ^{12}C. At relativistic energies Coulomb excitation is dominated by electric dipole radiation, and such a large cross section would be explained by the ground-state $E1$ distribution strength being strongly enhanced at low excitation energies. An estimate of the enhancement needed suggests that a soft giant dipole mode exists in ^{11}Li, in which about 20% of the total ground-state $E1$ strength lies at 1–2 MeV of excitation. $E1$ strengths between low-lying nuclear states are normally greatly suppressed compared to single-particle estimate with most of the dipole sum rule being accounted for by the giant resonance at higher excitation. Low-lying dipole transitions, which are typically $<10^{-2}$ W.u., then necessarily involve strong cancellations between the different single-particle contributions to the matrix element. However, if the last particle is loosely bound to the core, so that the corresponding single-particle wave function has a long tail, we might expect to find a low-lying state for which the cancellation is broken and the resulting $E1$ strongly enhanced.

The purpose of this paper is to investigate the properties of ^{11}Li within a large basis shell model which has been shown to give a consistent description of the neighboring nuclei and, in particular, of low-lying enhanced $E1$ transitions. The strongest known $E1$ transition between discrete states has been observed[2] in ^{11}Be where $B(E1{:}1/2_1^- \rightarrow 1/2_{\text{g.s.}}^+)=0.36$ W.u. The enhancement of the transition has been understood[2] in terms of the weak binding energy of the last neutron; the $1/2^-$ state lies at 0.32 MeV of excitation while the neutron threshold is at 0.5 MeV. Millener *et al.*[2] have examined this and other strongly enhanced, low-lying $E1$ transitions in p-shell nuclei and have shown that they can be explained within the shell model if the binding energies of the orbitals involved are taken into account correctly. A recent random-phase approximation (RPA) calculation[3] for ^{11}Li found some enhancement of $E1$ distribution at low energies, but not enough to explain the observed Coulomb excitation cross section. In this work we calculated the Coulomb cross section for ^{11}Li from the distribution of ground-state electric dipole strength predicted from a shell-model calculation which includes up to $3\hbar\omega$ of excitation, and examined in detail the structure of the low-lying $E1$ states.

In the simplest shell model ^{11}Li is described as a single $p_{3/2}$ proton with the neutrons forming a closed shell. The ground state (g.s.) magnetic moment,[4] $\mu=3.667\mu_n$, is close to the Schmidt $p_{3/2}$ value ($3.793\mu_n$), lending strong support to a shell-model description of the nucleus. However, to obtain a realistic ground-state wave function it is necessary to include $2\hbar\omega$ excitations in the model space. In particular, ground-state correlations play an important role in determining the total dipole strength and the excitation energy of the giant resonance. Also, two-neutron $p^{-2}(sd)^2$ excitations lie low in the theoretical spectrum and can mix strongly into the ground-state wave function. We have used a complete $(0+2)\hbar\omega$ model space for the negative-parity states and a full $1\hbar\omega$ and truncated $3\hbar\omega$ basis for the positive-parity states of ^{11}Li. The truncation of our $3\hbar\omega$ basis was chosen so as to include states which are favored by the two-body interaction and included the configurations with SU(3) symmetries $(\lambda,\mu)=(6,2)$, (5,1) (7,0), (4,3), (0,5). For the two-body interaction we used the Cohen-Kurath[5] (CK) p-shell, Chung-Wildenthal[6] sd-shell, and Millener-Kurath[7] (MK) particle-hole (ph) interactions. The p-shell and sd-shell single-particle energies were taken to be the CK and MK values, and $\hbar\omega$ was set at 13.0 MeV.

In large $(0+2)\hbar\omega$ shell-model calculations it is difficult to determine the mixing of the $2\hbar\omega$ configurations into the ground state. Indeed, one of remaining questions in shell-model theory is how to treat 2p-2h and 1p-1h excitations which can couple to the $0\hbar\omega$ state through the ph interaction that transforms as the SU(3) tensor $(\lambda,\mu)=(2,0)$. The problems which arise are described, for example, in Ref. 8. However, it is essential to include the (2,0) excitations to ensure reasonable $E1$ and $E2$ distributions; e.g., they are responsible for the increase of the energy of the giant dipole resonance and the reduction in the total $E1$ strength over the simple $(0\to1)\hbar\omega$ model. To handle the (2,0) excitations we adopted the procedure proposed in a study[8] of the giant dipole resonances built on the g.s. and 0^+(6.05 MeV) state of ^{16}O. In this method the strength of the (2,0) interaction is treated as a parameter; i.e., $V(2,0)=\epsilon V_{\rm MK}(2,0)$, where $V_{\rm MK}$ is the full MK strength and $0\le\epsilon\le1$. When $\epsilon\neq0$ the spectrum is restored using Ellis's[9] method for eliminating unlinked diagrams for shell-model calculations, i.e., a constant Δ was added to the unperturbed $0\hbar\omega$ matrix element prior to diagonalizing the Hamiltonian to ensure that the g.s. binding energy returns to its $\epsilon=0$ value. In the present calculations we used $\epsilon=0.75$, since this gives the best description of the ^{16}O g.s. $E1$ distribution. While this is but one prescription for determining the $0\hbar\omega$ and $2\hbar\omega$ mixing in large shell-model calculations, it is nonetheless physically reasonable. For example, the calculated[8] dipole distribution and $E1$-$E1$ two-photon decay rate of the 4p-4h 0^+(6.05 MeV) state in ^{16}O show strong sensitivity to the interaction used, and require the same value of ϵ ($=0.75$) as for the ^{16}O g.s.

Another important consideration in any structure calculation is the problem of spurious center-of-mass excitations. In an SU(3) harmonic-oscillator (HO) basis such as used here, these are eliminated exactly. However, our main aim in this work was to determine the dipole distribution for ^{11}Li, for which we need to use more realistic single-particle wave functions. Replacing $E1$ single-particle matrix elements obtained using oscillator functions with those obtained using more realistic functions unavoidably introduces some spurious contributions.

Diagonalization of the shell-model Hamiltonian in the $(0+2)\hbar\omega$ space yielded a $3/2^-$ ground state which is dominated by the $p_{3/2}$ proton configuration, with 2p-2h configurations making up 25% of the wave function. The predicted magnetic moment is $\mu=3.62\mu_n$, which is in good agreement with the observed[4] value. The $p_{1/2}$ spin-orbit partner of the ground state is strongly mixed with the first $2\hbar\omega 1/2^-$ state, and two $1/2^-$ states are predicted at 3.61 MeV and 5.0 MeV containing 39% and 41% of the $0\hbar\omega$ strength, respectively. The lowest dominantly $2\hbar\omega$ $3/2^-$ state lies at 2.64 MeV, while the pure $2\hbar\omega$ states start with the $5/2^-$ (5.86 MeV), $7/2^-$ (4.6 MeV), and $9/2^-$ (7.43 MeV). The low-lying positive-parity states are dominated by $p^{-1}(sd)$ neutron excitations, and $3\hbar\omega$ configurations typically make up $<10\%$ of the wave functions of states below 10 MeV. We adjusted the MK p-sd intershell spacing slightly so as to reproduce accurately the $1/2^+_{\rm g.s.}$-$1/2^-_1$ splitting in ^{11}Be. This then leads to $3/2^+_1$, $5/2^+_1$, and $1/2^+_1$ states in ^{11}Li at 1.45 MeV, 2.8 MeV, and 3.3 MeV, respectively. We note that our excitation spectrum and percentage of $2\hbar\omega$ and $3\hbar\omega$ configurations predicted in the wave functions of the states in ^{11}Li differ from those obtained by Sagawa *et al.*[10] These differences arise from the fact that (a) we have taken the Millener-Kurath approach[7] and used HO (as opposed to Hartree-Fock) wave functions to evaluate the two-body ph matrix elements from the MK potential and (b) we have treated (2,0) excitations differently.

We first calculated the electric dipole distribution using HO single-particle wave functions. The giant resonance, which is somewhat fragmented, is predicted to lie between 15 and 20 MeV, and $<2\%$ of the total $E1$ strength lies below 5 MeV of excitation. Thus, it is clear that if the loose binding of the last neutron is not taken into account, no soft $E1$ mode is predicted; a similar result was found[2] for ^{11}Be. This result is not unexpected, since the isovector ph interaction pushes most of the available $E1$ strength up in energy so that the different $p\to(sd)$ amplitudes add constructively in the resonance region, exhausting a large fraction of the sum rule. In the case of low-lying $E1$ transitions the $p\to1s$ and $p\to d$ amplitudes are generally opposite in sign and cancel strongly. There are also $0s\to p$ contributions to the $E1$ matrix elements, but for ^{11}Li and the neighboring nuclei these contributions are relatively small at low excitation energies.

We examined in detail the $3/2^+_1\to3/2^-_{\rm g.s.}$ transition since, with the use of Woods-Saxon (WS) functions, it was found to be a main contributor to the anomalous Coulomb excitation cross section. For the sake of simplicity we restrict our discussion to a $(0\to1)\hbar\omega$ model; adding the $2\hbar\omega$ and $3\hbar\omega$ configurations does not alter the situation greatly. In standard calculations the $E1$ matrix elements are expressed in terms of the one-body density matrix elements (OBDME's), which contain the nuclear structure information, and the single-particle matrix elements (SPME's). The dominant structure of the $3/2^+_1$ state is similar to the $1/2^+_{\rm g.s.}$ of ^{11}Be, and involves a neutron excitation from the p shell to the $1s_{1/2}$ level, so that the $p\to1s$ OBDME's are large. On the other hand, the use of HO SPME's for the destructive $p\to d$ amplitudes cause the $p\to d$ and $p\to1s$ amplitudes to be comparable, yielding a small total $B(E1)$ value (see Table I).

To include binding energies in evaluating the $E1$ matrix elements we note that it is the one-neutron and/or -proton separation energies which are significant in shell-model calculations of one-body operators. Following Ref. 2 we express the $E1$ matrix elements in terms of a summation over the $A-1$ core states as

$$\langle J_f T_f M_{T_f}||E1||J_i T_i M_{T_i}\rangle$$

$$=(-)^{j_1+j_2-\Delta J}\sum_{J_cT_c}\sum_{j_1j_2}U(J_fj_1J_ij_2;J_c\Delta J)\langle J_fT_f|||a^\dagger_{j_1}|||J_cT_c\rangle(-)^{T_c+(1/2)-T_i}\frac{\hat{T}_c}{\hat{T}_i}\langle J_cT_c|||\tilde{a}_{j_2}|||J_iT_i\rangle$$

$$\times\sum_{m_t(m_t'M_{T_c})}\langle T_cM_{T_c}\tfrac{1}{2}m_t|T_fM_{T_f}\rangle\langle T_cM_{T_c}\tfrac{1}{2}m_t'|T_iM_{T_i}\rangle\frac{\hat{j}_1}{\Delta\hat{J}}\langle j_1\tfrac{1}{2}||E1||j_2\tfrac{1}{2}\rangle\ . \qquad (1)$$

The usual shell-model OBDME's have been replaced by a sum over the parentage coefficients to all physical non-spurious states $|A-1;J_c\rangle$. To calculate the $E1$ matrix elements using Woods-Saxon (WS) wave functions we followed Millener's prescription[2] in which the SPME's in Eq. (1) are evaluated for the binding energy relative to each core state $|J_c\rangle$. We obtained a spectrum for ^{10}Li by diagonalizing the Cohen-Kurath[4] interaction in a $0\hbar\omega$ basis. Since the calculated excitation of the $3/2_1^+$ state of ^{11}Li is 1.45 MeV, it lies above neutron threshold; however, we have treated it as bound and examined the sensitivity of the predicted transition strength to the assumed binding. Evaluating the SPME's in Eq. (1) with WS wave functions led to a very enhanced $E1$ transition, $B(E1)=0.66$ W.u., which is a factor of 18 larger than the HO result. In this calculation we assumed that the $3/2_1^+$ state was bound by 0.2 MeV, but similar results were found for binding energies of 0.5 and 0.02 MeV. The main reason for the strong enhancement is the large increase of the $p\to 1s$ SPME relative to the $p\to d$ SPME, thus breaking the strong cancellation between the two contributions. The $1s_{1/2}$ parentage is strongest to the 1^+ g.s. and 2_1^+(0.956 MeV) state of ^{10}Li, while the $d_{5/2}$ strength is concentrated in the ^{10}Li(3_1^+, 3.4 MeV) state. Thus, the $d_{5/2}$ orbital is about 2 MeV more bound than the $s_{1/2}$ orbital. We carried out similar calculations for the $E1$ transitions to all $1/2^+$, $3/2^+$, and $5/2^+$ excited states of ^{11}Li below 5.0 MeV of excitation, and found the dipole distribution to be strongly enhanced for several states. Since HO wave functions give a reasonable description of the normal giant resonance in p-shell nuclei, we have used HO SPME's to calculate the $E1$ strength above 5.0 MeV. The predicted dipole distribution is shown in Figs. 1 and 2, where the large enhancement of the low-lying $E1$ strength in the WS calculation is apparent. It can also be seen that including the g.s. correlations and $3\hbar\omega$ states increases the energy of the giant resonance and reduces the total $E1$ strength, while keeping the energy-weighted sum rule constant.

It should be noted that in using WS wave functions to

TABLE I. The $3/2_1^+\to 3/2_{\rm g.s.}^-$ $E1$ transition in ^{11}Li.

j_1j_2	Core states ^{10}Li J_n (E_x in MeV)	OBDME[a]	SPME WS[b]	SPME HO[c]
$1s_{1/2}p_{1/2}$	$1_{\rm g.s.}^+$	−0.5426	−1.909	−0.7911
	2_1^+(0.956)	−0.2106	−1.622	
	2_2^+(3.52)	0.0618	−1.294	
$1s_{1/2}p_{3/2}$	$1_{\rm g.s.}^+$	−0.0954	−2.70	−1.12
	1_2^+	−0.0521	−1.734	
	2_1^+	−0.1813	−2.294	
	2_2^+	−0.1577	−1.831	
	2_3^+	0.0492	−1.508	
$d_{5/2}p_{3/2}$	$1_{\rm g.s.}^+$	0.0122	3.533	2.373
	1_2^+	−0.0139	2.768	
	2_1^+	−0.0028	3.208	
	2_2^+	−0.0796	2.852	
	3_1^+(3.39)	−0.3108	2.864	
$d_{3/2}p_{3/2}$	$1_{\rm g.s.}^+$	−0.0871	−1.178	−0.7911
	2_1^+	−0.0107	−1.0584	
	2_2^+	0.0122	−0.9467	
	0_1^+(5.75)	0.1052	−0.8971	
$d_{3/2}p_{1/2}$	$1_{\rm g.s.}^+$	−0.0236	2.634	1.7687
	2_1^+	−0.0784	2.391	
	2_2^+	−0.0343	2.126	
	2_3^+	0.0179	1.907	
$p_{1/2}0s_{1/2}$	20.0	−0.054	−1.041	−0.968
$p_{3/2}0s_{1/2}$	20.0	0.114	1.472	1.370

[a]The OBDME's are larger than the normal shell-model values by a factor $(A/A-1)^{Q_1+Q_2/2}$, as required when using a relative coordinate system (Ref. 2). Small contributions form higher lying states in ^{10}Li are included in the OBDME for the highest J_c.
[b]$a_0=0.65$ fm, $r_0=1.4$ fm, $r_c=1.6$ fm, $V_{so}=0$.
[c]$b_0=1.76$ fm, and $b_{rel}=(A/A-1)^{1/2}b_0$ (Ref. 2).

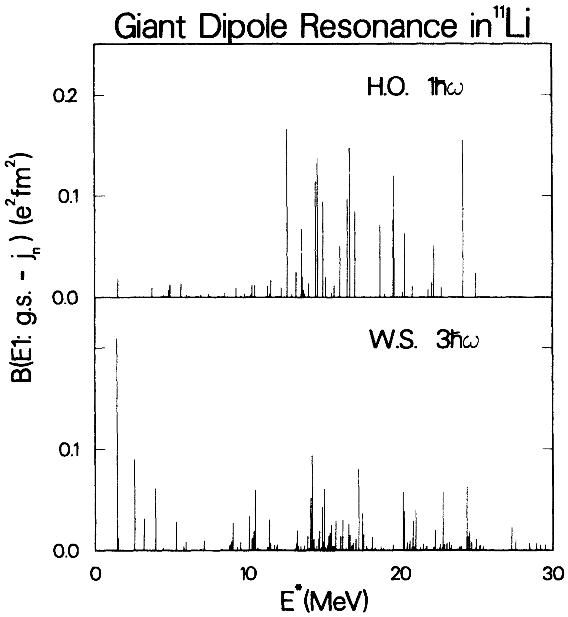

FIG. 1. The dipole distribution predicted in a $(0\rightarrow1)\hbar\omega$ model using HO wave functions and in a $(0+2\rightarrow1+3)\hbar\omega$ model using WS wave functions for states below 5.0 MeV of excitation.

evaluate the $E1$ matrix elements some inconsistencies have been introduced. We have assumed that the $E1$ transitions are purely isovector, but have allowed different radial dependences for the neutron and proton single-particle wave functions. Also, although we have attempted to minimize spurious center of mass contributions, they cannot be eliminated exactly in a WS basis. Despite these shortcomings our main result clearly represents a physical effect. Indeed, the enhancement of the low-lying dipole strength in ^{11}Li is exactly analogous to the strong enhancement of the $1/2_1^-\rightarrow1/2_{\rm g.s.}^+$ and $3/2_1^+\rightarrow1/2_{\rm g.s.}^-$ transitions in ^{11}Be and ^{9}Be.[2]

In the equivalent photon method, which is a good approximation at relativistic energies, the Coulomb excitation cross section can be calculated from the photoabsorption cross section as

$$\sigma_C=\sum_{\pi l}\int n_{\pi l}(\omega)\sigma_{\pi l}^{\gamma}(\omega)\frac{d\omega}{\omega}\ , \qquad (2)$$

where $n_{\pi l}(\omega)$ is the equivalent number of photons for multipolarity πl and energy $\hbar\omega$. Analytic expressions for $n_{\pi l}(\omega)$ have been derived by Bertunali and Baur.[11] The magnetic contributions to σ_C are negligible since the

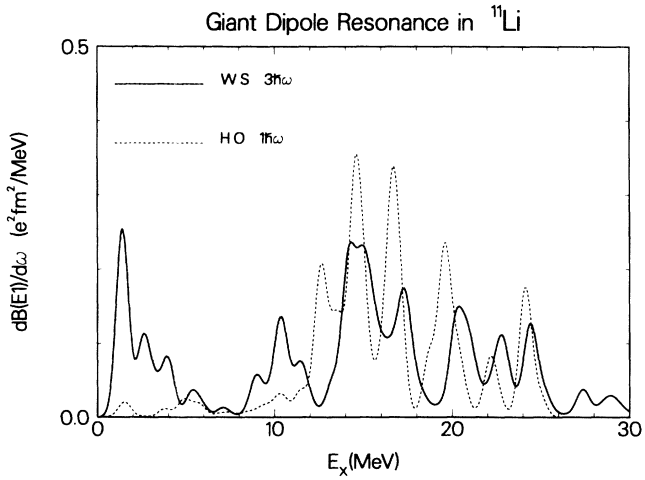

FIG. 2. The same as for Fig. 1, but with an artificial Gaussian width of 0.5 MeV applied to each state.

operators involve powers of $(1/Mc)^2$. For an assumed minimum impact parameter of 11 fm we obtained Coulomb cross sections of $\sigma_c(E2)=5.5$ mb and $\sigma_C(E1)=0.38$ b. This is to be compared with the observed Coulomb interaction cross section $\sigma_I^C=1.3\pm0.1$ b, or with the recent analysis of Bertsch *et al.*[12] which yields a Coulomb fragmentation cross section of $\sigma_{-2n}^C=0.65\pm0.1$ b. Thus, our calculation underestimates experiment by about a factor of 2. However, this result does not necessarily mean that excitations omitted from standard shell-model calculations are needed to explain the observed cross section. The $E1$ matrix elements to the low-lying $1/2^+$, $3/2^+$, and $5/2^+$ states are very sensitive to the $1s_{1/2}$ vs $d_{5/2}$ amplitudes in the wave functions, so that a small adjustment of the ph interaction could cause a significant increase in the predicted dipole strengths.

Finally, we emphasize the difference between the normal giant dipole resonance and the soft $E1$ mode. At high excitation energy the resonance involves constructive interference between the $p\rightarrow d$, $p\rightarrow1s$, and $0s\rightarrow p$ $E1$ amplitudes. In contrast, the soft dipole mode arises when the weak binding energy of the last neutron and corresponding long tail of the single-particle wave functions reduces the cancellation between large contributions to the matrix elements. The low-lying strength is very sensitive to the detailed structure of the states involved, and consequently is difficult reproduce accurately within any microscopic model.

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