Neutron-proton mass difference in nuclei and the Okamoto-Nolen-Schiffer anomaly

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(Received 21 May 1990)

Models that are successful in hadronic spectroscopy, viz., the nonrelativistic quark model, bag models, and QCD sum rules, are used to shed light on the Okamoto-Nolen-Schiffer anomaly in nuclear physics.

I. INTRODUCTION

The Okamoto-Nolen-Schiffer[1,2] (ONS) anomaly is a long-standing problem in nuclear physics. The anomaly is the persistent discrepancy between experiment and theory of binding energy differences of mirror nuclei (of the same atomic number A and with the exchange of proton number Z and neutron number N). Explicitly, the binding energy difference is

$$\Delta E = M_{z>} - M_{z<} + \delta_{np} , \tag{1.1}$$

where $M_{z>}$ is the atomic mass of the greater charge nucleus and $M_{z<}$ is the same of the lesser charge nucleus, and $\delta_{np} = 0.782$ MeV is the neutron-proton atomic mass difference. Experimental determinations of ΔE are 0.764, 3.54, 7.28, and 18.83 MeV for ^{3}H, ^{17}O, ^{41}Ca, and ^{208}Pb, respectively. For heavy nuclei, ΔE is greater than the binding energy of particles ~ 8 MeV. The calculated energy differences $(\Delta E)_{\text{theor}}$ between mirror nuclei (and other analog states) are $\sim 5-10\%$ smaller than $(\Delta E)_{\text{expt}}$. The difference $(\Delta E)_{\text{expt}} - (\Delta E)_{\text{theor}}$ increases with the mass number A and amounts to ~ 900 keV for the heaviest nucleus considered above.

Nuclear-structure effects, a long list of which may be found in the review article by Shlomo,[3] are thought individually to give small (1–2 %) contributions. As many of the effects considered give opposite contributions to $(\Delta E)_{\text{theor}}$, their total contribution does not resolve the anomaly. Recently, the role of charge-symmetry-breaking effects[1,4,5] has been reexamined,[6] with the conclusions that sizable positive contributions to $(\Delta E)_{\text{theor}}$ can arise from isospin breaking nuclear forces, especially those from ρ-ω mixing,[7] and that these contributions are helpful in partly reducing the discrepancy between experiment and theory.

Henley and Krein[8] (HK) have recently explored the possibility that the resolution of the ONS anomaly may be related to the partial restoration of chiral symmetry in nuclei. Specifically, they study the variation of the neutron-proton mass difference $\Delta M \equiv M_n - M_p$ in the nuclear medium. Their calculations are based on the Nambu–Jona-Lasinio model[9,10] for chiral symmetry breaking and are supplemented by a nonrelativistic quark model[11] for the nucleon. They find that the neutron and proton masses decrease from their free-space values with increasing nuclear density in such a manner that the variation of ΔM is in the right direction to remove the ONS discrepancy. Their numerical results even overshoot the values required to remove the anomaly. It is therefore of great interest to establish the degree to which the HK results are model dependent and also the degree to which chiral restoration is effective in reducing the discrepancy between experiment and theory.

The purpose of this work is to study the behavior of ΔM in the medium using different kinds of models for the nucleon all of which incorporate relativistic kinematics. The models we have considered are (i) the constituent quark model with minimal relativity, (ii) the MIT bag model, (iii) the chiral bag model, and (iv) an approach based on QCD sum rules. Our results are that ΔM decreases in the medium in all cases except the MIT bag model in which there is practically no variation with density. In each case, the decrease of ΔM occurs due to the change in the structure of the ground state of matter. We find, however, that the inclusion of relativistic effects leads to smaller variations of ΔM than obtained by HK who did not consider such effects in their model of the nucleon.

There exists a very natural concern about using effective hadron models to calculate the tiny $\Delta M \sim 1$ MeV. Are such models, which do not have such high precision, reliable for the calculation of ΔM? The answer is yes. The origin of the small ΔM is the small isospin breaking. In any reasonable model of the nucleon, once one turns off this small perturbation, ΔM is exactly zero. With isospin breaking, one can calculate the effect at least qualitatively since the perturbation is small. In other words, the smallness of ΔM is due to the smallness of the perturbation and one does not always need 1 MeV precision in the isospin-symmetric sector of the hadron models. This situation is similar to the calculation of weak processes involving hadrons.

In Sec. II, we reanalyze the Henley-Krein model by taking into account relativistic effects. These effects reduce the too large variation of ΔM found in the nonrelativistic approximation. In Sec. III, we present calculations of ΔM using the simple MIT bag model, which does not show any variation with density. In Sec. IV, results from the chiral bag model are given. Here effects due to the pion cloud are chiefly responsible for the result that ΔM decreases with density. In Sec. V, QCD sum rules are used to establish a direct relationship between the variations of ΔM and the quark condensate in the medium. Section VI is devoted to summary and conclusions.

II. THE CONSTITUENT QUARK MODEL

In the nonrelativistic quark model used by Henley and Klein, the neutron-proton mass difference ΔM is given by an expression due to Isgur[11]

$$\Delta M=(1-AM^{-3/2}-BM^{-9/4}-0.08)\delta M -CM^{1/4}+DM^{-5/4} , \quad (2.1)$$

where

$$M=(M_d+M_u)/2 \text{ and } \delta M=M_d-M_u . \quad (2.2)$$

M_d (M_u) is the constituent quark mass of the $d(u)$ quark. The coefficients A, B, C, D, and δM are determined from fits to $\Delta M=1.3$ MeV and $M=333$ MeV for nucleons in free space using harmonic oscillator potentials. In Ref. 8, the spring constant K for quarks bound by harmonic forces is assumed to be independent of nuclear density, and therefore A, B, C, and D are simple functions involving K. With $K=(219.6\text{ MeV})^3$, the numerical values of the other coefficients are

$$A=\sqrt{3K/4}\cong 2815\text{ MeV}^{3/2} , \quad B=7.63\times 10^4\text{ MeV}^{9/4} ,$$
$$C=0.146\text{ MeV}^{3/4} , \quad D=136\text{ MeV}^{9/4} . \quad (2.3)$$

As the variation of the electric ($CM^{1/4}$) and magnetic ($DM^{-5/4}$) terms with M is insignificant, we shall focus our attention on the parts coming from quark mass differences and strong interactions in ΔM. When the constituent quark masses vary from $M=333$ MeV down to 280 MeV, the main variation of ΔM is given by the variation of the terms proportional to A and B. δM itself varies little with density.

We discuss now the origin of these terms. As the term $AM^{-3/2}$ is the most important, we take this first. In both the proton and the neutron, we have two identical quarks, which we label 1 and 2, and denote their masses by m. Quark number 3 is the u (d) quark in the neutron (proton) with mass m'. The total rest mass is $\mu=2m+m'$. The nonrelativistic Hamiltonian is

$$H_0=\frac{1}{2m}(\mathbf{p}_1^2+\mathbf{p}_2^2)+\frac{1}{2m'}\mathbf{p}_3^2+\frac{K}{2}\sum_{i<j}(\mathbf{r}_i-\mathbf{r}_j)^2 . \quad (2.4)$$

Introducing the Jacobi coordinates

$$\rho=\tfrac{1}{2}(\mathbf{r}_1-\mathbf{r}_2), \quad \lambda=\frac{1}{\sqrt{6}}(\mathbf{r}_1+\mathbf{r}_2-2\mathbf{r}_3) ,$$
$$\mathbf{R}=\frac{m}{\mu}(\mathbf{r}_1+\mathbf{r}_2)+\frac{m'}{\mu}\mathbf{r}_3 , \quad (2.5)$$

one can rewrite Eq. (2.4) as

$$H_0=\frac{1}{2m_\rho}\mathbf{p}_\rho^2+\tfrac{3}{2}K\rho^2+\frac{1}{2m_\lambda}\mathbf{p}_\lambda^2+\tfrac{3}{2}K\lambda^2+\frac{1}{2\mu}\mathbf{P}_{\text{c.m.}}^2 , \quad (2.6)$$

where $m_\rho=m$, $m_\lambda=3mm'/\mu$, $\mathbf{p}_\rho=m_\rho\rho$, $\mathbf{p}_\lambda=m_\lambda\lambda$, and $\mathbf{P}_{\text{c.m.}}=\mu\mathbf{R}$. The Hamiltonian in the rest frame consists now of two independent three-dimensional harmonic oscillators both with the same spring constant K. The difference between the oscillator energies in the proton and the neutron comes about because in the proton the two u quarks are in the ρ oscillator whereas in the neutron the two d quarks are in the ρ oscillator, i.e.,

$$m_\rho=M_u=M-\tfrac{1}{2}\delta M, \quad m_\lambda=M+\tfrac{1}{6}\delta M \quad \text{(proton)} ,$$
$$m_\rho=M_d=M+\tfrac{1}{2}\delta M, \quad m_\lambda=M-\tfrac{1}{6}\delta M \quad \text{(neutron)} . \quad (2.7)$$

The different oscillator energies of the proton and the neutron contribute to the energy difference of the proton and neutron the amount

$$E_p-E_n=\tfrac{3}{2}[\omega_\rho(p)+\omega_\lambda(p)-\omega_\rho(n)-\omega_\lambda(n)]$$
$$=\sqrt{3K/4}M^{-3/2}\delta M=AM^{-3/2}\delta M , \quad (2.8)$$

where $\omega_{\rho,\lambda}=\sqrt{3K/m_{\rho,\lambda}}$. If we include the rest masses of the quarks in the nonrelativistic Hamiltonian Eq. (4), the net neutron-proton mass difference becomes

$$\Delta M=(M_d-M_u)(1-AM^{-3/2}) . \quad (2.9)$$

Now the factor $AM^{-3/2}$ can also be written as

$$AM^{-3/2}=T_3M^{-1} , \quad (2.10)$$

where T_3 is the nonrelativistic kinetic energy for the third quark

$$T_3=\frac{1}{2m'}\langle 0|\mathbf{p}_3^2|0\rangle , \quad (2.11)$$

whence

$$\left\langle 0\left|\frac{\mathbf{p}_3^2}{m'^2}\right|0\right\rangle=2m'T_3\cong 2AM^{-3/2} . \quad (2.12)$$

For $M=333$ MeV, $\langle 0|\mathbf{p}_3^2/M^2|0\rangle=0.93$, which is uncomfortably close to one. The situation becomes worse if we keep the string tension K constant and decrease the constituent mass M. For example, for $M=290$ MeV, $\langle 0|\mathbf{p}_3^2/M^2|0\rangle=1.14$ so that the nonrelativistic velocity is greater than that of light. One is then concerned about the degree to which a strong variation of ΔM with the constituent mass is an artifact of the nonrelativistic model. Relativistic corrections to variations of ΔM with nuclear density must be of some importance.

We estimate now the importance of relativistic corrections by using minimal relativity in the sense that we use the Hamiltonian

$$H_R=\sum_{i=1}^{3}\sqrt{m_i^2+\mathbf{p}_i^2}+\frac{K}{2}\sum_{i<j}(\mathbf{r}_i-\mathbf{r}_j)^2 , \quad (2.13)$$

for the nucleons and take the expectation value of H_R between the eigenstates of the nonrelativistic approximation

$$H_{\text{NR}}=\sum_i m_i+\sum_i\frac{\mathbf{p}_i^2}{2m_i}+\frac{K}{2}\sum_{i<j}(\mathbf{r}_i-\mathbf{r}_j)^2 . \quad (2.14)$$

Since

$$\mathbf{p}_1=\frac{1}{\sqrt{2}}\mathbf{p}_\rho+\frac{1}{\sqrt{6}}\mathbf{p}_\lambda+\frac{m}{\mu}\mathbf{P}\ ,$$
$$\mathbf{p}_2=-\frac{1}{\sqrt{2}}\mathbf{p}_\rho+\frac{1}{\sqrt{6}}\mathbf{p}_\lambda+\frac{m}{\mu}\mathbf{P}\ , \qquad (2.15)$$
$$\mathbf{p}_3=-\frac{2}{\sqrt{6}}\mathbf{p}_\lambda+\frac{m'}{\mu}\mathbf{P}\ ,$$

expectation values can be calculated by using eigenfunctions of $H_{\rm NR}$ in the momentum representation where the total momentum $\mathbf{P}$ is quantized to zero. Then the neutron-proton mass difference to first order in perturbation theory will be

$$\Delta M=\langle n|H_R|n\rangle-\langle p|H_R|p\rangle\ ,$$
$$=\left\langle n\left|\sum_{i=1}^{3}\sqrt{m_i^2+\mathbf{p}_i^2}\right|n\right\rangle-\left\langle p\left|\sum_{i=1}^{3}\sqrt{m_i^2+\mathbf{p}_i^2}\right|p\right\rangle$$
$$-\tfrac{1}{2}AM^{-3/2}\delta M\ , \qquad (2.16)$$

where the last term is calculated to first order in (M_d-M_u). The expectation value in the ground state of the neutron is

$$\mathcal{O}_n=\left\langle n\left|\sum_{i=1}^{3}\sqrt{m_i^2+\mathbf{p}_i^2}\right|n\right\rangle$$
$$=\frac{8}{\pi\sqrt{3}(\nu_\rho\nu_\lambda)^{3/2}}\int\int dk_\rho^2\,dk_\lambda^2\,e^{-k_\rho^2/\nu_\rho-k_\lambda^2/\nu_\lambda}$$
$$\times\left[\left[M_d^2+\frac{k_\rho^2}{2}+\frac{k_\lambda^2}{6}+\frac{k_\rho k_\lambda}{\sqrt{3}}\right]^{3/2}-\left[M_d^2+\frac{k_\rho^2}{2}+\frac{k_\lambda^2}{6}-\frac{k_\rho k_\lambda}{\sqrt{3}}\right]^{3/2}\right]+\frac{2}{\sqrt{3\pi}}M_u\sqrt{y}\,e^{y}K_1(y)\ . \qquad (2.17)$$

Here $y=(\sqrt{\tfrac{2}{3}}M_u)^{3/2}/4A$, $\nu_\rho=m_\rho\omega_\rho$ and $\nu_\lambda=m_\lambda\omega_\lambda$. The corresponding expression for

$$\mathcal{O}_p=\langle p|\sum_{i=1}^{3}\sqrt{m_i^2+\mathbf{p}_i^2}|p\rangle$$

is obtained from $\mathcal{O}_n$ by interchanging M_u and M_d in all factors entering Eq. (2.17).

In the upper panel of Fig. 1, we show the variation of ΔM from the nonrelativistic approximation Eq. (2.9) (dashed line) and the expression in Eq. (2.16), which contains corrections from minimal relativity (solid line). At this level, ΔM contains only the part involving the quark mass difference and the oscillator energy difference. In order to have the same normalization for ΔM for free particles (with $M=333$ MeV), we use $\delta M=6$ MeV in Eq. (2.9) and $\delta M=4.76$ MeV in Eq. (2.16). It is clear that the variation is much smaller when we use Eq. (2.16) (minimal relativity) than when we use Eq. (2.9) (the nonrelativistic approximation). We believe that the variation would be even more reduced if we included relativistic corrections to the wave function. Sizable variations of ΔM with nucleon mass are very difficult to obtain from kinetic terms of the Hamiltonian.

[We note in passing that if for Eq. (2.14) we naively adopted a "relativistic" version, namely $R=\sqrt{M^2+2MT_3}-M$, where

$$T_3=AM^{-1/2}=\langle 0|\mathbf{p}_3^2|0\rangle/2M\ ,$$

Eq. (2.16) would take the form

$$\Delta M=(M_d-M_u)(2-\sqrt{1+2AM^{-3/2}})\ , \qquad (2.18)$$

and in the relevant range of M, ΔM varies very little with M.]

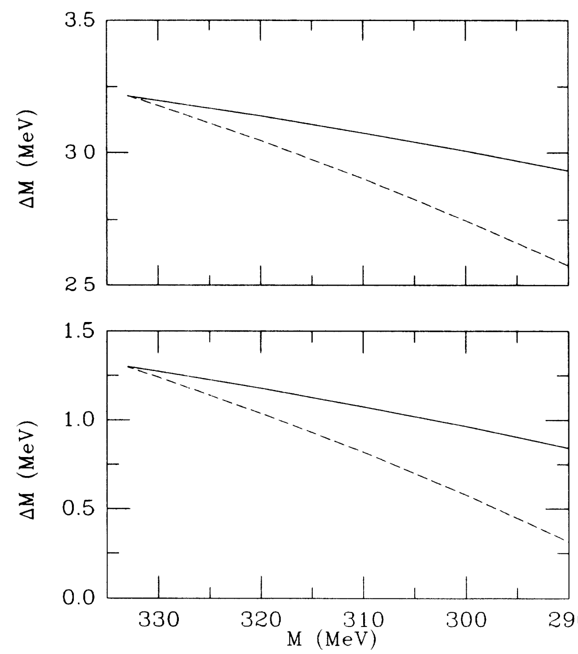

FIG. 1. Upper panel: $\Delta M=M_n-M_p$ (involving the quark mass difference and the oscillator energy difference) versus the average constituent quark mass M. The solid line shows the result from Eq. (2.16) which includes relativistic corrections ($\delta M=4.76$ MeV). The dashed line is the result of the nonrelativistic approximation Eq. (2.9), with $\delta M=6$ MeV. Lower panel: Full mass difference from Eq. (2.1). Solid line is the result including minimal relativity corrections. Dashed line is from the nonrelativistic approximation.

As the numerical results of Henley and Krein overshoot the required decrease of ΔM in the medium, the relativistic corrections are welcome. There is an additional variation of ΔM in Eq. (2.1) due to the term $BM^{-9/2}$, which originates from the first-order chromomagnetic interaction. It might, however, be that this is an artifact of the nonrelativistic model. In bag-type relativistic models, the chromomagnetic neutron-proton mass difference is very small and independent of the bag radius for a fixed α_s.

It might nevertheless be worth mentioning that if we correct the first two terms of Eq. (2.1) by using minimal relativity, and retain all other terms, then we would have to use $\delta M=4.2$ MeV (with $M=333$ MeV) to obtain the empirical mass difference of 1.3 MeV. The lower panel of Fig. 1 compares results of this approach (solid line) to the nonrelativistic results (dashed line) of Eq. (2.1) ($\delta M\cong 6.2$ MeV and $M=333$ MeV). When the constituent quark mass varies in the range 333–290 MeV, the decrease of ΔM is ~ 0.5 MeV using minimal relativity, which is to be compared with the decrease of ~ 1 MeV from the nonrelativistic approximation.

We have shown that the relativistic corrections are important when one wants to compute variations of the neutron-proton mass difference as a function of the constituent quark masses. It should also be mentioned that Eq. (2.1) for the neutron-proton mass difference implicitly assumes that the string constant K for the quark interaction does not change when the constituent mass changes due to a decrease of the quark condensate in the medium. The mean-square radius $\sqrt{\langle r^2\rangle}$ for nucleons is therefore bigger for bound nucleons than for free nucleons. For example, when m changes from 333 to 290 MeV, $\sqrt{\langle r^2\rangle}$ changes by the factor $(\frac{333}{290})^{1/2}\cong 1.07$. In medium swelling of the nucleon is implicit when Eq. (2.1) is applied to explore ΔM in medium. We turn now to other models that have been used in baryon spectroscopy to see if a sufficient variation of ΔM can be obtained in order to explain the ONS anomaly, at least qualitatively.

III. THE MIT BAG MODEL

There is some evidence for the property that the electromagnetic mean-square radius of a nucleon increases when the nuclear density increases.[12] In the literature, this feature is termed the swelling of the nucleon. We can simulate this effect in the MIT bag model[13,14] by allowing the bag radius R to change from its canonical value. The dynamical reason for this behavior can be a decrease of the vacuum pressure, i.e., a decrease of the bag constant B relevant for the calculation of the nucleon mass in the nuclear medium compared to a similar calculation in vacuum. One may then inquire how the neutron-proton mass difference varies when $\sqrt{\langle r^2\rangle_N}$ (or equivalently R) changes due to effects induced by the nuclear medium. The mass difference (m_d-m_u) of the current quarks would be reflected in the mass difference for the neutron and proton partly through a kinetic term and partly through a term proportional to the quark gluon coupling constant α_s. The energy of each quark with mass m is

$$E_Q(m,R)=\frac{1}{R}\sqrt{x^2+(mR)^2}\equiv\frac{\omega(mR)}{R}\ , \tag{3.1}$$

where the eigenmodes $x=x(mR)$ are solutions of the equation

$$\tan x=\frac{x}{1-mR-\sqrt{x^2+(mR)^2}}\ . \tag{3.2}$$

Expansion of the kinetic part of ΔM gives

$$E_Q(m,R)=E_Q(0,R)+m\left.\frac{d\omega(y)}{dy}\right|_{y=0}+\cdots\ , \tag{3.3}$$

where $y=mR$. Note that the first-order correction does not depend on R. Thus, to first order in isospin breaking the bag radius of the neutron and the proton are the same, $R_n=R_p$. Therefore, ΔM from the kinetic energy difference reads

$$\begin{aligned}\delta K&\equiv E_Q(m_d,R)-E_Q(m_u,R)\\&=(m_d-m_u)\left.\frac{d\omega(y)}{dy}\right|_{y=0}\ ,\end{aligned} \tag{3.4}$$

with

$$\left.\frac{d\omega(y)}{dy}\right|_{y=0}=\frac{0.5}{x(0)-1}=0.48\ . \tag{3.5}$$

To first order in δm, we find that the chromomagnetic energy $\delta E_{\rm CM}=(-\frac{2}{3})\times 0.047\alpha_s$ which is also independent of R. Thus the MIT bag result of the neutron-proton mass difference from the strong interaction is

$$\Delta M=\delta K+\delta E_{\rm CM}=(m_d-m_u)(0.48-0.031\alpha_s)\ . \tag{3.6}$$

As there is no R dependence in this expression (neglecting a variation of α_s as a function of R in the small second term), it is clearly impossible to induce a substantial variation of ΔM in this model by nucleon swelling only. The results shown in Table I substantiate this conclusion clearly. These results were calculated using a zero point energy constant $Z_0=1.869$, the quark-gluon coupling constant $\alpha_s=2.2$, $m_u=3$ MeV, and $m_d=7.4$ MeV.

The magnitude of ΔM which has the correct value due to hadronic interactions in free space,[14] is seen to be relatively insensitive to the nucleon size. We note in passing that Eq. (3.6) is valid over a wide range of R. The dynamics leading to the ONS effect must therefore be more complicated than provided by the MIT bag model with changing radii. We therefore turn now to the MIT bag model in its chiral version.[15-18]

IV. THE CHIRAL BAG MODEL

The chiral bag model[15-18] is a more realistic model of the nucleon than the original MIT bag model as it gives partially conserved axial currents (PCAC) and thereby takes into account the pionic degrees of freedom in nuclei. The pion-nucleon coupling constant is calculable as the effects of the pionic cloud on the energy of the free nucleon. The pionic contribution will be different for the neutron and the proton due to the isospin breaking of the

TABLE I. Variation of masses in the MIT bag model with increasing nucleon size.

$B^{1/4}$ (MeV)	M_p (MeV)	M_n (MeV)	R (fm)	ΔM (MeV)
145	937.80	939.62	0.99	1.82
100	647.39	649.22	1.43	1.83

π^0-N coupling induced by the u-d quark mass difference. We first describe how this comes about before we estimate how this isospin breaking influences the neutron-proton mass difference in nuclei.

We initially consider the perturbative approach using the big-bag model. For simplicity, we present the arguments using free-quark wave functions inside the bag. To get numerically reasonable pion-nucleon coupling constants, we shall later use the cloudy bag[19] where the pion is permitted to penetrate inside the bag. The isovector axial current to lowest order in the pion field is

$$\mathbf{A}_\mu=\bar{\Psi}\frac{\tau}{2}\gamma_\mu\gamma_5\Psi\Theta(R-r)+f_\pi\partial_\mu\pi \quad (4.1)$$

where π is the static pion field normalized so that

$$\pi=\frac{g\tau}{8\pi M}\frac{\sigma\cdot\hat{\mathbf{r}}}{r^2}(1+\mu r)e^{-\mu r}\,, \quad (4.2)$$

outside the nucleon. Above, M is the nucleon mass and μ is the pion mass. The pion-nucleon coupling constant is fixed by PCAC constraint of the conserved axial current through the surface. In the isospin-symmetric limit where the quarks have the same mass, the quark wave functions $\Psi(r)=[iF(r),G(r)\sigma\cdot\hat{\mathbf{r}}]^t$ inside the bag are the same for u and d quarks. The neutral pion-nucleon coupling constant is

$$g=\tfrac{5}{3}C[|F|^2+|G|^2]_{r=R}\,. \quad (4.3)$$

The constant C is given by[20]

$$C=\frac{e^{\mu R}}{2+2\mu R+\mu^2R^2}\frac{MR^3}{f_\pi}\frac{1}{1+X}\,, \quad (4.4)$$

where X is a function of μR that is very close to 0.5 for all values of R that interest us here.

When isospin is broken through the difference of u and d quark masses, the continuity of axial currents through the bag surface leads to isospin breaking in the coupling of pion to nucleons. As in the isospin symmetric case, we use as wave functions

$$|p\uparrow\rangle=\frac{1}{\sqrt{6}}[2|u_\uparrow u_\uparrow d_\downarrow\rangle-|u_\uparrow u_\downarrow d_\uparrow\rangle-|u_\downarrow u_\uparrow d_\uparrow\rangle]$$

$$|n\uparrow\rangle=-\frac{1}{\sqrt{6}}[2|d_\uparrow d_\uparrow u_\downarrow\rangle-|d_\uparrow d_\downarrow u_\uparrow\rangle-|d_\downarrow d_\uparrow u_\uparrow\rangle] \quad (4.5)$$

$$\pi^+=\bar{d}u,\quad \pi^-=\bar{u}d,\text{ and }\pi^0=\frac{1}{\sqrt{2}}(\bar{u}u-\bar{d}d)\,.$$

If one takes into account the small s quark mixing for the neutral pion, one has[21]

$$\pi^0=a\bar{u}u-b\bar{d}d-c\bar{s}s$$
$$\sim 0.711\bar{u}u-0.703\bar{d}d-0.008\bar{s}s\,. \quad (4.6)$$

The neutral pion coupling to the proton and neutron becomes

$$g_{\pi^0pp}=\sqrt{2}C\left[\frac{4a}{3}(|F_u|^2+|G_u|^2)+\frac{b}{3}(|F_d|^2+|G_d|^2)\right]$$
$$\simeq[\tfrac{4}{3}(|F_u|^2+|G_u|^2)+\tfrac{1}{3}(|F_d|^2+|G_d|^2)]\,, \quad (4.7)$$

and

$$g_{\pi^0nn}=\sqrt{2}C\left[-\frac{a}{3}(|F_u|^2+|G_u|^2)-\frac{4b}{3}(|F_d|^2+|G_d|^2)\right]$$
$$\simeq-[\tfrac{1}{3}(|F_u|^3+|G_u|^2)+\tfrac{4}{3}(|F_d|^2+|G_d|^2)]\,. \quad (4.8)$$

Here F_q and G_q are the upper and lower components taken at the surface. These formulae are instructive since they show that if the pions couple to the nucleon at some surface, then $g_{\pi^0pp}>|g_{\pi^0nn}|$. The reason is purely quantum mechanical: the u quark is lighter than the d quark and will therefore be more spread out in space than the d quark. Hence

$$F_u^2+G_u^3>F_d^2+G_d^2\,. \quad (4.9)$$

It is of course implicit in our argument that the "radius of the bag" is nearly identical for the proton and neutron. The particle (proton) that contains the most of the lightest flavor will be coupled the strongest to the neutral pion. We remark that this simple result is quite consistent with the result of the other approaches[22] even quantitatively.

This isospin breaking for pion coupling now has an influence on the mass difference of the neutron and the proton which is of interest if the sizes of nucleon increase in nuclei. Pionic contributions to the proton and to the neutron mass are now unequal. It is instructive to see how this works in the limit of large bags where perturbative arguments hold and where the pionic contribution to the nucleon mass is equivalent to calculating self-energy diagrams for the nucleons.[23,24]

With nucleons and Δ as intermediate states, the pionic contribution to the nucleon mass in the isospin symmetric limit is

$$\Delta M_N^\pi\simeq-C\frac{g_{\pi N}^2}{R^3}\,, \quad (4.10)$$

where C is a slowly varying function of R

$$C=\frac{567}{2400\pi M^2}e^{-2\mu R}(1+\mu R)(2+2\mu R+\mu^2R^2)\,. \quad (4.11)$$

In ΔM_N^π, two-thirds of the contribution comes from the self-energy of charged pions and one third comes from the neutral pion. The contribution to the neutron-proton mass difference due to the pionic interaction will therefore be positive

$$\Delta M_{np}^\pi=\frac{C}{3}\frac{g_{\pi^0pp}^2-g_{\pi^0nn}^2}{R^3}\,, \quad (4.12)$$

and it decreases with increasing R (or charge radius) of the nucleon. It is instructive to substitute the value of C from Eq. (4.11) when $R \sim 1$ fm, whence

$$\Delta M_{np}^{\pi} = \frac{C}{3} g_{\pi^0 pp}^2 \left[1 - \frac{g_{\pi^0 nn}^2}{g_{\pi^0 pp}^2} \right] \frac{1}{R^3}$$
$$\simeq \frac{8\ \mathrm{GeV}^{-2}}{R^3} \left[1 - \frac{g_{\pi^0 nn}^2}{g_{\pi^0 pp}^2} \right] . \quad (4.13)$$

As bag models give typically $g_{\pi^0 nn}^2 / g_{\pi^0 pp}^2 \sim 0.99$, an order of magnitude estimate is

$$\Delta M_{np}^{\pi} \simeq \frac{0.1\ \mathrm{GeV}^{-2}}{R^3} , \quad (4.14)$$

so that, for $R = 5\ \mathrm{GeV}^{-1}$, $\Delta M_{np}^{\pi} \sim 1$ MeV. The order of magnitude so obtained is therefore relevant when one discusses the ONS anomaly. This discussion has been rather qualitative; its aim has been to show that in chiral models there is a contribution to ΔM coming from the broken isospin invariance in the pion-nucleon coupling.

We now turn to a quantitative analysis by calculating the properties of the neutron and the proton in the chiral bag model. Our calculations will be quite similar to those in Ref. 20 except that we use a mass of the d quark which is 3.6 MeV larger than the mass of the u quark and modify the chiral boundary condition to allow for the broken isospin symmetry. The parameters we use are $B^{1/4} = 0.125$ GeV for the bag pressure, $Z_0 = 0.612$ for the zero-point energy, $\alpha_s = 3$ for the strong coupling constant, $m_u = 0$ and $m_d = 3.6$ MeV.[25] We summarize the properties obtained using this solution that are of interest to us in Table II.

The hadronic mass difference between a neutron and proton in free space is 1.8 MeV as it should be (the Coulomb energy difference[14] amounts to ~ -0.5 MeV). The model is clearly not giving ridiculous results when compared to well-known data. How well the model does in its determination of isospin breaking in the pion coupling we cannot say, but isospin breaking at the 1% level as above is not in conflict with any experimental data.

We study now how the masses change when we decrease the bag pressure so that the mean square radius of the proton increases. If we decrease B from $B^{1/4} = 0.125$ to 0.1 GeV, $\langle r^2 \rangle_p^{1/2}$ increases by about 24%. The masses of the nucleons then decrease to $M_n = 754.10$ MeV, $M_p = 752.42$ MeV and $g_A = 1.19$. We see that we have obtained a small decrease in ΔM (of 0.12 MeV) and g_A from their values in free space.

The results of this section are that ΔM and g_A decrease for nucleons bound in nuclei. This decrease is somewhat smaller than that obtained using the constituent quark model with minimal relativity (see Sec. II) and can at most explain a fraction of the ONS anomaly. It is of course conceivable that the bag model underestimates the charge symmetry breaking of the π-N system and that a swelling of the nucleons is indeed an important dynamical mechanism for the decrease of ΔM in the medium.

We turn now to a completely different approach—QCD sum rules—that shows the same qualitative variation of ΔM as did the models in Secs. II and IV, when the nuclear density increases.

TABLE II. Observables in the chiral bag model with $m_d = 3.6$ MeV and $m_u = 0$.

Particle	Neutron		Proton
Mass (MeV)	939.15		937.16
$\langle r^2 \rangle^{1/2}$ (fm)	−0.07		0.9
Magnetic moment (Nuc. magnetons)	2.51		−1.80
$g_{\pi^0}^2 / 4\pi$	14.37		14.21
$g_A(n \to p)$		1.26	
$g_{\pi^+ pn}^2 / 4\pi$		14.29	

V. QCD SUM RULES

The QCD sum rules[26–28] are another relativistic formulation—of fundamental origin—that can be used to analyze how baryon masses change in the medium. In this approach, the neutron-proton mass difference may be related to the variation of the quark and the gluon condensates in the medium. We are not aware that isospin breaking in the baryon sector has been analyzed before, but it is clear that we can use the result of calculations due to $\mathrm{SU}_f(3)$ breaking.[29,30] In fact, the neutron (proton) composite operator is obtained from the Ξ^0 (Σ^+) operator by a simple substitution $s \to d$;

$$\Psi_{\Xi} = \epsilon_{abc} [(s^a(x) C s^b(x)) \gamma_5 u^c(x) + t (s^a(x) C \gamma_5 s^b(x)) u^c(x)] \to \Psi_n ,$$
$$\Psi_{\Sigma} = \epsilon_{abc} [(u^a(x) C u^b(x)) \gamma_5 s^c(x) + t (u^a(x) C \gamma_5 u^b(x)) s^c(x)] \to \Psi_p , \quad (5.1)$$

where C denotes the charge-conjugation operator and t is a mixing strength of the two independent operators having the same quantum number as the baryons under consideration. Thus we can use the sum rules for Ξ and Σ with the replacement $m_s \to m_d$ ($m_u = 0$) to calculate ΔM.

The QCD sum rule is based on the use of the operator-product expansion (OPE) of the correlation function for Ψ above

$$\Pi^{\alpha\beta}(q) \equiv i \int d^4x\, e^{iqx} T[\Psi(x)^{\alpha} \overline{\Psi}(0)^{\beta}] = \sum_n C_n(q) \mathcal{O}_n , \quad (5.2)$$

where α and β are spinor indices. The short-range part of the correlation is summarized in the Wilson coefficients C_n, while the long-range part is summarized in the local composite operators $\mathcal{O}_n$. It is assumed that in vacuum the expectation values of $\mathcal{O}_n$ is nonvanishing, viz., $\langle 0 | \mathcal{O}_n | 0 \rangle \neq 0$. In medium, one should take the ground-state expectation value instead. Then all the medium (finite density) corrections are controlled by the density dependence of the expectation values of the composite operators. An alternate approach is to rearrange the OPE in the medium to an expansion that contains medium-dependent Wilson coefficients.[31,32] The former approach is more convenient in the low-density and/or low-temperature regime.

In taking expectation values in a medium, the choice of a specific frame, e.g., the rest frame of matter, gives rise to noncovariant condensates. For example, in the OPE one obtains the condensate

$$\langle :u^{\alpha}\bar{u}^{\beta}: \rangle = -\tfrac{1}{4}\sum_{\delta}\Gamma_{\delta}\langle :\bar{u}\Gamma^{\delta}u: \rangle \ , \tag{5.3}$$

where Γ_{γ}'s are the independent γ matrices and the symbol : denotes normal ordering with respect to the perturbative vacuum. In the rest frame of nuclear matter, $\Gamma_{\alpha}=1$ as well as γ_0 on the right-hand side give rise to the condensates $\langle \bar{q}q \rangle$ and $\langle q^{\dagger}q \rangle$, respectively. For $|\langle \bar{q}q \rangle|$, the relevant range of variation is

$$(230\ \text{MeV})^3 < |\langle \bar{u}u \rangle| < (250\ \text{MeV})^3 \ . \tag{5.4}$$

The condensate $\langle q^{\dagger}q \rangle$, which is actually the quark number density, is also restricted:

$$\langle u^{\dagger}u \rangle < \frac{N_c \rho_0}{N_f} \sim \frac{N_c}{N_f} \times 0.16 \times (200\ \text{MeV})^3 \ , \tag{5.5}$$

and amounts to $\sim 20\%$ of the scalar condensate at the nuclear matter equilibrium density of $\rho_0 = 0.16\ \text{fm}^{-3}$. Since the relevant densities lie below ρ_0 and the noncovariant condensates always appear associated with the scalar condensate, we will not explicitly consider the noncovariant condensate but will regard the scalar condensate as effectively including both effects.

On the phenomenological side of the sum rule, one commonly assumes a specific form for the spectral function $\rho^{\alpha\beta}(s)$. For example,

$$\rho^{\alpha\beta}(s) = F[\gamma_{\mu}p^{\mu} + M_N]^{\alpha\beta}\delta(s - M_N^2) + \text{ continuum} \ . \tag{5.6}$$

This is then used in the dispersion relation in the medium[31,32]

$$\text{Re}\Pi^{\alpha\beta}(\omega, \mathbf{p}=0) = \frac{\omega^{2n}}{\pi}\int_0^{\infty} \frac{\rho^{\alpha\beta}(s)}{s^n(s-\omega^2)} ds - \text{ subtractions} \ . \tag{5.7}$$

Here we take a nucleon having three-momentum $\mathbf{p}=0$ and deep Euclidean energy $\omega = i\omega_I$ ($\omega_I \gg 1$). After Borel transformation of Eqs. (5.2) and (5.7), one gets an expression for the mass as a function of the condensates and the Borel parameter τ. For light hadrons, it is known that the quark condensate, rather than the gluon condensate, is most effective in determining physical properties. We can now examine hadronic properties associated with reasonable changes of the quark condensate in the medium.

Before presenting results of detailed calculations using the Borel-transformed version of the sum rule, we first present some qualitative arguments for the variation of ΔM. At zero density, a simple formula for ΔM can be obtained by expanding the mass formula for Σ and Ξ (see Ref. 28) up to first order in symmetry breaking. For $t=-1$ and neglecting continuum contribution for simplicity,

$$\Delta M = \frac{M_0}{a\tau^4}[-(\tau^4 + \tfrac{4}{3}aM_0)a\gamma + (\tfrac{2}{3}a^2 - \tau^6 - aM_0\tau^2)(m_d - m_u)] \ , \tag{5.8}$$

where

$$a \equiv -(2\pi)^2\langle \bar{q}q \rangle \simeq (2\pi)^2(250\ \text{MeV})^3 \ ,$$

$$b \equiv \pi^2\langle (\alpha_s/\pi)G^2 \rangle \simeq \pi^2(330\ \text{MeV})^4 \ ,$$

and

$$\gamma \equiv \langle \bar{d}d \rangle / \langle \bar{u}u \rangle - 1 \ .$$

The quantity γ provides a measure of isospin breaking in the condensates and is expected to be negative. M_0 and τ are the nucleon mass in the chiral limit and the Borel mass, respectively. These are determined by the stability condition $\partial M_0(\tau)/\partial\tau = 0$. By using the solution in Eq. (5.8), one gets

$$\Delta M = -4.79\gamma|\langle \bar{q}q \rangle|^{1/3} - 1.56(m_d - m_u) \ . \tag{5.9}$$

In deriving Eq. (5.9), we have used a simple scaling relation, viz., every mass scale, except for $(m_d - m_u)$, is proportional to $\langle \bar{q}q \rangle^n$. This dependence may be justified using the finite-energy sum rule.[33] By using the standard value $(m_d - m_u) = 4$ MeV, and, taking $\gamma = -0.0067$ which we will justify later, one gets 1.5 MeV for ΔM at zero density. Equation (5.9) shows that the neutron-proton mass difference is chiefly governed by the isospin breaking of the condensate.[34] The above relation holds also at finite density provided we restrict ourselves to densities where the noncovariant condensates are small.

These considerations suggest a decrease of ΔM with density if the density dependence of γ is small compared to that of $\langle \bar{q}q \rangle$. We emphasize that Eq. (5.9) is valid only when the quark condensate is sufficiently large, viz., at rather low densities. Close to the critical density of chiral restoration, effects of the higher-order condensates become important.

Some remarks on the electromagnetic contribution to the mass difference in the present approach are in order. For dimensional reasons, electromagnetic effects modify only the first term of the above formula to lowest order in isospin symmetry breaking. This leads to the replacement

$$(-4.79\gamma)|\langle \bar{q}q \rangle|^{1/3} \rightarrow (-4.79\gamma - \text{const} \times e^2)|\langle \bar{q}q \rangle|^{1/3} \ .$$

As ΔM should be positive, this term cannot be negative. Therefore, our arguments below are not affected when we neglect electromagnetic effects.

To predict ΔM in the medium, we need the density dependences of γ and the quark condensate as inputs. Since lattice QCD results for these quantities are not yet available, we will estimate their behavior using the SU(2) Nambu–Jona-Lasinio (NJL) model. The NJL Lagrangian is

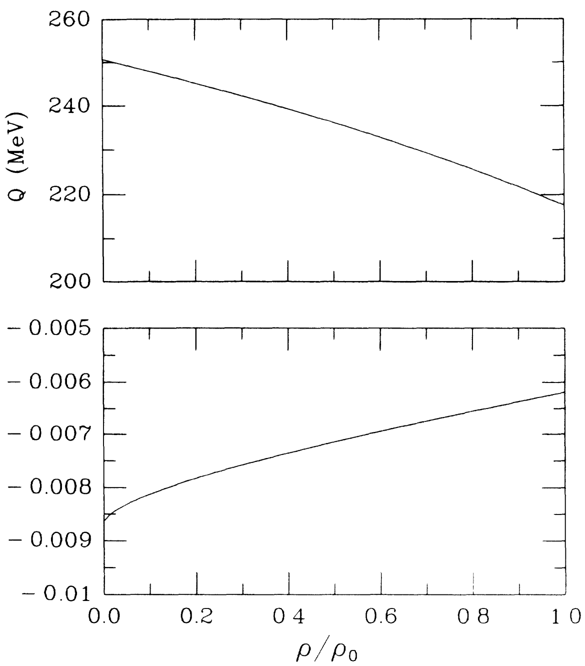

FIG. 2. The nonperturbative part of the quark condensate $Q \equiv |\langle \bar{q}q \rangle|^{1/3}$ and the symmetry-breaking parameter γ in the SU(2) Nambu–Jona-Lasinio model as a function of the nuclear density with $\rho_0 = 0.16\ \text{fm}^{-3}$.

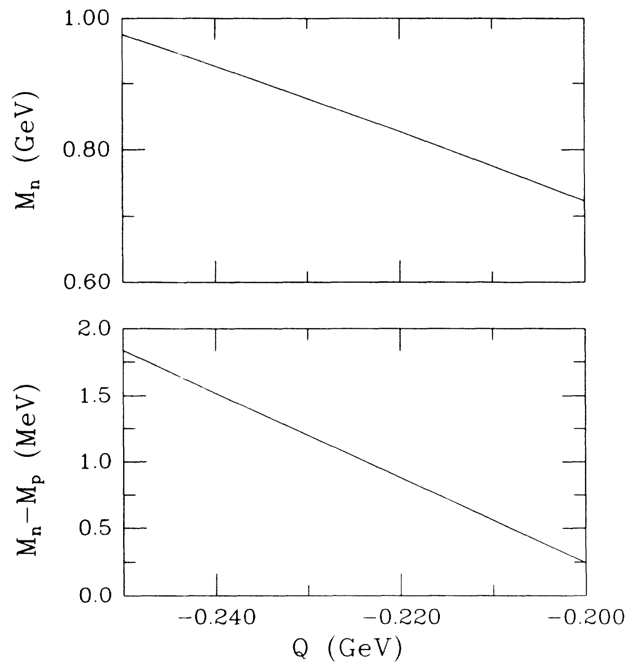

FIG. 4. M_n and $\Delta M = M_n - M_p$ in the QCD sum rules approach as a function of $Q \equiv |\langle \bar{q}q \rangle|^{1/3}$.

$$\mathcal{L} = \bar{\psi}(i\gamma\cdot\partial - m)\psi + G[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\tau\psi)^2] , \qquad (5.10)$$

for which the nonperturbative part of the quark condensate is given by[35]

$$\langle \bar{u}u \rangle = -i\,\text{Tr}[S_F(p;M_u) - S_F(p;m_u)] ,$$
$$\langle \bar{d}d \rangle = -i\,\text{Tr}[S_F(p;M_d) - S_F(p;m_d)] , \qquad (5.11)$$

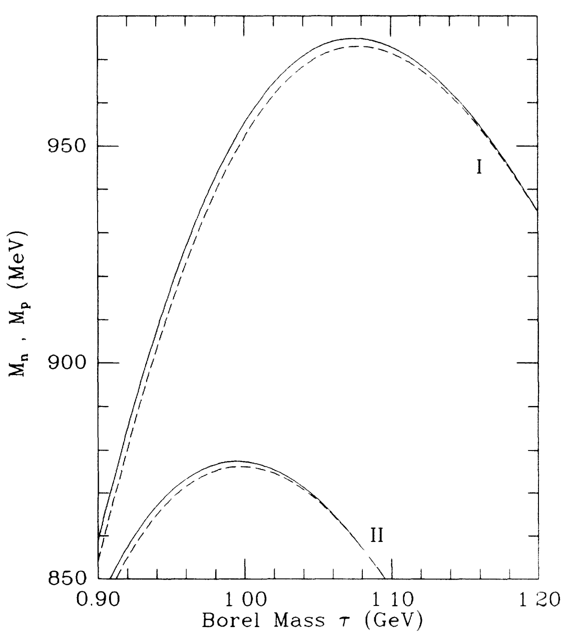

FIG. 3. The proton and neutron masses as a function of the Borel mass τ. The solid lines (dashed lines) refer to the neutron (proton). $Q \equiv |\langle \bar{q}q \rangle|^{1/3}$ is 250 MeV for curve I and 230 MeV for curve II.

where $S_F(p;X)$ is the quark propagator in the medium with mass X.

In Fig. 2, we show the density dependence of γ and the average condensate in the medium. The numerical results were obtained using the following parameters: $\Lambda = 663$ MeV and $G\Lambda^2 = 1.93$, $m_u = 3$ MeV and $m_d = 7$ MeV. With these values one obtains

$$(\langle \bar{u}u \rangle + \langle \bar{d}d \rangle)/2 = 251\ \text{MeV}$$

and

$$(M_u + M_d)/2 = 311\ \text{MeV}$$

at zero density. One can see that (i) $\gamma = -0.0086$ at $\rho = 0$ and increases as the density increases, and (ii) $|\langle \bar{q}q \rangle|$ decreases with ρ and approaches zero at high densities, suggesting the partial restoration of chiral symmetry.[35] Combining these observations with the result in Eq. (5.9) we can see that ΔM will decrease in medium.

We give now more quantitative results for the variation of ΔM using the Borel-transformed version of the sum rule. These results represent an improvement over the mass formula in Eq. (5.9) which is an expansion in terms of γ and the current masses.

In the vacuum, the best fit to the baryon octet is obtained by the parameters (the continuum contribution is neglected)

$$m_u = m_d = 0 \quad \text{and} \quad m_s = 170\ \text{MeV} ,$$
$$\frac{\langle \bar{s}s \rangle}{\langle \bar{u}u \rangle} - 1 = -0.33 \quad \text{and} \quad t = -1.15 . \qquad (5.12)$$

(These numbers are quoted in the note added in proof of Ref. 30.) Values of $\langle \bar{u}u \rangle^{1/3} = 0.25$ GeV and the gluon condensate $(\alpha_s/\pi)\langle GG\rangle=(0.33\ \text{GeV})^4$ are commonly used in the literature.

To illustrate the behavior of M_n and M_p when isospin is broken $(m_d \neq m_u)$ and when the quark condensate changes, we use the above value of t and γ by assuming a linear current quark mass dependence of the quark condensate:

$$\langle \bar{q}q\rangle=\langle \bar{u}u\rangle\left[1-\frac{0.33}{170\ \text{MeV}}m_q\right]. \tag{5.13}$$

For $(m_d-m_u)=4$ MeV, we obtain

$$\gamma=-7.76\times10^{-3}, \tag{5.14}$$

which is consistent with the result from the NJL model above. Other independent analyses[22] have yielded $\gamma=-0.007$ and $\gamma=-0.0063\pm0.0030$. The resulting neutron-proton mass difference is 1.84 MeV. Here the optimum Borel parameter τ_{opt} is chosen again by the stability condition $\partial M_{n,p}/\partial\tau=0$, which is a standard method when one neglects the continuum contribution.

In Fig. 3, the neutron and proton masses are shown as a function of the Borel mass τ. One observes that in a reasonable range of τ, ΔM is positive. The nucleon mass becomes small associated with the partial restoration of chiral symmetry as expected. Furthermore, the Borel masses corresponding to the extremum for neutron and proton are roughly the same, which allows us to extract ΔM using a single Borel parameter. The situation is not the same if the current quark mass is large, in which case the optimal Borel parameters for Ξ and Σ are sufficiently different to prohibit the use of a single parameter to extract the mass difference.

In Fig. 4, M_n and ΔM are shown as a function of the quark condensate. When $|\langle\bar{u}u\rangle|^{1/3}$ decreases from 0.25 to 0.23 GeV, ΔM decreases from 1.84 to 1.24 MeV, while M_n decreases from 974.8 to 877.5 MeV. A decrease of the effective mass of the nucleon by 10% is just what is expected close to nuclear densities. Our sum-rule calculation seems to give a correct correlation between the nucleon mass and the neutron-proton mass difference. We obtain nearly identical results with the second set of parameters given in Ref. 30: $\langle\bar{u}u\rangle^{1/3}=-0.22$ GeV, $t=-1.32$, $\gamma=-0.43$, $m_s=200$ MeV and $(m_d-m_u)\simeq6.2$ MeV.

VI. CONCLUSIONS

In this article we have looked at what some popular models for hadrons have to say about the variation of the neutron-proton mass difference ΔM when these particles are in a medium of varying density.

For the nonrelativistic quark model and the bag model, one can obtain a decrease of ΔM which is associated with a swelling of nucleons in the medium. We have shown how the inclusion of corrections due to minimal relativity decreases the medium dependence of ΔM in the nonrelativistic quark model. The results so obtained are compatible with what is needed to explain the ONS anomaly. We suspect, however, that additional relativistic corrections could further diminish the medium dependence of ΔM. The chiral bag model as we have treated it also yields the desired decrease of ΔM and is associated with the diminishing bag pressure in the medium. The magnitude of the decrease is, however, too small to explain the whole anomaly.

Of the models we have looked at we find the QCD sum-rule approach to be the most satisfactory from a fundamental point of view. From the expected behavior of the different quark condensates in the nuclear medium one arrives at a very natural explanation why ΔM decreases when nuclear densities increase. We are a little uncomfortable, however, because the theory is still at a qualitative level. For example, the nucleon masses are somewhat larger than the empirical masses. Furthermore, the variation of the condensates in medium has to be injected from another source.

We think nevertheless that the QCD sum rule can shed new light on the ONS anomaly. The formula (5.9) will probably overestimate the effects of the nuclear medium. A more realistic formula for the hadronic part of the neutron-proton mass difference can be obtained by regarding Fig. 4 as

$$\Delta M\simeq6.0(M^*/M)-4.2\ \text{MeV},$$

where M^* (M) is the nucleon mass in the medium (vacuum).

ACKNOWLEDGMENTS

We would like to acknowledge G. E. Brown, T. H. Hansson, and I. Zahed for fruitful discussions and comments. H.H. thanks K. Olaussen for discussion. T.H. thanks G. A. Miller for informing us about Ref. 6 and M. Ericson for discussions. This work was supported by the U.S. Department of Energy under Grant No. DE-FG02-88ER40388.

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$$(m_d - m_u)[\partial f(x,y)/\partial x - \partial f(x,y)/\partial y]|_{x=y=0} .$$

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