Odd-odd interacting-boson-approximation calculations: Experimental agreement for doubly decoupled and highly distorted bands in $^{176-180}$Re

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We present results of interacting-boson-fermion-fermion approximation (IBFFA) calculations for deformed odd-odd ^{176}Re and ^{178}Re. Such calculations yield accurate structural information for many different rotational band types, including highly distorted and doubly decoupled bands. Thus, they can help in interpreting experimental data, predicting spin-parity assignments that agree with those obtained by other methods, and giving rather good predictions for interband transition energies. We also compare our previously published IBFFA predictions for ^{180}Re with new experimental data.

I. INTRODUCTION

Successes of the interacting-boson-approximation (IBA) model in describing the properties of many different types of even-even nuclei can be found in many references.[1-7] So can those of the interacting-boson-fermion approximation (IBFA), which can reproduce properties of different types of odd-mass nuclei in phenomenological calculations.[8-10] The interacting-boson-fermion-fermion approximation (IBFFA), extension of IBA techniques to odd-odd nuclei, is a relatively new approach. Nevertheless, studies of $N=83$ isotones[11,12] and odd-odd Re isotopes[13,14] have already demonstrated the power of IBFFA calculations.

This paper supplements and complements our previous paper,[13] in which we discussed our techniques for extending IBA calculations to odd-odd nuclei. The reader is referred to that paper for details of procedure, including best values of parameters and efficient truncation of basis spaces, etc. In the present paper we extend our calculations to the more neutron-deficient isotopes, ^{176}Re and ^{178}Re. These isotopes provide a more severe test of the IBFFA model, inasmuch as they exhibit even more highly-distorted bands, including doubly decoupled bands, than did the isotopes ($^{180-184}$Re) considered in our original paper.

In Sec. II we give a quick review of the observed and likely low-lying single-particle states available for the odd-odd Re isotopes in which we are interested. We also investigate how the properties of these states affect the results of IBFA and IBFFA calculations. In Sec. III we compare our calculations with experimental results for ^{178}Re,[15,16] which were published after our calculations had been completed. Both Santos *et al.*[15] and our group[17,18] have studied ^{176}Re via in-beam γ-ray spectroscopy, but its level scheme is not so extensive as for the heavier odd-odd Re isotopes and there is some disagreement between the two groups. In Sec. IV we compare the differing interpretations with IBFFA results and try to use the calculations as an aid in choosing the correct configurations.

While preparing our original paper,[13] we could not make any sensible comparison with the rather preliminary experimental results for ^{180}Re.[19,20] Now that new and much more complete data are available,[21,22] it is worthwhile comparing our IBFFA calculations with them, which we do in Sec. V.

Finally, we summarize our findings in Sec. VI. The IBFFA calculations do a remarkably good job of duplicating the "geometrical" properties of rotational bands in deformed odd-odd nuclei, even to the extent of reproducing extreme coriolis distortions, such as those encountered in the doubly decoupled bands. However, they suffer from the same difficulties as other odd-odd calculations, viz., predicting band-head positions and triplet-singlet splittings, problems related to positions of the Nilsson states and our understanding of the proton-neutron residual interaction. Nevertheless, they appear to be a potentially useful tool for helping to sort out the complexities of odd-odd rotational bands.

II. SINGLE-PARTICLE STATES

In preparation for IBFFA calculations, we need first to calculate the even-even core states via IBA. For ^{176}Re this is ^{176}Os (six proton holes, eighteen neutrons, for twelve active bosons); for ^{178}Re it is ^{178}Os (p^{-6},n^{20}, 13 bosons); and for ^{180}Re it is ^{182}Os (p^{-6},n^{-20}, 13 bosons).[13,17] Next we calculate (IBFA) the appropriate odd-mass nuclei and couple these states together (IBFFA) to form the odd-odd states. As a redundancy check on our truncation scheme, we performed this in two ways—coupling the odd-proton states to the appropriate odd-mass Os nuclei, then coupling the odd-neutron states to the odd-mass Re nuclei. Errors tend to accumulate: We adjust the IBA parameters first and try to match them

TABLE I. Energies of single-particle states used in IBFFA calculations.

	Nilsson state $\Omega^{\pi}[Nn_z\Lambda]$	Isotope	E (keV) Exp.	Calc.
		Odd-neutron states		
Fig. 1(a)	$\frac{7}{2}^+[633]$	^{177}Os	300.6	lowest+state
	$\frac{9}{2}^+[624]$	^{179}Os	243.0	lowest+state
Fig. 1(b)	$\frac{1}{2}^+[521]$	^{177}Os	0	0
	$\frac{1}{2}^+[521]$	^{179}Os	0	0
	$\frac{5}{2}^-[512]$	^{177}Os	152.3	242.9
	$\frac{7}{2}^-[514]$	^{179}Os	145.5	268.9
		Odd-proton states		
Fig. 1(c)	$\frac{9}{2}^-[514]$	^{177}Re	?	86.8
	$\frac{1}{2}^-[541],J=\frac{5}{2}$	^{177}Re	?	0
	$\frac{5}{2}^+[402]$	^{177}Re	84.7 above $\frac{9}{2}^-$	lowest+state
Fig. 1(d)	$\frac{9}{2}^-[514]$	^{179}Re		180.6
	$\frac{1}{2}^-[541],J=\frac{5}{2}$	^{179}Re		0
	$\frac{5}{2}^+[402]$	^{179}Re		lowest+state

systematically with experimental results, then we perform a similar operation with the IBFA parameters. Thus, the IBA fits affect the IBFA fits, which in turn affect the IBFFA results. What we know about the odd-mass nuclei in particular can produce significant effects, and there are two potential problems we may face in IBFFA calculations for ^{176}Re and ^{178}Re.

First, uncertainties in the choices and positions of single-particle states. The experimental (and calculated—see below) energies of the bandheads of the single-particle states chosen are listed in Table I; the bands are also shown in Fig. 1. For odd-mass Re isotopes with $A=177$–185, three common bands have been observed,[23–27] those based on the $\frac{1}{2}^-[541]$, $\frac{9}{2}^-[514]$, and $\frac{5}{2}^+[402]$ states. There is little doubt that these are the most-available odd-proton states. On the other hand, the situation is considerably more complicated for odd-neutron states. $\frac{1}{2}^-[521]$ seems to be a universal state for the 101st [e.g., ^{175}W (Ref. 28) and ^{177}Os (Ref. 29)] and 103rd [^{177}W (Ref. 30) and ^{179}Os (Ref. 29)] neutron. However, although both ^{177}W and ^{179}Os have 103 neutrons, we find $\frac{9}{2}^+[624]$ in ^{179}Os but $\frac{7}{2}^+[633]$ in ^{177}W. Similarly, $\frac{7}{2}^-[514]$ but not $\frac{5}{2}^-[512]$ appears in ^{179}Os, while the reverse is true for ^{177}W. This makes for ambiguity in the ^{178}Re calculation, and we take only ^{179}Os for the appropriate odd-neutron states. This leads to $\frac{9}{2}^+[624]$, $\frac{7}{2}^-[514]$, and $\frac{1}{2}^-[521]$ as the choices for the ^{178}Re low-lying single-neutron states.

Second, we should bear in mind that heavy-ion induced reactions bring in large amounts of angular momentum to the compound system and consequently preferentially populate rotationally-aligned (decoupled) states or states with large coriolis matrix elements.[31,32] Once such states are populated, the subsequent γ deexcitation will feed primarily into similar states, populating to greatest degree only a selected subset of the available states. These are what we see in the existing experimental data. When we fit odd-mass nuclei in IBFA calculations, we concentrate only on what can be observed in the experiments; additionally, to keep the calculations tractable, we calculate only those states lying below roughly 2 MeV in excitation.

The appropriate odd-mass nuclei for the IBFFA calculations of ^{176}Re are ^{175}Re and ^{177}Os; for ^{178}Re they are ^{177}Re and ^{179}Os. Results for these nuclei in the IBFA model are shown in Fig. 1. The IBA parameters for the even-even cores are given in Table II; IBFA parameters for the single-particle states, in Table III. (For details about how these were obtained, see Ref. 13.) Among these nuclei, no experimental data are available for ^{175}Re. However, since odd-mass $^{177-185}$Re have all been studied and fitted in the IBFA model,[13,17] we simply extrapolate the parameters for ^{175}Re from these heavier nuclei. As will be seen, this appears to give reasonable results.

III. ^{178}Re RESULTS

A. The doubly decoupled band

In-beam γ-ray studies on ^{178}Re were carried out by Kreiner *et al.*[15,16] using the reactions, ^{169}Tm(^{12}C,$3n\gamma$)^{178}Re and ^{165}Ho(^{18}O,$5n\gamma$)^{178}Re. Four rotational bands were identified and are shown in the left portion of Fig. 2. Because it is not clear what the relative positions of these bands are, we simply plot them all as starting at 0 MeV. For comparison, we plot six calculated low-lying bands in the right portion of the figure.

Of the four observed bands, one is the well-known

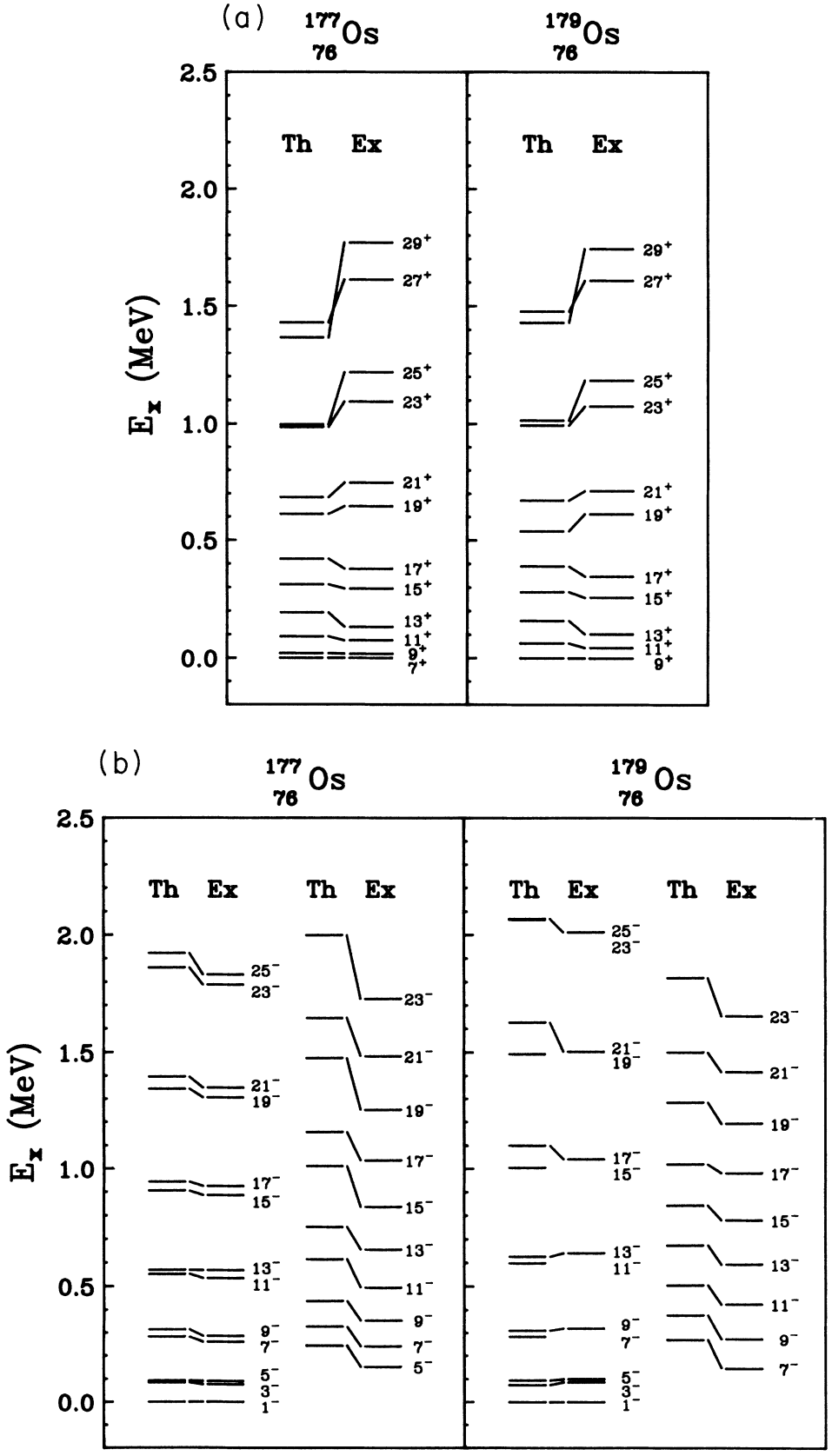

FIG. 1. IBFA-calculated excitation energies compared with experimental data. States are labeled with $2J$. (a) Positive-parity states in odd-mass Os isotopes. (b) Negative-parity states in odd-mass Os isotopes. (c) States of both parities in ^{177}Re. (d) States of both parities in ^{175}Re—no experimental data are available.

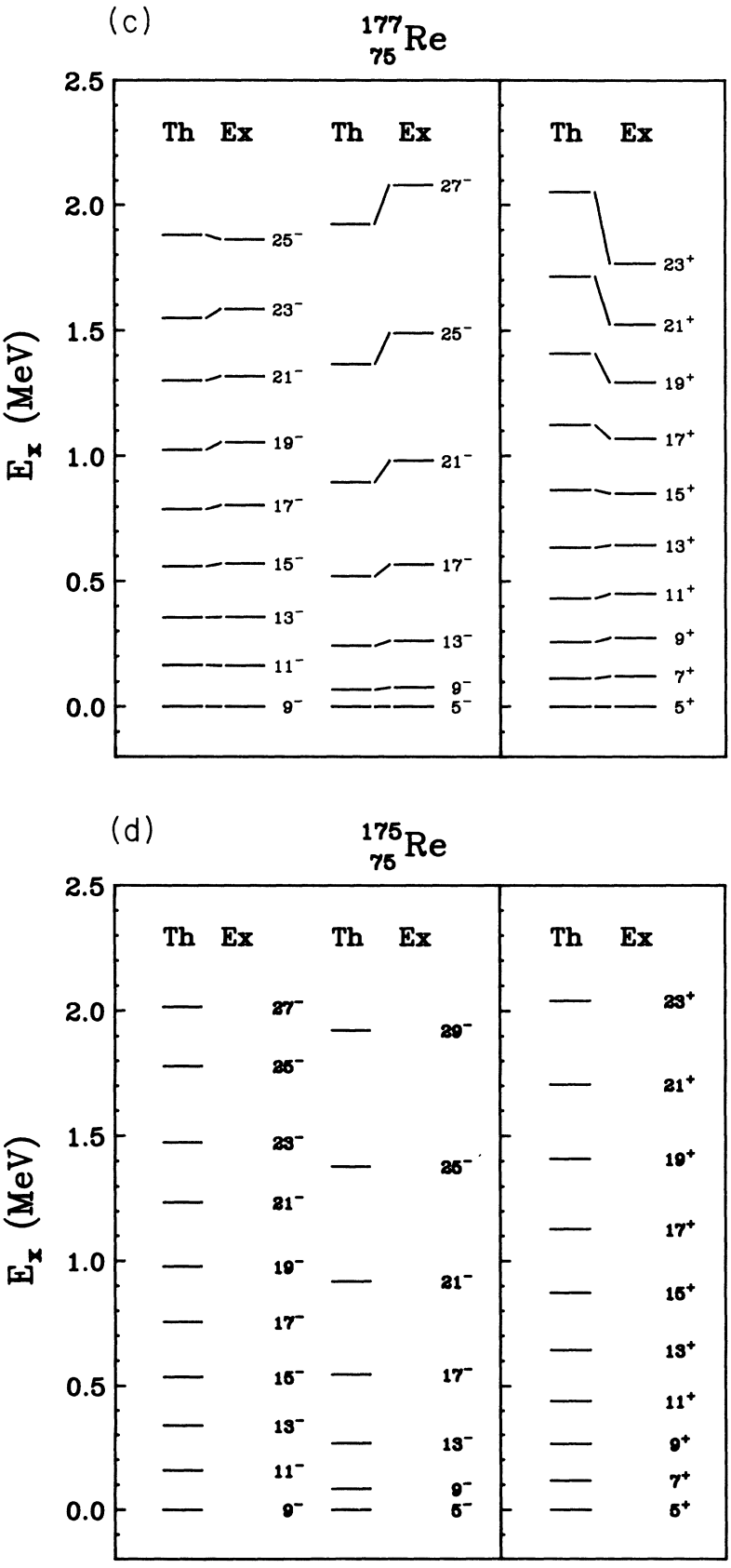

FIG. 1. (*Continued*).

TABLE II. IBA parameters for the even-even Os core nuclei.

	^{176}Os	^{178}Os
ϵ_d (MeV)	0.54	0.53
K' (MeV)	0.000	0.0015
K (MeV)	−0.014	−0.015
χ (dimensionless)	−1.163	−0.929

$\Delta J=2$ doubly decoupled band (band 1 in Fig. 2), formed by coupling the extremely distorted $h_{9/2}\ \frac{1}{2}^-[541]$ proton state with the less-distorted $p_{3/2}\ \frac{1}{2}^-[521]$ neutron state. The J^π values of the members of this band are well established.[33] A 3^+ state is believed to be the band head, according to the two-quasiparticle-plus-rotor model (TQPRM).[15] However, such a state has not been observed experimentally.

Our IBFFA calculations also produce 3^+ as the lowest member of the band (band A in Fig. 2), predicting a 71.0-keV $5^+\rightarrow 3^+$ transition. The calculations yielded excellent agreement for the higher-energy transitions in the doubly decoupled band, which gives us some faith in the prediction for the $5^+\rightarrow 3^+$ transition. Its relatively low-energy and high-($E2$) conversion coefficient would have precluded its having been seen in the experiments reported.

In addition to producing good agreement with the observed transition energies, the IBFFA calculations predict that the unfavored—and unobserved—signature members of the band, those of even spin, are shifted to higher energy in relation to the favored, odd-spin ones. Also, it predicts that the "singlet $K=0$" coupling is shifted up with respect to the "triplet $K=1$" coupling. (K is not strictly a good quantum number in such decoupled bands, but it remains a useful label.[34]) Exactly the same situation is seen for ^{176}Re and will be discussed in more detail in Sec. IV.

TABLE III. IBFA parameters for single-particle states.

	Odd-proton negative-parity states	
	^{176}Re	^{177}Re
Λ_0^-	1.28	1.36
Γ_0^-	1.25	1.02
A_0^-	−0.10	−0.10
χ	−1.20	−1.20
$\nu^2 h_{11/2}$	0.58	0.59
$\nu^2 h_{9/2}$	0.05	0.05
E(MeV)$h_{9/2}$	5.20	4.50
	Odd-proton positive-parity states	
	^{175}Re	^{177}Re
Λ_0^+	0.795	0.418
Γ_0^+	0.751	0.666
A_0^+	0.05	0.00
χ	−1.20	−1.20
$\nu^2 g_{7/2}$	0.974	0.974
$\nu^2 d_{5/2}$	0.919	0.918
$\nu^2 d_{3/2}$	0.722	0.723
$\nu^2 s_{1/2}$	0.705	0.713
	Odd-neutron negative-parity states	
	^{177}Os	^{179}Os
Λ_0^-	0.941	0.809
Γ_0^-	0.180	0.228
A_0^-	−0.10	−0.10
χ	−1.00	−1.00
$\nu^2 f_{7/2}$	0.801	0.846
$\nu^2 h_{9/2}$	0.683	0.734
$\nu^2 p_{3/2}$	0.139	0.190
$\nu^2 f_{5/2}$	0.135	0.166
$\nu^2 p_{1/2}$	0.0965	0.115
	Odd-neutron positive-parity states	
	^{177}Os	^{179}Os
Λ_0^+	2.760	2.450
Γ_0^+	0.380	0.441
A_0^+	−0.10	−0.10
χ	−1.00	−1.00
$\nu^2 i_{13/2}$	0.300	0.350

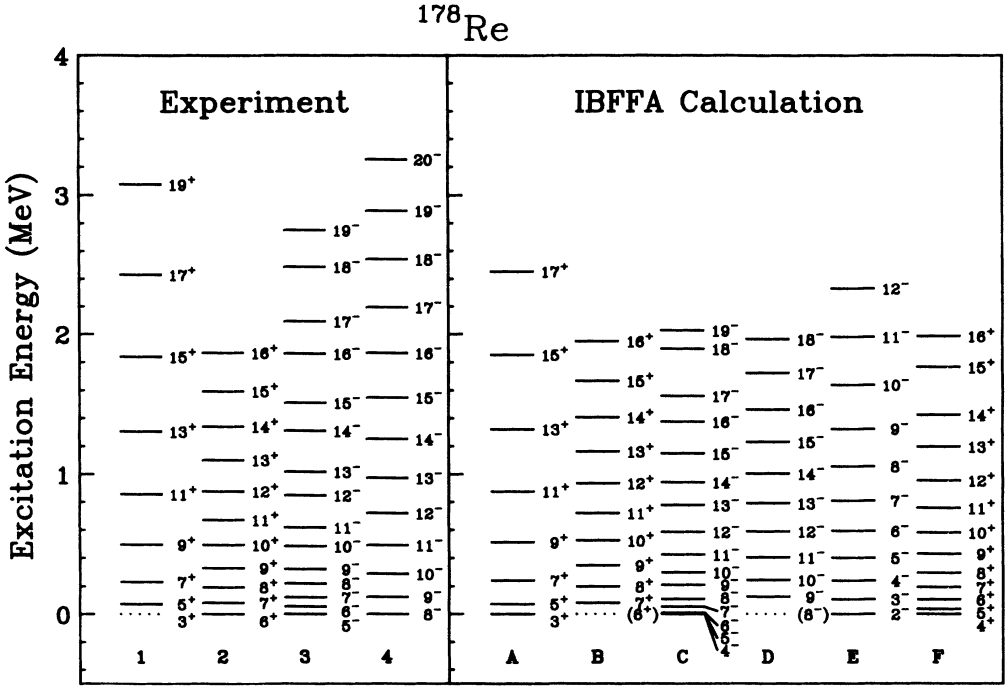

FIG. 2. Comparison of experimental bands in ^{178}Re with IBFFA calculations. Since the relative positions of the rotational bands are not clear, all are plotted as starting at 0 MeV. A dotted line indicates a nonobserved (nonexistent?) state.

B. The $\pi\frac{5}{2}^+[402]\otimes\nu i_{13/2}$ band

Band 2 in Fig. 2 was assigned the configuration, $\pi\frac{5}{2}^+[402]\otimes\nu i_{13/2}$. A TQPRM calculation[15] suggested that $\nu\frac{7}{2}^+[633]$ lies closest to the Fermi level; therefore, the above coupling produces a $K^\pi=6^+$ band. Because the $\frac{7}{2}^+[633]$ state is strongly coupled to other $i_{13/2}$ states, $\frac{9}{2}^+[624]$ in particular, such a band would exhibit compressed, but not staggered, spacings.

Our IBFFA calculations generate a band (band B) with properties very similar to the above band. However, the band head of band B is 7^+ instead of 6^+, as observed and predicted by the TQPRM calculation. The reason, as stated in Sec. II, is that we could not decide whether $\frac{7}{2}^+[633]$ or $\frac{9}{2}^+[624]$ is the $i_{13/2}$-parentage single-neutron state closest to the Fermi surface. Also, ^{179}Os (the odd-neutron nucleus we chose for our input) has the $\frac{9}{2}^+[624]$ state at a very-low excitation energy. Thus, when this $\frac{9}{2}^+[624]$ state couples with the $\pi\frac{5}{2}^+[402]$ state, we obtain a $K^\pi=7^+$ band and there is no $7^+\rightarrow6^+$ transition. Interestingly enough, this disagreement should not significantly affect energy predictions for states having $J^\pi=7^+$ and greater. Indeed, agreement is within ±10 keV for states up to $J^\pi=16^+$.

Note that our calculation was completed before the experimental results had been reported. Of course, if we alter the parameters in our IBFA calculation for ^{179}Os, we could easily lower the $\frac{7}{2}^+[633]$ band, obtaining the desired $K^\pi=6^+$ band at a lower excitation in ^{178}Re. However, that is not the point of this exercise. We fitted the odd-mass nuclei only from what seemed to be obvious properties and then used the odd-mass states to form odd-odd systems without additional parameters. Only in this manner—without any preconceived ideas about the odd-odd states—can we reach disinterested conclusions about the strengths and weaknesses of the IBFFA model.

C. The $\pi h_{9/2}\otimes\nu i_{13/2}$ band

Band 3 in Fig. 2 was assigned as $\pi h_{9/2}\otimes\nu i_{13/2}$, a so-called "semidecoupled" band,[33–36] in which the proton is decoupled ($K=\frac{1}{2}$) and the neutron is in a high-j (here $i_{13/2}$) state with Ω much greater than $\frac{1}{2}$. Characteristics of such a band are the odd-even staggerings in spacings caused by the large coriolis couplings, similar to the high-j neutron band in the odd-mass nucleus.

The Fermi levels lie closest to the $\pi\frac{1}{2}^-[541]$ and $\nu\frac{7}{2}^+[633]$ states. According to the simple Gallagher-Moszkowski coupling rules,[37] from these states we should expect to see a $K^\pi=3^-$ band. However, Kreiner *et al.*[15] suggest it to be a 4^- band, with the 4^- state not seen because the energy of the $5^-\rightarrow4^-$ transition was too low to be observed in their experiments.

We should not be too concerned about the ambiguity in spin assignment, but focus on the structure itself. It is clear that band C shows considerable odd-even staggering and is the candidate for such a semidecoupled band. The staggering is overestimated by the calculations, with the result that the 4^- state is shifted to higher energy. This behavior is similar to that encountered for the 4^- band in ^{182}Re,[13,17] and the reason for exaggeration of the staggering is that we did not provide for independent attenua-

tion of the coriolis force in our IBFA calculations. Thus, the deviations accumulate when both the proton and neutron states have large coriolis matrix elements.

D. A possible $\pi\frac{9}{2}^-[514]\otimes\nu i_{13/2}$ band

The last band identified is band 4, assigned as $\pi\frac{9}{2}^-[514]\otimes\nu i_{13/2}$. As previously discussed in Sec. II B, we predict only a $K^\pi=9^-$ band, with $\nu\frac{9}{2}^+[624]$ as the $i_{13/2}$ component. The IBFFA result (band *D*) is not in particularly good agreement with experimental data.

It has been our experience[13,17] that whenever we obtain good fits with IBFA calculations, we can expect good fits for the odd-odd composites. The difference between bands *B* and *D* is the proton state, $\frac{5}{2}^+[402]$ for *B* and $\frac{9}{2}^-[514]$ for *D*, and the fits for these two bands in ^{177}Re are equally good (cf. Fig. 1). Thus, we would expect to obtain equally good fits for the two odd-odd bands. Band *B* can be all but superimposed on experimental band 2, so we are led to question the assigned configuration for band 4.

From their assigned configurations, both bands 2 and 4 should be highly compressed, especially the first spacing(s). Yet band 4 is not so compressed as band 2, and the two also differ in the apparent blocking effects on backbending.[16] We consider this again in Sec. IV B, in connection with a band in ^{176}Re.

E. Two additional calculated bands

To demonstrate that the IBFFA calculations can handle somewhat less-distorted bands as well, we plot two additional expected low-lying bands in Fig. 2. Band *E* results from the coupling, $\pi\frac{5}{2}^+[402]\otimes\nu\frac{1}{2}^-[521]$, and should have properties characteristic of bands with $\Omega\neq\frac{1}{2}$ in the odd-proton system—e.g., the $\frac{5}{2}^+[402]$ band in ^{177}Re (Fig. 1). Band *F* has the configuration, $\pi\frac{1}{2}^-[541]\otimes\nu\frac{7}{2}^-[514]$, expected to have compressed spacings. The calculation predicts this configuration but adds a little extra staggering.

IV. ^{176}Re RESULTS

A. The doubly decoupled band

Both Santos *et al.*[15] and we[17,18] have studied ^{176}Re via in-beam γ-ray spectroscopy, they by the ^{169}Tm(^{12}C,5$n\gamma$)^{176}Re reaction and we by the ^{159}Tb(^{22}Ne,5$n\gamma$)^{176}Re and ^{165}Ho(^{16}O,5$n\gamma$)^{176}Re reactions. Two rotational bands were identified extensively, one of which is another doubly decoupled band. In Fig. 3 we compare the lower members of this band with our IBFFA calculations. The $5^+\rightarrow3^+$ transition has not been observed, although again both TQPRM[15] and IBFFA predict a 3^+ bandhead. We thus place the 5^+ member at 82.9 keV, the IBFFA-predicted position. There is some uncertainty in this energy, for the $\frac{9}{2}^-\rightarrow\frac{5}{2}^-$ spacing of the $\frac{1}{2}^-[541]$ band in unknown ^{175}Re will strongly influence the predicted $5^+\rightarrow3^+$ spacing in ^{176}Re. However, we systematically extrapolated the IBFA parameters down to ^{175}Re, and the resulting

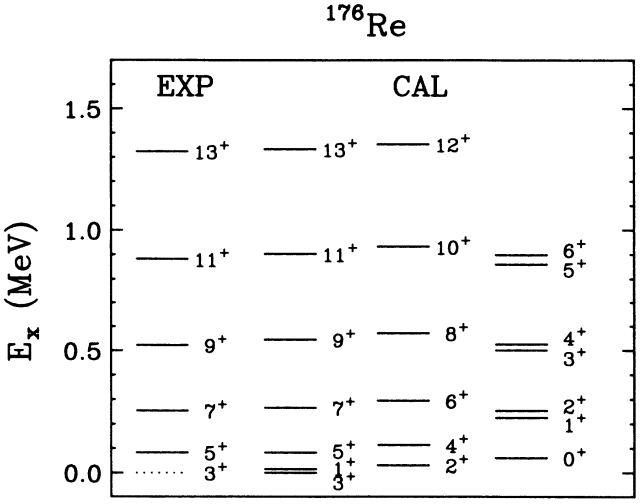

FIG. 3. Comparison of the experimentally observed lower-lying members of the doubly decoupled band in ^{176}Re with IBFFA calculations. The $5^+\rightarrow3^+$ transition has not been observed, so we place the 5^+ member of the band at 82.9 keV, the predicted position.

description of the ^{176}Re doubly decoupled band is in general excellent, as can be verified by Fig. 3. As in the other Re isotopes, only the favored-signature (odd-spin) members of the band are observed, the even-spin members having been shifted up in energy. Also, the unobserved singlet ("$K=0$") coupling is predicted to be raised in energy and strongly staggered, as indicated at the right side of the figure.

B. A normal $\Delta J=1$ rotational band

The two experimental groups do not agree on the assignment of the second rotational band. The disagreement comes about primarily because our group[17,18] found a half-life of 21 ± 7 ns for the 99.6-keV transition, whereas Santos *et al.*[15] did not perform data analysis and consider it to be an intraband transition.

Now, the disagreement could possibly be resolved by more detailed experiments, but by their very nature these would prove tedious and likely of diminishing returns for the use of accelerator time. As an alternative, we can consider likely candidates for bandhead configurations, compare these with our IBFFA calculations, and perhaps choose or at least narrow down the assignment for this band.

We chose the eight low-lying bands most likely to be populated by heavy-ion bombardments and extracted them from our IBFFA calculations. We compare these with the experimental band in Fig. 4. Remember that the IBFA parameters for ^{175}Re had to be extrapolated, so we should not rely too much on detailed spacings but instead consider the general features of these calculated bands.

We can immediately eliminate bands 5 and 6 because they show considerable staggering resulting from the coriolis contribution of the $\pi\frac{1}{2}^-[541]$ and/or $\nu\frac{7}{2}^+[633]$

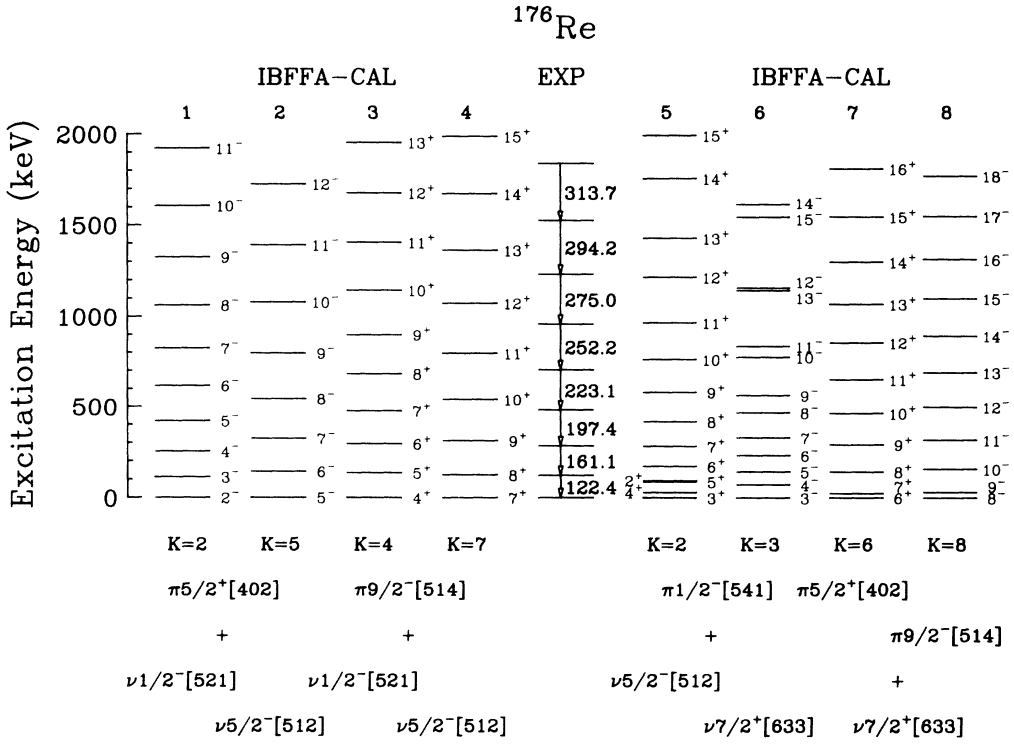

FIG. 4. The eight most likely candidates for the $\Delta J=1$ band in ^{176}Re as calculated by the IBFFA model. Only triplet couplings are shown. The experimental band is shown in the middle.

states. The experimental band does not exhibit such behavior. Bands 7 and 8 can probably be eliminated, although not quite so obviously, on a similar basis. Also, band 4 does not match the spacings very well.

An effective way to emphasize distortions is the so-called trumpet plot, as shown in Fig. 5. In this figure the IBFFA-calculated spacings are connected by dotted lines and the experimental spacings by solid lines. (The experimental value for K was varied, matching that of the calculated band plotted with the same symbol.) Here the $K^\pi=2^-$ band (band 1), exhibits some staggering, induced by the $\nu\frac{1}{2}^-[521]$ state. The staggering in the $K^\pi=8^-$ band (band 8) is concentrated in the compression of the first spacing, characteristic of a $\nu i_{13/2}$ component. (Band 7, again not included, is worse.) We are left with $K^\pi=5^+$ or 7^+ as the most likely candidates, and the latter gives somewhat better agreement with this data.

This reopens the question of the band in ^{178}Re previously discussed in Sec. III D. That the bands in ^{176}Re and ^{178}Re have remarkably similar spacings is attested to by the comparison given in Table IV. Perhaps this indicates an essentially identical structure?

V. COMPARISON WITH NEW ^{180}Re RESULTS

At the time we published our original calculations[13] on ^{180}Re, only preliminary experimental data existed,[19,20]

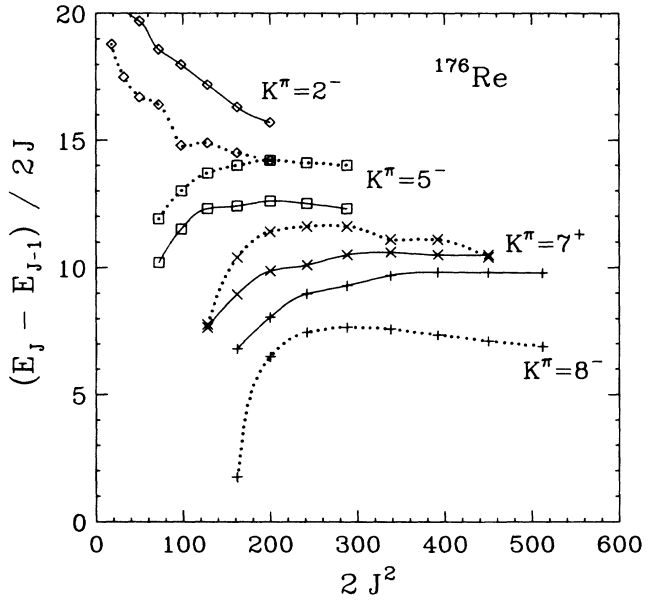

FIG. 5. Trumpet plots of likely candidates for the $\Delta J=1$ band in ^{176}Re. The bands are plotted in pairs, with the IBFFA-calculated spacings corrected by dotted lines and the experimental spacings (assuming the same K values) corrected by solid lines.

TABLE IV. Comparison of band transitions energies in ^{176}Re and ^{178}Re (in keV).

^{176}Re[a]	^{178}Re[b]
99.6	
122.4	123.8
161.1	166.1
197.4	201.5
223.1	229.7
252.2	253.3
275.0	277.7
294.2	296.7
313.7	316.9
(327)	328.4

[a]References 17 and 18.
[b]Reference 16.

and we were unable to make meaningful comparisons with them. Now, however, ^{180}Re has been studied rather extensively by Venkova *et al.*,[21] using the ^{181}Ta($\alpha,5n\gamma$)^{180}Re and ^{170}Er(^{14}N,$4n\gamma$)^{180}Re reactions for in-beam γ-ray spectroscopy. Because no linking transition(s) to the ground state could be clarified, configuration assignments could be based only on g_K factors, alignments, and rotational spacings. Another study by Kreiner *et al.*,[22] again in-beam γ-ray spectroscopy using the ^{181}Ta($\alpha,5n\gamma$)^{180}Re and ^{176}Yb(^{10}B,$6n\gamma$)^{180}Re reactions, agrees in general with that of Venkova *et al.* on energies. However, they reach quite different conclusions with respect to configurations, again based on alignments and rotational spacings.

In Fig. 6 we plot the experimental data, together with the J^π assignments given by the two groups. Absolute energies are not known for band heads. Thus, they are plotted with respect to the 6^- and 3^- states on the left. The spacing between these two states is unknown, as is the connection of either state to the ground state. On the right the 6^+ state has the lowest observed excitation energy, and it may or may not be the ground state. The 8^+ and 8^- states are placed correctly with respect to it, but the 5^+ state lies above it at an unknown energy δ.

Eight of the appropriate low-lying bands from our IBFFA calculations are shown in Fig. 7. Bands 1 and 2 are the triplet and singlet couplings of $\pi\frac{5}{2}^+[402]\otimes\nu\frac{7}{2}^-[514]$. The 1^- band is believed to be the ground-state band,[38] but, interestingly enough, the IBFFA calculations predict the reverse. Whether true or false, this ought not to affect the general position of the other predictions.

We list the configurations assigned for the bands by the two groups in Table V, indicating also the corresponding IBFFA bands in Fig. 7 for these assigned configurations. Band *A* is the same as band *E*, as are band *B* and band *F*. They have been given the same neutron state but different proton states by the two groups. Also, band *C* and band *G* appear to be the same and are assigned the same proton but different neutron states. Band *D* and band *H* are undoubtedly different bands.

Unfortunately, nature has not been kind in allowing one to distinguish between the two sets of assignments. The differences in excitation energies of these commonly observed bands results from unobserved (low-energy and/or delayed) or differently interpreted linking transitions between the bands. Further experiments will be necessary to determine which are correct. An example of the difficulty can be seen by comparing band *A* with band

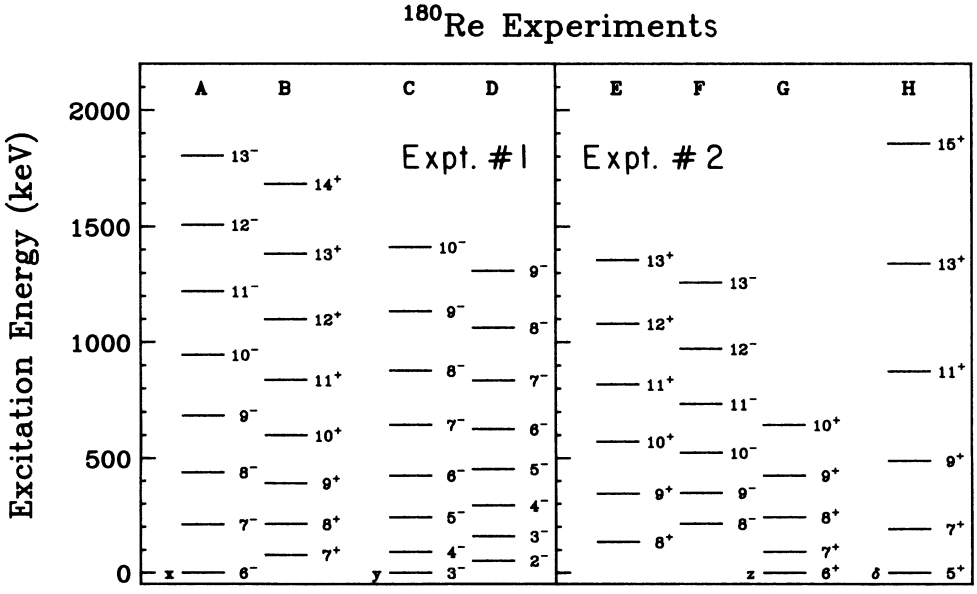

FIG. 6. Recent experimental results for bands in ^{180}Re. Left: bands and assignments of Venkova *et al.* (Ref. 21). Right: bands and assignments of Kreiner *et al.* (Ref. 22). Absolute energies of the band heads are unknown. They are plotted with respect to the 6^- and 3^- states on the left and the 6^+ and 5^+ states on the right.

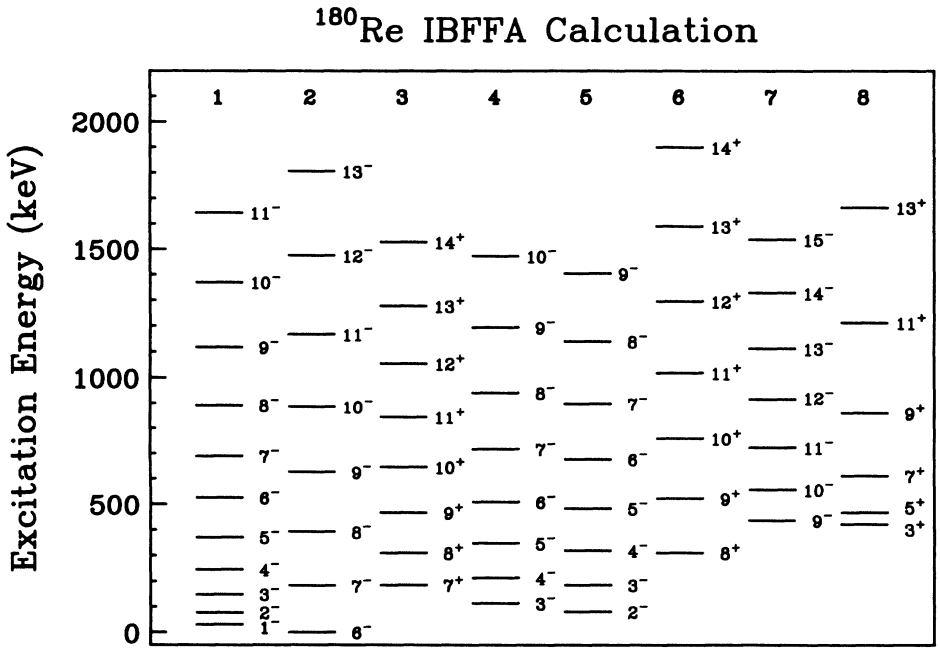

FIG. 7. Eight low-lying bands in ^{180}Re as predicted by IBFFA calculations.

E (the same experimental band). If it is indeed the ground-state band or nearby, then it agrees with our IBFFA results for $K^{\pi}=6^{-}$; on the other hand, if it lies at several hundred keV, it could agree with our result for 8^{+}. The same ambiguities exist for the other comparisons. And none of these observed bands has particularly unique or distinguishing spacing characteristics. Most odd-odd assignments remain anything but trivial.

VI. SUMMARY

IBFFA calculations of states in odd-odd nuclei provide a stringent test of the IBA method, for odd-odd nuclei do not exhibit the same orderly systematics as even-even or odd-mass nuclei. Nevertheless, we have encountered considerable success in predicting odd-odd rotational bands.

Errors tend to accumulate, and IBFFA calculations are especially influenced by the IBFA results. Whenever a good fit can be obtained from IBFA, a decent result can be expected from IBFFA. IBFFA seems to be especially good at predicting "geometrical" properties (such as rotational spacings) of bands, less good in predicting bandhead positions and triplet-singlet splittings. However, if both the proton and neutron states suffer from severe coriolis distortions, we usually obtain too compressed spacings, since we make no provision for attenuation of the coriolis force in our present calculations. On the other hand, IBFFA does an excellent job in duplicating the very severely distorted doubly decoupled bands.

From our comparisons with experimental data for the

TABLE V. Comparison of experimental band assignments in ^{180}Re by two different groups and with IBFFA predictions.

Venkova *et al.*[a]	IBFFA Band no.	IBFFA K^{π}	Kreiner *et al.*[b]	IBFFA Band no.	IBFFA K^{π}
A:$\pi\frac{5}{2}^{+}[402]\otimes\frac{7}{2}^{-}[514]$	2	6^{-}	E:$\pi\frac{9}{2}^{-}[514]\otimes\nu\frac{7}{2}^{-}[514]$	6	8^{+}
B:$\pi\frac{5}{2}^{+}[402]\otimes\nu\frac{9}{2}^{+}[624]$	3	7^{+}	F:$\pi\frac{9}{2}^{-}[514]\otimes\nu\frac{9}{2}^{+}[624]$	7	9^{-}
C:$\pi\frac{5}{2}^{+}[402]\otimes\nu\frac{1}{2}^{-}[521]$	4	3^{-}	G:$\pi\frac{5}{2}^{+}[402]\otimes\nu\frac{9}{2}^{+}[624]$	3	7^{+}
D:$\pi\frac{5}{2}^{+}[402]\otimes\nu\frac{1}{2}^{-}[521]$	5	2^{-}			
			H:$\pi\frac{1}{2}^{-}[541]\otimes\nu\frac{1}{2}^{-}[521]$	8	3^{+}

[a]Reference 21.
[b]Reference 22.

odd-odd Re isotopes, we find that IBFFA gives almost surprisingly good agreement. We can even use its predictions as a guide—within reason—in helping us to assign configurations. However, odd-odd nuclei, although becoming more tractable to understanding, remain varied and tricky. IBFFA is a reasonably powerful aid but by no means a panacea for the headaches we may encounter in trying to sort out their complexities.

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[1]A. Arima and F. Iachello, Phys. Rev. C **14**, 761 (1976).
[2]A. Arima and F. Iachello, Ann. Phys. (N.Y.) **99**, 253 (1976).
[3]A. Arima and F. Iachello, Ann. Phys. (N.Y.) **111**, 201 (1976).
[4]A. Arima and F. Iachello, Ann. Phys. (N.Y.) **123**, 468 (1976).
[5]O. Scholten, F. Iachello, and A. Arima, Ann. Phys. (N.Y.) **115**, 325 (1979).
[6]R. F. Casten and J. A. Cizewski, Nucl. Phys. **A309**, 477 (1978).
[7]I. Talmi, in *Short-Distance Phenomena in Nuclear Physics,* edited by D. H. Boal and R. H. Woloshyn (Plenum, New York, 1983), p. 311.
[8]F. Iachello and O. Scholten, Phys. Rev. Lett. **43**, 679 (1979).
[9]O. Scholten, in *Contemporary Research Topics in Nuclear Physics,* edited by D. J. Feng, M. Vallieres, M. W. Guidry, and L. L. Riedinger (Plenum, New York, 1985), p. 503.
[10]O. Scholten and N. Blasi, Nucl. Phys. **A380**, 509 (1982).
[11]O. Scholten, in *Recent Advances in Nuclear Physics,* Poiana Brasov, 1988, edited by M. Petrovici and N. V. Zamfir (World Scientific, Singapore, 1990).
[12]O. Scholten, H. C. Wu, and A. E. L. Dieperink, Z. Phys. A **332**, 1 (1989).
[13]W.-T. Chou, Wm. C. McHarris, and O. Scholten, Phys. Rev. C **37**, 2834 (1988).
[14]W.-T. Chou, Wm. C. McHarris, and O. Scholten, in *Exotic Nuclear Spectroscopy,* edited by Wm. C. McHarris (Plenum, New York, 1990), Chap. 34.
[15]D. Santos, A. J. Kreiner, J. Davidson, M. Davidson, M. Debray, D. Hojman, and G. Falcone, Phys. Rev. C **39**, 902 (1989).
[16]A. J. Kreiner, V. R. Vanin, F. Beck, Ch. Bourgeois, Th. Byrski, D. Curien, G. Duchene, B. Haas, J. C. Merdinger, M. G. Porquet, P. Romain, S. Rouabah, D. Santos, and J. P. Vivien, Phys. Rev. C **40**, R487 (1989).
[17]W.-T. Chou, Ph.D. thesis, Michigan State University, 1989 (unpublished).
[18]W. A. Olivier, W.-T. Chou, Aracelys Rios, Wm. C. McHarris, R. Aryaeinejad, E. S. Paul, D. B. Fossan, R. Ma, N. Xu, and Y. Liang, Michigan State University-National Superconducting Cyclotron Laboratory Report No. MSUCL-697, 1990.
[19]R. M. Lieder, in *Proceedings of the XXIII International Winter Meeting on Nuclear Physics,* Bormio, 1985 (unpublished).
[20]Ts. Venkova, R. M. Lieder, T. Morek, W. Gast, G. Hebbinghaus, A. Kramer-Flecken, J. Schaffler-Krah, W. Urban, H. Kluge, K. H. Maier, A. Maj, and G. Sletten, Kernfisisch Vernsneller Julich Annual report, 1985.
[21]Ts. Venkova, T. Morek, R. M. Lieder, W. Gast, G. Hebbinghaus, A. Kramer-Flecken, J. Schaffler-Krah, W. Urban, G. Sletten, J. Kluge, and K. H. Maier, Nucl. Phys. A (to be published).
[22]A. J. Kreiner, J. Davidson, M. Davidson, D. Abriola, C. Pomar, and P. Theiberger, Phys. Rev. C **36**, 2309 (1987); **37**, 1338(E) (1988).
[23]C. X. Yang, J. Kownacki, J. D. Garrett, G. B. Hagemann, B. Herskind, J. C. Bacelar, J. R. Leslie, R. Chapman, J. C. Lisle, J. N. Mo, A. Simcock, J. C. Wilmott, W. Walus, L. Carlen, S. Jonsson, J. Lyttkens, H. Ryde, P. O. Tjom, and P. M. Walker, Phys. Lett. **133B**, 39 (1983).
[24]E. Browne, Nucl. Data Sheets **55**, 483 (1988).
[25]R. B. Firestone, Nucl. Data Sheets **43**, 289 (1984).
[26]Y. A. Ellis-Akovali, Nucl. Data Sheets **52**, 789 (1987).
[27]E. Browne, Nucl. Data Sheets **58**, 441 (1989).
[28]P. M. Walker, G. D. Dracoulis, A. Johnson, J. R. Leigh, M. G. Slacombe, and I. F. Wright, J. Phys. G **4**, 1655 (1978).
[29]G. D. Dracoulis, C. Fahlander, and A. P. Byrne, Nucl. Phys. **A401**, 490 (1983).
[30]Y. A. Ellis and B. Harmatz, Nucl. Data Sheets **16**, 135 (1975).
[31]M. F. Slaughter, R. A. Warner, T. L. Khoo, W. H. Kelly, and Wm. C. McHarris, Phys. Rev. C **29**, 114 (1984).
[32]Wm. C. McHarris, W.-T. Chou, J. Kupstas-Guido, and W. A. Olivier, in *Nuclei Off the Line of Stability,* American Chemical Society Symposium Series No. 324 (American Chemical Society, Washington, D.C., 1986), p. 329.
[33]A. J. Kreiner, D. E. DiGregorio, A. J. Fendrik, J. Davidson, and M. Davidson, Phys. Rev. C **29**, 1572 (1984).
[34]A. J. Kreiner, in *Exotic Nuclear Spectroscopy,* edited by Wm. C. McHarris (Plenum, New York, 1990), Chap. 26.
[35]A. J. Kreiner, D. E. DiGregorio, A. J. Fendrik, J. Davidson, and M. Davidson, Nucl. Phys. **A432**, 451 (1985).
[36]A. J. Kreiner, J. Davidson, M. Davidson, P. Thieberger, E. K. Warburton, S. Andre, and J. Genevey, Nucl. Phys. **A489**, 525 (1988).
[37]C. J. Gallagher and S. A. Moszkowski, Phys. Rev. **111**, 1282 (1958).
[38]A. Arima, A. Gelberg, and O. Scholten, Phys. Lett. B **185**, 259 (1987).