Multichannel approach to the Glauber model for heavy-ion collisions

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A formalism is developed in order to describe, within the Glauber model, the scattering processes between heavy ions in situations involving several coupled channels. The approach is based on a suitable truncation of the number of nuclear states which can be excited at each microscopic nucleon-nucleon collision. The set of coupled equations for the S-matrix elements of the conventional reaction theory is replaced by simple matrix relations, only involving the nucleon-nucleon scattering amplitude and the nuclear densities and transition densities. This method avoids the difficulties arising from the combinatorial aspects of the multiple scattering theories, the slow convergence of the series, and the problems of center-of-mass correlations. We discuss some specific examples of multichannel collisions where the multiple-scattering series can be summed to give analytic expressions for the scattering amplitude. We finally explicate the formalism for the perturbative treatment of mutual excitation and charge-exchange processes.

I. INTRODUCTION

A large amount of work has been devoted to the application of the Glauber model[1,2] to the description of high-energy hadron-nucleus and nucleus-nucleus scattering processes. In the variety of the contributions, all exploiting the Glauber basic idea of describing the global process in terms of the coherent effect of microscopic nucleon-nucleon collisions, one can evidence two fronts of advance. The first is the successful analysis of a large variety of experimental data, within the original formulation[1,2] or its numerous approximations[3-5] (for a recent review see, e.g., Ref. 6). The spectrum of applications also includes the excitation of discrete low-energy nuclear levels in heavy-ion reactions.[7-14] A second front can be viewed in the works devoted to justify and rederive the Glauber model and its approximations, in terms of the Goldberger-Watson multiple-scattering theory.[15-18]

Along this line, and encouraged by the successful results quoted above,[11-14] we devote this paper to put into a wider conceptual framework the method we have introduced in the previous works. The formalism is based on a suitable truncation of the completeness of the nuclear states which can be thought to be virtually excited in each elementary collision. In the case of elastic scattering, this amounts, in lowest order of approximation, to describe the process as resulting from sequences of elementary collisions in which the colliding nuclei are frozen in their ground state. This starting idea has been pointed out by several authors (see, e.g., Refs. 7 and 19), including the authors of the present work.[11,14] This approximation leads, for the elastic scattering, to the same analytical results as the so-called optical limit to the Glauber model.[3]

The capability of this approach to describe inelastic-scattering processes[7,14] suggests the possibility of generalizing the method in order to obtain a formalism suitable to describe a wider class of reactions, including those characterized by strong channel coupling. The system of differential equations obtained in the conventional coupled-channel approach is replaced by simple matrix operations. We underline that the basic approximation, and the consequent simplifications introduced by the optical limit, is that all the multiple sequences in an assigned order of scattering give the same contribution to the scattering amplitude. Similar approximations and simplifications occur in the more general formalism we consider here. In specific conditions of coupling between different channels, the multiple-scattering series can be summed giving analytic expressions for all scattering amplitudes.

Section II is devoted to illustrating the drawbacks of the standard Glauber model in the description of heavy-ion scattering process. More precisely, we give some concrete examples of the difficulties arising from the combinatorial aspects of the multiple-scattering approach and show the slow convergence of the series when the uncorrelated-particle model is assumed to describe the nuclear states. The standard derivation of the optical limit is also recalled. In Sec. III the philosophy of the multichannel approach is discussed and the general formalism is built up. Sections IV and V are devoted to a different sort of examples. In Sec. IV we discuss rather general models of coupling between channels, but disregard the excitation of the projectile and the angular-momentum algebra. In Sec. V the specific case of mutual excitation and of charge-exchange reactions are described in a more realistic framework. Some concluding remarks are pointed out in Sec. VI.

II. THE GLAUBER FORMALISM FOR NUCLEUS-NUCLEUS COLLISIONS

A. Basic formulas and classification of scattering sequences

Let us recall for later reference the basic concepts and the formalism of the Glauber model for the scattering of complex systems as developed in Refs. 3 and 4. The scattering amplitude for the collision of a projectile of mass A_P on a target of mass A_T assumes the expression

$$F_{sq,rp}(\bar{\Delta})=\frac{ik}{2\pi}\int d^2b\, e^{i\bar{\Delta}\cdot\bar{b}}\langle \Phi_s^T\Phi_q^P|\Gamma(\bar{b})|\Phi_r^T\Phi_p^P\rangle \ , \tag{1}$$

where the total profile $\Gamma(\bar{b})$ is expressed in terms of the nucleon-nucleon profiles γ in the form

$$\Gamma(\bar{b})=1-\prod_{i=1}^{A_P}\prod_{j=1}^{A_T}[1-\gamma(\bar{b}-\bar{s}_j^T+\bar{s}_i^P)] \ , \tag{2}$$

s,q and r,p denote the final and the initial states of the target and projectile, respectively. In the above expressions k is the incident momentum in the ion-ion center-of-mass system, $\bar{\Delta}$ is the transferred momentum, $\bar{b}$ is the impact parameter, and $\bar{s}_i^P$ and $\bar{s}_j^T$ are the projections of the coordinate vectors of the ith and jth nucleons on the plane perpendicular to the incident momentum. The elementary profile γ is the two-dimensional Fourier transform of the NN scattering amplitude $f_{NN}(\bar{q})$

$$\gamma(\bar{b})=\frac{1}{2\pi ik_{NN}}\int e^{-i\bar{b}\cdot\bar{q}}f_{NN}(\bar{q})d\bar{q} \ . \tag{3}$$

If not otherwise stated, the scattering amplitude $f_{NN}(\bar{q})$ is assumed to be completely general, therefore including spin and isospin degrees of freedom.

By expanding the products in (2), we can group all the resulting terms according to the order of scattering, i.e., the number of nucleon-nucleon collisions involved. We can therefore express Γ in the form

$$\Gamma(\bar{b})=\sum_n\Gamma^{(n)}(\bar{b})=\sum_{n,g}\Gamma^{(n,g)}(\bar{b}) \ , \tag{4}$$

where we have explicitly expressed the nth-order contribution $\Gamma^{(n)}$ as the sum of the terms associated with the

$$\begin{bmatrix} A_P A_T \\ n \end{bmatrix}$$

possible sequences of n collisions. For each n, the additional index g individuates a specific sequence of that order, namely, the set of n pairs of indices (one for the target and one for the projectile) characterizing each microscopic collision. One has, in fact, the possibility of different combinations of the scattering processes, with allowed multiple collision by the same nucleon and excluding only a second collision between the same pair of nucleons. For any scattering order n a generic sequence g of n collisions contributes to the profile with the term

$$\Gamma^{(n,g)}(\bar{b})=\underbrace{\gamma(\bar{b}-\bar{s}_{j_1}^T+\bar{s}_{k_1}^P)\gamma(\bar{b}-\bar{s}_{j_2}^T+\bar{s}_{k_2}^P)\cdots\gamma(\bar{b}-\bar{s}_{j_n}^T+\bar{s}_{k_n}^P)}_{n\text{ factors}} \ . \tag{5}$$

For a given scattering order n, not all combinations give, however, equivalent contributions to the scattering amplitude, even treating all nucleons as indistinguishable. This is due to the obvious consideration that the same number n of microscopic collisions can be obtained from the combination of different groups of nucleons, as illustrated in Table I. An actual calculation within the Glauber model therefore requires, as a preliminary step, the solution of a combinatorial problem. One has to individuate, at each scattering order, the nonequivalent sequences of collisions and determine their multiplicities which arise from the indistinguishability of the nucleons. This problem, which has been worked out when the number of constituents of the colliding partners is relatively small,[20,21] may become a nontrivial task in the case of heavier systems. As an example, in Table I we display all the different sequences associated with the first six orders of collisions for the scattering of an alpha particle by a target of mass A. Their number, which is 1, 3, 6, 16, 27, and 62 for the first six values of n, grows rather rapidly with the scattering order. For each case we also give the corresponding value of the multiplicity. To get an idea of the difficulty of a complete solution of the problem we remind that, taking as an example the process $\alpha+{}^{12}\mathrm{C}$, we have to sum the multiple-scattering series up to the maximum scattering order $n=48$. The complexity of cases gets maximum around $n=24$, where the number of sequences to be classified is

$$\begin{bmatrix} 48 \\ 24 \end{bmatrix}\approx 3\times 10^{13} \ .$$

Note that, for scattering orders beyond the value $n=24$, because of a sort of particle-hole symmetry of the problem, we reobtain just the cases encountered for $n<24$, now in decreasing order of complexity.

B. The independent-particle approximation

A second class of problems is associated with the explicit evaluation of the contribution of each (nonequivalent) scattering sequence. Each contribution involves a multidimensional integral over all the nucleon coordinates, the knowledge of the nucleon-nucleon scattering amplitude and of the many-body nuclear wave functions. After integration over the variables not appearing in a given sequence of profiles, one is left with an integral in which the knowledge of the nuclear correlation functions is demanded. A common way of overcoming this point is based on the independent-particle description of the nuclear states, an assumption which allows us to express all

TABLE I. Schemes for all the nonequivalent sequences associated with the first six orders of collision for the scattering of an alpha particle (a) by a nucleus of mass number A. Each dot is associated with an elementary nucleon-nucleon collision. The nucleons involved are individuated by the column (target) and the row (projectile). The multiplicity of each scattering sequence is listed on the right of each scheme as a product of combinatorial factors. The sequences are ordered according to decreasing compactness index Q.

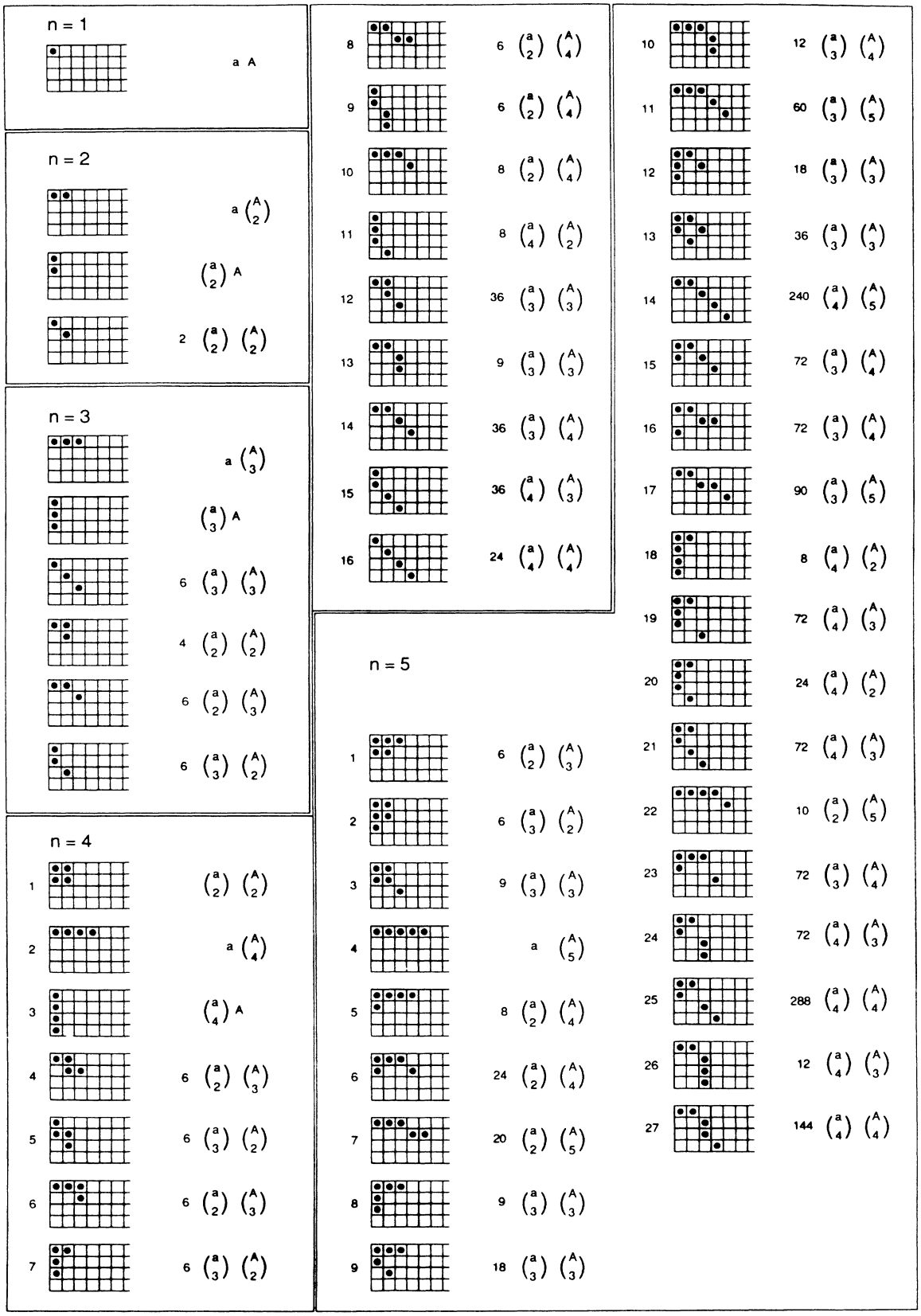
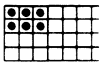
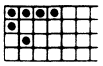
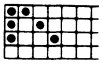
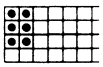
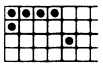
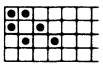
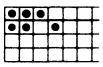
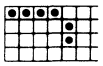
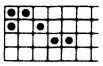
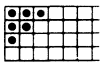
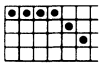
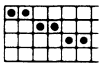
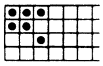
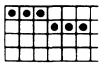
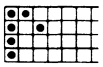
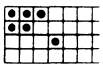
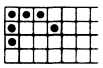
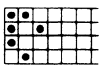
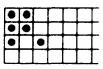
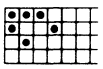
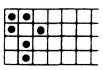
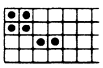
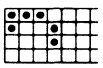
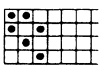
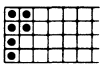
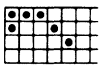
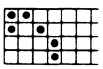
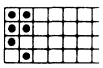
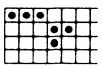
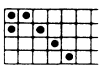
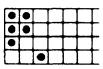
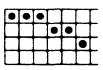
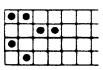
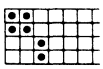
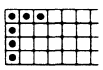
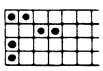
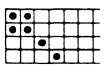
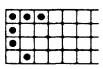
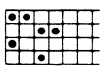
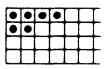


TABLE I. (*Continued*).

n = 6

1		$\binom{a}{2}\binom{A}{3}$	22		36 $\binom{a}{3}\binom{A}{4}$	43		24 $\binom{a}{3}\binom{A}{4}$
2		$\binom{a}{3}\binom{A}{2}$	23		120 $\binom{a}{3}\binom{A}{5}$	44		72 $\binom{a}{3}\binom{A}{4}$
3		12 $\binom{a}{2}\binom{A}{4}$	24		15 $\binom{a}{3}\binom{A}{5}$	45		180 $\binom{a}{3}\binom{A}{5}$
4		36 $\binom{a}{3}\binom{A}{3}$	25		90 $\binom{a}{3}\binom{A}{6}$	46		90 $\binom{a}{3}\binom{A}{6}$
5		18 $\binom{a}{3}\binom{A}{3}$	26		20 $\binom{a}{2}\binom{A}{6}$	47		36 $\binom{a}{4}\binom{A}{3}$
6		72 $\binom{a}{3}\binom{A}{4}$	27		72 $\binom{a}{3}\binom{A}{4}$	48		144 $\binom{a}{4}\binom{A}{3}$
7		18 $\binom{a}{3}\binom{A}{3}$	28		144 $\binom{a}{3}\binom{A}{4}$	49		72 $\binom{a}{4}\binom{A}{3}$
8		18 $\binom{a}{3}\binom{A}{4}$	29		72 $\binom{a}{3}\binom{A}{4}$	50		72 $\binom{a}{4}\binom{A}{3}$
9		12 $\binom{a}{4}\binom{A}{2}$	30		360 $\binom{a}{3}\binom{A}{5}$	51		144 $\binom{a}{4}\binom{A}{4}$
10		12 $\binom{a}{4}\binom{A}{2}$	31		120 $\binom{a}{3}\binom{A}{5}$	52		720 $\binom{a}{4}\binom{A}{5}$
11		72 $\binom{a}{4}\binom{A}{3}$	32		360 $\binom{a}{3}\binom{A}{6}$	53		144 $\binom{a}{4}\binom{A}{4}$
12		18 $\binom{a}{4}\binom{A}{3}$	33		12 $\binom{a}{4}\binom{A}{3}$	54		144 $\binom{a}{4}\binom{A}{4}$
13		72 $\binom{a}{4}\binom{A}{4}$	34		72 $\binom{a}{4}\binom{A}{3}$	55		288 $\binom{a}{4}\binom{A}{4}$
14		12 $\binom{a}{2}\binom{A}{4}$	35		24 $\binom{a}{4}\binom{A}{3}$	56		180 $\binom{a}{4}\binom{A}{5}$
15		6 $\binom{a}{3}\binom{A}{3}$	36		144 $\binom{a}{4}\binom{A}{4}$	57		1080 $\binom{a}{4}\binom{A}{6}$
16		$a\binom{A}{6}$	37		288 $\binom{a}{4}\binom{A}{4}$	58		180 $\binom{a}{3}\binom{A}{5}$
17		10 $\binom{a}{2}\binom{A}{5}$	38		144 $\binom{a}{4}\binom{A}{4}$	59		1440 $\binom{a}{4}\binom{A}{5}$
18		12 $\binom{a}{2}\binom{A}{6}$	39		720 $\binom{a}{4}\binom{A}{5}$	60		576 $\binom{a}{4}\binom{A}{4}$
19		40 $\binom{a}{2}\binom{A}{5}$	40		16 $\binom{a}{4}\binom{A}{4}$	61		288 $\binom{a}{4}\binom{A}{4}$
20		30 $\binom{a}{2}\binom{A}{6}$	41		240 $\binom{a}{4}\binom{A}{5}$	62		30 $\binom{a}{2}\binom{A}{5}$
21		12 $\binom{a}{3}\binom{A}{4}$	42		480 $\binom{a}{4}\binom{A}{6}$			

these correlation functions simply in terms of the nuclear densities. Note that the assumption of uncorrelated wave functions gives origin in each contribution to a common center-of-mass correction factor only depending on the transferred momentum and the masses of the colliding nuclei (cf. Refs. 3–5). If one further assumes a Gaussian form for both the nucleon-nucleon scattering amplitude and for the nuclear densities, one eventually obtains ana-

lytic expressions for all the contributing factors to the elastic scattering (cf. Ref. 3).

As an example of the predictions based on the independent-particle Glauber model, in Figs. 1 and 2 we consider the elastic scattering of an alpha particle on ^{12}C and ^{40}Ca at an incident energy of 1.37 GeV. Standard Gaussian parametrizations have been assumed for the nuclear densities of both projectile and target (cf. captions to the figures), as well as for the NN scattering amplitude

$$f_{NN}(q)=\frac{k_{NN}}{4\pi}(\alpha_{NN}+i)\sigma_{NN}e^{-aq^2} \tag{6}$$

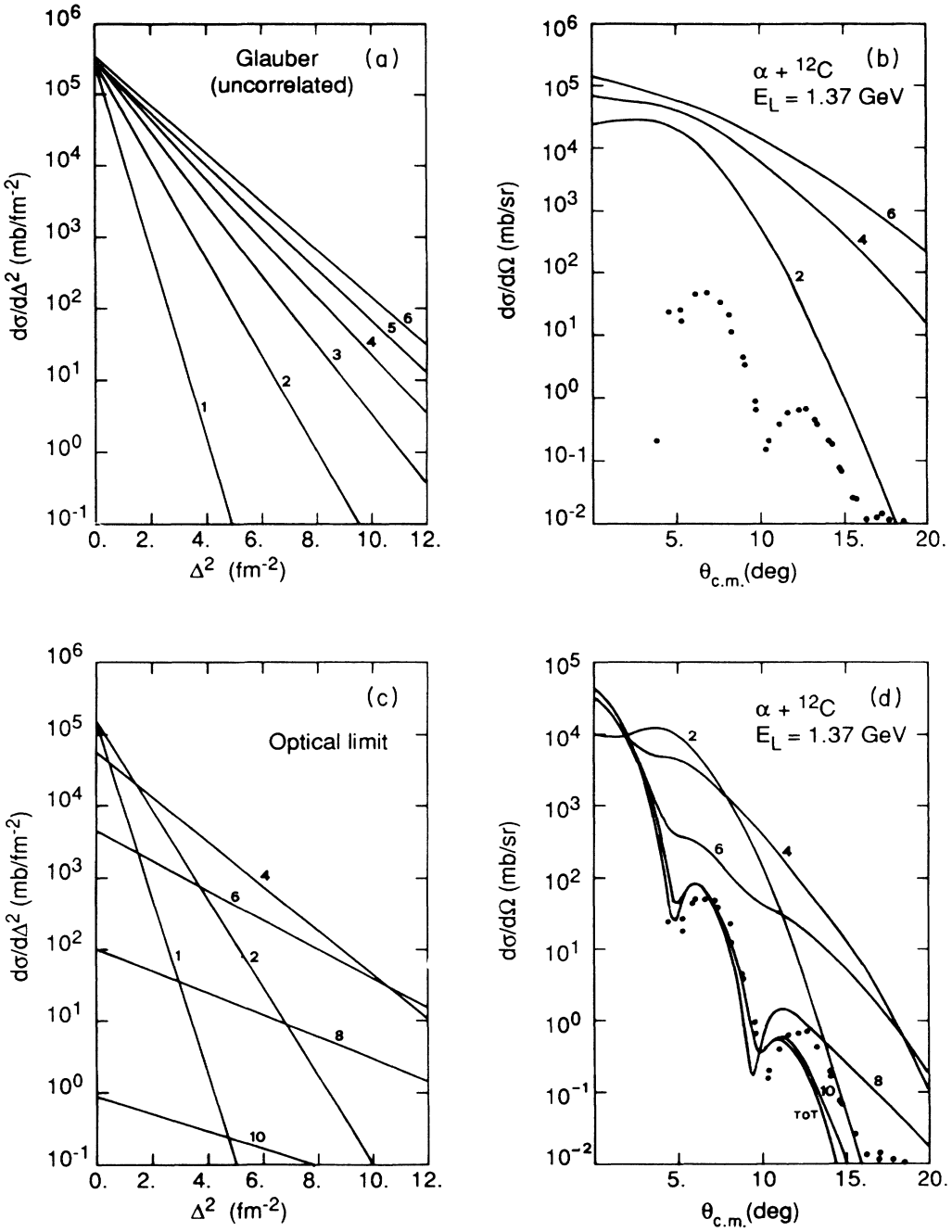


FIG. 1. Elastic scattering of an alpha particle on ^{12}C at $E_{lab}=1.37$ GeV. (a) and (c) display the separate contributions of different scattering orders to the cross section versus the square of the momentum transferred for the uncorrelated Glauber model and the optical limit, respectively. (b) and (d) give the summed values for the angular distributions versus the scattering angle for the two methods. Densities of Gaussian shape are used, normalized to unity: $\rho(r)=\rho(0)e^{-r^2/R^2}$ with $R_\alpha=1.37$ fm and $R_{^{12}C}=1.935$ fm. Experimental data from Ref. 26 are displayed only for a qualitative comparison of the convergence of the methods.

with parameters $\sigma_{NN}=2.84$ fm^2, $\alpha_{NN}=0.26$, and $a=0.045$ fm^2 relevant to the nucleon-nucleon collision at the corresponding energy of 342.5 MeV.[9]

In Figs. 1(a) and 2(a) we give, for the two reactions, the separate contributions of the different scattering orders (up to $n=6$) to the elastic cross sections, while the summed values are given in Figs. 1(b) and 2(b). The exponential dependence of the contribution at each order on the square of the transferred momentum Δ is a direct consequence of the Gaussian form for amplitudes and densities. This behavior allows a more transparent estimate of the importance of the different scattering orders. In particular, the different slopes of the curves are a signal of the expected dominance of the high-order terms at large scattering angles. More surprising, these turn out to be also largely dominant in the forward direction.

The situation that emerges from the analysis of the figures is rather discouraging. The contributions of the lowest scattering orders do not seem to display any indication of a possible rapid convergence of the full series.

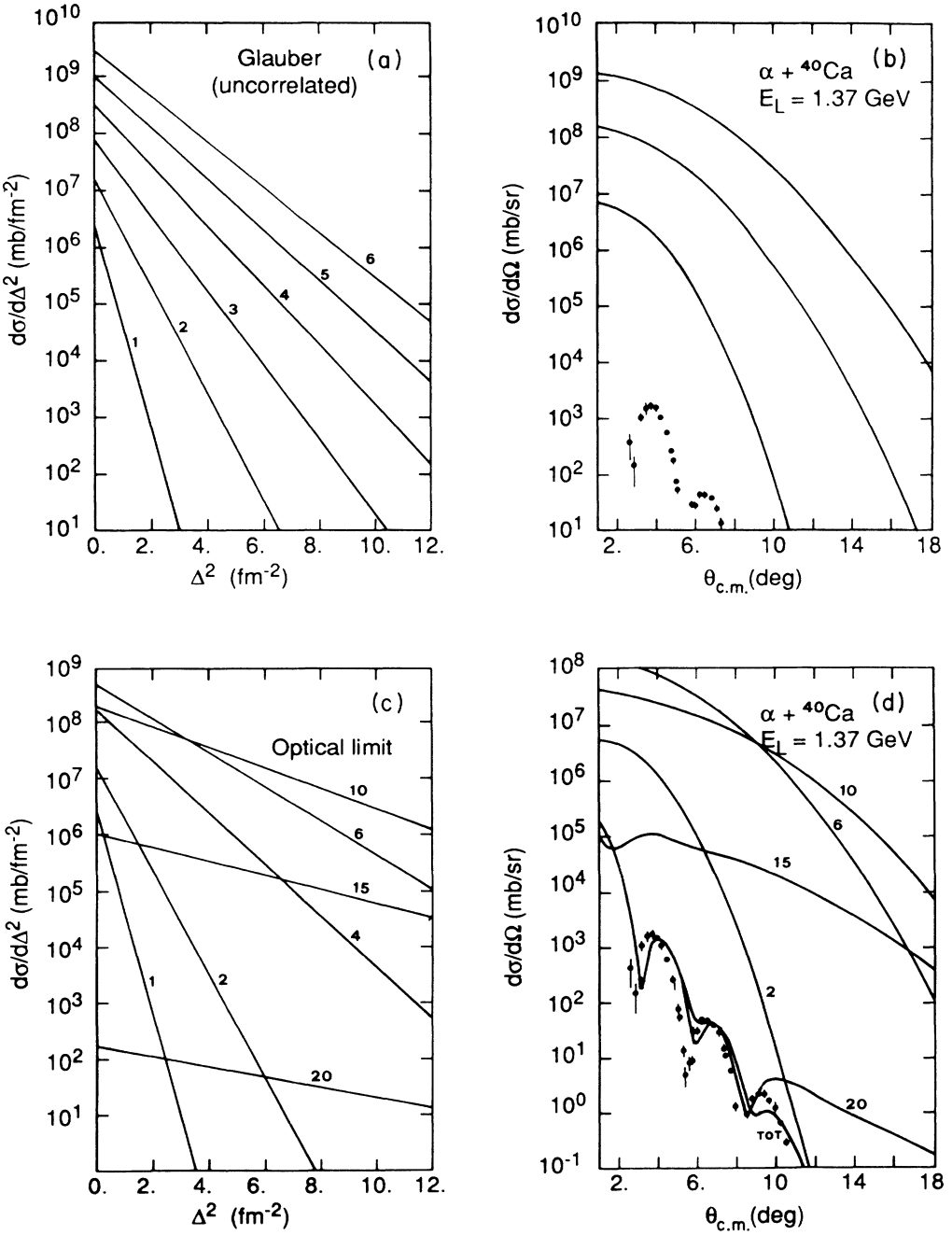


FIG. 2. The same as in Fig. 1 for the elastic scattering of an alpha particle on ^{40}Ca at $E_{\rm lab}=1.37$ GeV. The Gaussian density parameter for ^{40}Ca is $R_{^{40}\rm Ca}=2.52$ fm. Experimental data from Ref. 27 are displayed only for a qualitative comparison of the convergence of the methods.

In particular, no resemblance yet appears with the experimental data, both in the magnitude, largely overestimated, and in the shape which does not display the characteristic diffraction pattern.

C. Order-of-magnitude estimate and the optical limit

The inclusion of higher-order terms, although necessary, looks rather cumbersome without further simplifying assumptions. An order-of-magnitude estimate of the different contributions was worked out by Czyz and Maximon[3] based on the concepts of *separability* and *compactness*. Within a given sequence of elementary collisions, one may individuate subsequences which are separated, in the sense that no coordinate of the profiles of the one appears in the others. We denote n_i the scattering order of the ith subsequence ($\sum_i n_i = n$). As an example, the sequence 16 of the fourth order in Table I is characterized by four separated subsequences of order 1, while the sequence 1 is irreducible. In the independent-particle approximation, the total contribution factorizes in the product of terms, each corresponding to a separated subsequence. For each irreducible configuration, one can introduce the compactness index $Q_i = n_i - v_i + 1$, v_i being the number of different variables (of both nuclei) appearing in the ith subsequence. Finally, the total compactness index Q of a definite sequence is defined as the sum of the partial indices Q_i, i.e., $Q = \sum_i Q_i$. For example, within the sequences of order $n = 4$, the sequence 1 has $Q = 1$, all the others are characterized by $Q = 0$. Similarly, for $n = 6$, the sequences 1 and 2 have $Q = 2$, those from 3 to 14 have $Q = 1$, all the others have $Q = 0$. Now the basic result which can be obtained along the Czyz and Maximon approach is that the contribution to the forward-elastic-scattering amplitude coming from the definite sequence g only depends on the order n and compactness Q_g being approximately given by

$$F_{00}^{(n,g)}(\Delta=0) \approx \frac{-ikR^2}{2n} \left[\frac{\sigma_{NN}}{R^2} \right]^n \left[\frac{R^2}{a} \right]^{Q_g} , \tag{7}$$

where $R^2 = R_T^2 + R_P^2 + 2a$, a being the slope of the elementary profile (6). By weighting each sequence of order n and compactness Q by the multiplicity factors $T_n(Q)$, we deduce the final formula

$$\begin{aligned} F_{00}(\Delta=0) &= \sum_{n,g} F_{00}^{(n,g)}(\Delta=0) \\ &\approx \frac{-ikR^2}{2} \sum_{n=1}^{A_P A_T} \frac{(-1)^{n+1}}{n} \left[\frac{\sigma_{NN}}{R^2} \right]^n \\ &\quad \times \left[\sum_{Q=0}^{Q_{\max}} T_n(Q) \left[\frac{R^2}{a} \right]^Q \right] . \end{aligned} \tag{8}$$

Note that the relative importance of the sequences with different compactness is basically governed by the value of the slope parameter a. We can immediately see that, for relatively low bombarding energies where a is becoming small as the nucleon-nucleon amplitude tends to become isotropic, the more compact sequences, although associated with lower multiplicities, become dominant. This result rules out any statistical approach. As shown by Czyz and Maximon, this effect is not a consequence of the semiquantitative estimate, but of the independent-particle description of the nuclear wave functions disappearing with the introduction of correlations.

These order-of-magnitude estimates can be used to derive the so-called optical limit to the Glauber theory.[3] In the limit of a very large number of constituent objects ($A_T, A_P \to \infty$, with the constraint that the product $A_T A_P \sigma_{NN} \to$ const), the numbers $T_n(Q \neq 0)$ become vanishingly small with respect to

$$T_n(Q=0) \approx \begin{bmatrix} A_P A_T \\ n \end{bmatrix} .$$

It is therefore assumed that all the contributions at a given order are equal to that obtained for the case of a completely separable sequence, a choice that also clears all problems connected with possible divergences. In this particular case, we can release the approximated expression (7). The corresponding contribution is, in fact, completely factorized and simply given by

$$\langle \Phi_0^T \Phi_0^P | \Gamma^{(n,g)} | \Phi_0^T \Phi_0^P \rangle = \lambda(b)^n , \tag{9}$$

where λ is the scattering term associated with a single collision

$$\begin{aligned} \lambda(b) &= \langle \Phi_0^T \Phi_0^P | \gamma(\bar{b} - \bar{s}_j^T + \bar{s}_k^P) | \Phi_0^T \Phi_0^P \rangle \\ &= \int \rho_0^P(\bar{r}_k^P) \rho_0^T(\bar{r}_j^T) \gamma(\bar{b} - \bar{s}_j^T + \bar{s}_k^P) d\bar{r}_k^P d\bar{r}_j^T . \end{aligned} \tag{10}$$

Summing to all orders one gets the final expression for the elastic amplitude in the optical limit

$$\begin{aligned} F(\bar{\Delta}) &= \frac{ik}{2\pi} \sum_{n=0}^{A_P A_T} \begin{bmatrix} A_P A_T \\ n \end{bmatrix} (-)^{n+1} \int d^2b \, e^{i\bar{\Delta}\cdot\bar{b}} \lambda(b)^n \\ &= \frac{ik}{2\pi} \int d^2b \, e^{i\bar{\Delta}\cdot\bar{b}} \{ 1 - [1 - \lambda(b)]^{A_T A_P} \} \end{aligned} \tag{11}$$

or, assuming $A_P A_T$ very large,

$$F(\bar{\Delta}) = \frac{ik}{2\pi} \int d^2b \, e^{i\bar{\Delta}\cdot\bar{b}} (1 - e^{-A_T A_P \lambda(b)}) . \tag{12}$$

The predictions of the optical limit for the two cases previously considered are also displayed in Figs. 1 and 2. As in the case of the full Glauber model, we show the partial contributions associated with each scattering order [Figs. 1(c) and 2(c)], and the summed values up to a definite order [Figs. 1(d) and 2(d)]. Due to the simplicity of the optical limit, we can easily consider scattering processes of any order at variance with the full Glauber model.

As is clear from the figure, the results of the optical limit are in good agreement with the experimental data, showing a faster convergence than the previous calculation. This is a systematic feature which has been pointed out in variety of systems and bombarding energies, including rather low values (cf. Refs. 12–14).

III. THE MULTICHANNEL FORMALISM

As shown in the previous section, the Glauber optical limit for heavy-ion scattering seems to display a number of advantages with respect to the uncorrelated Glauber model: it is extremely easy to compute, it does not present any pathological behavior with the slope of the N-N amplitude and, as the most important feature, its predictions are in good agreement with the experimental data. It is therefore important to find an alternative derivation of the optical limit from the full correlated Glauber model. The basic key in this scheme is provided by the truncation of the completeness of intermediate states which can be inserted between each pair of elementary profiles in the full expression (5), i.e.,

$$\mathbb{1}=\sum_{jk}|\Phi_j^T\Phi_k^P\rangle\langle\Phi_j^T\Phi_k^P| \ , \tag{13}$$

where Φ_j^T and Φ_k^P stand for the target and projectile states (in particular, $j=0$ and $k=0$ correspond to the ground states). This procedure, suggested in the framework of a rederivation of the optical limit,[7,11] is here extended to cover a wide range of reactions.

For the sake of simplicity we shall consider the case in which the projectile (with mass number A_P) is not excited during the collision. The generalization to the case of an excited projectile will be exemplified in Sec. V. To avoid complications in the formulas associated just with the angular-momentum algebra, we will, for the moment, assume that all the considered states of the target have angular momentum zero. The scattering amplitude in this case assumes the form

$$\begin{aligned}F_{s,r}(\bar{\Delta})&=F_{s0,r0}(\bar{\Delta})\\&=\frac{ik}{2\pi}\int d^2b\, e^{i\bar{\Delta}\cdot\bar{b}}\langle\Phi_s^T\Phi_0^P|\Gamma(b)|\Phi_r^T\Phi_0^P\rangle\\&\equiv\frac{ik}{2\pi}\int d^2b\, e^{i\bar{\Delta}\cdot\bar{b}}\Gamma_{s,r}(\bar{b}) \ .\end{aligned} \tag{14}$$

In the expression of the contribution of a definite sequence g of nth order

$$\begin{aligned}\Gamma_{s,r}^{(n,g)}(\bar{b})&=\langle\Phi_s^T\Phi_0^P|\Gamma^{(n,g)}(\bar{b})|\Phi_r^T\Phi_0^P\rangle\\&=\underbrace{\langle\Phi_s^T\Phi_0^P|\gamma(\bar{b}-\bar{s}_{j_1}^T+\bar{s}_{k_1}^P)\gamma(\bar{b}-\bar{s}_{j_2}^T+\bar{s}_{k_2}^P)\cdots\gamma(\bar{b}-\bar{s}_{j_n}^T+\bar{s}_{k_n}^P)|\Phi_r^T\Phi_0^P\rangle}_{n\text{ factors}}\end{aligned} \tag{15}$$

we insert between each pair of successive profiles a truncated completeness of projectile and target states in which only the ground state of the projectile and the lowest N levels for the target appear, i.e.,

$$\mathbb{1}\simeq\sum_{q=0}^{N}|\Phi_q^T\Phi_0^P\rangle\langle\Phi_q^T\Phi_0^P| \ . \tag{16}$$

We obtain by this way

$$\begin{aligned}\Gamma_{s,r}^{(n,g)}(\bar{b})\approx\langle\Phi_s^T\Phi_0^P|\gamma(\bar{b}-\bar{s}_{j_1}^T+\bar{s}_{k_1}^P)\sum_{q=0}^{N}|\Phi_q^T\Phi_0^P\rangle\langle\Phi_q^T\Phi_0^P|\gamma(\bar{b}-\bar{s}_{j_2}^T+\bar{s}_{k_2}^P)\\\times\sum_{q'=0}^{N}|\Phi_{q'}^T\Phi_0^P\rangle\langle\Phi_{q'}^T\Phi_0^P|\cdots|\gamma(\bar{b}-\bar{s}_{j_n}^T+\bar{s}_{k_n}^P)|\Phi_r^T\Phi_0^P\rangle \ .\end{aligned} \tag{17}$$

This expression evidences the limitations and the merits embodied in the method proposed here: for assigned initial and final states the right-hand side is independent on the particular sequence g of collisions, only depending on the scattering order n, as in the case of the optical limit and at variance with the exact expression. In fact, it factorizes in the product of terms, each involving a single profile and therefore not dependent on the two particular nucleons responsible of each microscopic collision. On this ground we can state that this approach is the natural extension of the optical-limit approximation to the analysis of many-channel processes.

To get a more explicit expression we can introduce the matrix $\mathbf{D}$

$$\mathbf{D}_{qq'}(\bar{b})=\langle\Phi_q^T\Phi_0^P|\gamma(\bar{b}-\bar{s}_j^T+\bar{s}_k^P)|\Phi_{q'}^T\Phi_0^P\rangle \ . \tag{18}$$

With this notation the contribution of each sequence of collision of order n assumes the form

$$\begin{aligned}\Gamma_{s,r}^{(n)}(\bar{b})&=\sum_{q',q'',\ldots}\underbrace{\mathbf{D}_{sq'}(\bar{b})\mathbf{D}_{q'q''}(\bar{b})\cdots\mathbf{D}_{q^{n-1}r}(\bar{b})}_{n\text{ factors}}\\&\equiv\{[\mathbf{D}(\bar{b})]^n\}_{s,r} \ ,\end{aligned} \tag{19}$$

namely, $\Gamma_{s,r}^{(n)}(\bar{b})$ is just given by the (s,r) element of the nth power of matrix $\mathbf{D}$. Note that we have consistently dropped the redundant index g.

The equivalence of all the sequences of the same order has obviously cleared all cumbersome combinatorial problems associated with the multiplicity of each sequence. Furthermore, as it appears from Eq. (19), only matrix elements of one-body operators have to be calculated, the knowledge of all correlation functions being replaced by the knowledge of all nuclear densities and transition densities. The two formulations are clearly equivalent when all nuclear states are included as intermediate states, but the latter suggests novel perspectives when physical considerations suggest a truncation in the

basis. We also note that the introduction of complete sets of states among each pair of successive collisions arises in a quite natural way in the derivations of the Glauber model given by Foldy and Valecka,[18] Harrington,[17] Eisenberg,[15] and Wallace[16] who derive the Glauber model from the Goldberger-Watson multiple-scattering theory.

The total profile function and the scattering amplitudes then assume the final expressions

$$\Gamma_{s,r}(\bar{b})=\delta_{s,r}-\sum_{n=1}^{A_P A_T}(-)^{n+1}\begin{bmatrix}A_P A_T\\ n\end{bmatrix}\Gamma_{s,r}^{(n)}$$

$$=\delta_{s,r}-\sum_{n=1}^{A_P A_T}(-)^{n+1}\begin{bmatrix}A_P A_T\\ n\end{bmatrix}\{[\mathbf{D}(\bar{b})]^n\}_{s,r} \quad (20)$$

and

$$F_{s,r}(\bar{\Delta})=\frac{ik}{2\pi}\int d^2b\, e^{i\bar{\Delta}\cdot\bar{b}} \times\sum_{n=0}^{A_P A_T}\begin{bmatrix}A_P A_T\\ n\end{bmatrix}\{[\mathbf{D}(\bar{b})]^n\}_{s,r}\ , \quad (21)$$

respectively, having taken into account that

$$\{[\mathbf{D}(\bar{b})]^n\}_{s,r}=\delta_{s,r}$$

for $n=0$.

More explicit expressions of $\mathbf{D}_{jj'}$ can be given in the form

$$\mathbf{D}_{jj'}(\bar{b})=\int\rho_0^P(\bar{r}_k^P)\rho_{jj'}^T(\bar{r}_j^T)\gamma(\bar{b}-\bar{s}_j^T+\bar{s}_k^P)d\bar{r}_k^P d\bar{r}_j^T\ , \quad (22)$$

where ρ_0^P is the ground-state density of the projectile and $\rho_{jj'}^T$ the target density of state j (for $j=j'$) or the transition density from j to j' (for $j\neq j'$). The same quantity can be written in the form

$$\mathbf{D}_{jj'}(\bar{b})=\frac{1}{2\pi ik_{NN}}\int\hat{\rho}_0^P(\bar{q})\hat{\rho}_{jj'}^T(\bar{q})f_{NN}(q)e^{i\bar{q}\cdot\bar{b}}d\bar{q} \quad (23)$$

in terms of the densities and transition densities $\hat{\rho}_0^P(\bar{q})$ and $\hat{\rho}_{jj'}^T(\bar{q})$ in the momentum space.

As a final remark we briefly discuss the problem of the center-of-mass correction factor. As mentioned before, this arises in connection with the use of uncorrelated wave functions.[3] In our approach the nuclear wave functions only enter in the evaluation of the matrix elements $\mathbf{D}_{jj'}$, i.e., expectation values of one-body operators which lead to the use of diagonal and nondiagonal densities. From a physical point of view, these densities are obviously related to the fully correlated wave functions and therefore no need arises to introduce center-of-mass corrections.

IV. DEVELOPMENT OF PARTICULAR COUPLING SCHEMES

A. One-channel case: The optical limit

As a first example of the formalism built up in the previous section, we consider the case of elastic scattering in the assumption of absence of virtual excitations of the other states. Under these conditions the matrix $\mathbf{D}$ has a single element $[\mathbf{D}(\bar{b})]_{0,0}$ which coincides with the matrix element λ defined in Eq. (10). In this case the sum in Eq. (21) coincides with Eq. (11). One so obtains, in a completely different framework, the expression of the optical limit. Note that this alternative derivation of the optical limit, suggested by a number of authors,[7,11,19] does not make use of the independent-particle model, being based on the converse on the frozen-nucleus approximation and so no center-of-mass correction is necessary. Furthermore, no role is played by the assumption of large number of constituents ($A_T, A_P\to\infty$) so justifying the good results obtained by the optical limit for relatively light nuclei. We recall that the use of the optical limit in connection with finite masses together with several versions of the center-of-mass correction problem have been discussed by Franco and Varma.[22]

B. Two-channel case

Let us consider the case of two strongly coupled channels, labeled by 0 and 1. In this case the relevant matrix $\mathbf{D}$ is a two-dimensional matrix with elements

$$\mathbf{D}_{00}(b)=\langle\Phi_0^T\Phi_0^P|\gamma(\bar{b}-\bar{s}_j^T+\bar{s}_k^P)|\Phi_0^T\Phi_0^P\rangle$$

$$=\lambda(b)\simeq\mathbf{D}_{11}(b) \quad (24)$$

and

$$\mathbf{D}_{10}(b)=\langle\Phi_1^T\Phi_0^P|\gamma(\bar{b}-\bar{s}_j^T+\bar{s}_k^P)|\Phi_0^T\Phi_0^P\rangle=\mu(b) \quad (25)$$

which can be explicitly evaluated in terms of the densities and transition densities as in Eqs. (22) and (23). To simplify the notation, we make explicit the assumption that all the states we consider have zero angular momentum, so that densities and transition densities have spherical symmetric distributions. With Eq. (24) we introduced the reasonable approximation[7] that the diagonal elements of the matrix $\mathbf{D}$ are the same in the ground and the excited state. With these notations the matrix $\mathbf{D}$ assumes the form

$$\mathbf{D}(b)=\begin{bmatrix}\lambda(b) & \mu(b)\\ \mu(b) & \lambda(b)\end{bmatrix}=\lambda(b)\mathbb{1}+\mathbf{B}(b)\ , \quad (26)$$

where the matrix $\mathbf{B}$ has the simple expression

$$\mathbf{B}(b)=\begin{bmatrix}0 & \mu(b)\\ \mu(b) & 0\end{bmatrix}. \quad (27)$$

The nth power is easily evaluated by the binomial formula

$$[\mathbf{D}(b)]^n=\sum_{m=0}^{n}\begin{bmatrix}n\\ m\end{bmatrix}\lambda^{n-m}(b)[\mathbf{B}(b)]^m \quad (28)$$

and this leads directly to the total profiles for elastic and inelastic scattering

$$\Gamma_{00}(b)=1-\sum_{M=0(\text{even})}^{A_T A_P}\begin{bmatrix}A_T A_P\\ M\end{bmatrix}\times[1-\lambda(b)]^{A_T A_P-M}\mu(b)^M$$

$$\approx 1-e^{-A_T A_P\lambda(b)}\cosh[A_T A_P\mu(b)] \quad (29)$$

and

$$\Gamma_{10}(b)=\sum_{M=1(\text{odd})}^{A_T A_P}\begin{pmatrix}A_T A_P\\ M\end{pmatrix}[1-\lambda(b)]^{A_T A_P-M}\mu(b)^M$$
$$\approx e^{-A_T A_P\lambda(b)}\sinh[A_T A_P\mu(b)]\ . \tag{30}$$

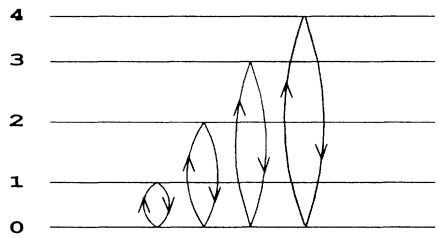
Note that, by a proper choice of phases in Eqs. (26) and (27), we assumed real transition densities. If a more general assumption is introduced, only the substitution of μ^2 with $\mu_{01}\mu_{10}$ is required.

These equations embody the expected features of a multiple-scattering problem. In the elastic scattering we have contributions from inelastic-multiple-scattering processes, but these always appear in even powers, corresponding to excitation-deexcitation sequences. In the inelastic case, on the other hand, the inelastic term μ appears in odd powers, as necessary to end up in the excited channel after a sequence of excitation-deexcitation processes.

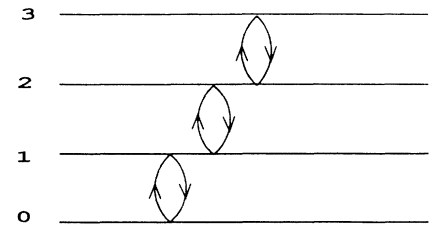
In the limit of weak coupling ($|\mu| \ll |\lambda|$), the contribution of an inelastic microscopic collision becomes negligible with respect to those of elastic nature. We can therefore keep the first term in the sum in Eqs. (29) and (30). In this way the expression for the elastic scattering reduces to the optical limit (12), while for the inelastic channel we get

$$\Gamma_{10}(b)\approx A_T A_P\mu(b)e^{-A_T A_P\lambda(b)}\ , \tag{31}$$

as expected from the fact that, in the weak-coupling limit, in each sequence of order n contributing to the multiple-scattering series, only one collision of inelastic nature can appear.



(a)



(b)

FIG. 3. Sketch of the coupling schemes involved in the cases considered in Secs. IV C (a) and IV D (b).

C. Multichannel case with no coupling within excited states

The above procedure can be easily extended to the case in which the ground state is directly coupled to an indefinite number of excited states, which are not coupled among them [cf. Fig. 3(a)]. In this case, by denoting $\mu_i(b)$, the coupling term between the ground state and the ith state

$$\mathbf{D}_{i0}(b)=\langle\Phi_i^T\Phi_0^P|\gamma(\bar{b}-\bar{s}_j^T+\bar{s}_k^P)|\Phi_0^T\Phi_0^P\rangle=\mu_i(b) \tag{32}$$

and introducing again the ansatz for the diagonal terms

$$\mathbf{D}_{ii}(b)\simeq\mathbf{D}_{00}(b)=\lambda(b)\ , \tag{33}$$

one obtains the following expressions for the elastic channel profile:

$$\Gamma_{00}(b)=1-\sum_{M=0(\text{even})}^{A_T A_P}\begin{pmatrix}A_T A_P\\ M\end{pmatrix}[1-\lambda(b)]^{A_T A_P-M}\times\left[\sum_{i\neq 0}\mu_i(b)^2\right]^{M/2} \tag{34}$$

and for the profile connecting the ground state to the ith excited state

$$\Gamma_{i0}(b)=\mu_i(b)\sum_{M=1(\text{odd})}^{A_T A_P}\begin{pmatrix}A_T A_P\\ M\end{pmatrix}[1-\lambda(b)]^{A_T A_P-M}\times\left[\sum_{i\neq 0}\mu_i(b)^2\right]^{(M-1)/2} \tag{35}$$

[see the comment after Eq. (30)]. On defining

$$\Delta(b)=\sum_{i\neq 0}\mu_i(b)^2=\sum_{i\neq 0}\langle\Phi_0^T\Phi_0^P|\gamma|\Phi_i^T\Phi_0^P\rangle\langle\Phi_i^T\Phi_0^P|\gamma|\Phi_0^T\Phi_0^P\rangle\ , \tag{36}$$

we can write the above expressions in the final form

$$\begin{aligned}\Gamma_{00}(b)&=1-\sum_{M=0(\text{even})}^{A_T A_P}\begin{pmatrix}A_T A_P\\ M\end{pmatrix}[1-\lambda(b)]^{A_T A_P-M}\times\Delta(b)^{M/2}\ ,\\ \Gamma_{i0}(b)&=\mu_i(b)\sum_{M=1(\text{odd})}^{A_T A_P}\begin{pmatrix}A_T A_P\\ M\end{pmatrix}[1-\lambda(b)]^{A_T A_P-M}\times\Delta(b)^{(M-1)/2}\ ,\end{aligned} \tag{37}$$

which can be approximated in the form

$$\begin{aligned}\Gamma_{00}(b)&\approx 1-e^{-A_T A_P\lambda(b)}\cosh[A_T A_P\Delta(b)^{1/2}]\ ,\\ \Gamma_{i0}(b)&\approx\mu_i(b)e^{-A_T A_P\lambda(b)}\sinh[A_T A_P\Delta(b)^{1/2}]\ .\end{aligned} \tag{38}$$

These equations have similar structure to Eqs. (29) and (30). They are also capable of the same perturbative expansions. The quantity $\Delta(b)$ can be expressed in the alternative form

$$\Delta(b)=\int C_2^T(\bar{r}_1^T,\bar{r}_2^T)\rho_0^P(\bar{r}_1^P)\rho_0^P(\bar{r}_2^P)\times\gamma(\bar{b}-\bar{s}_1^T+\bar{s}_1^P)\gamma(\bar{b}-\bar{s}_2^T+\bar{s}_2^P)\times d\bar{r}_1^P d\bar{r}_2^P d\bar{r}_1^T d\bar{r}_2^T \tag{39}$$

in terms of the target two-body correlation function

$$C_2^T(\bar{r}_1,\bar{r}_2)=\rho_2^T(\bar{r}_1,\bar{r}_2)-\rho^T(\bar{r}_1)\rho^T(\bar{r}_2)\ . \tag{40}$$

This case is capable of illustrating the relation between the present approach, stressing the role of the coupling between different states, and the one based on the truncation of the correlation functions. In fact, Eq. (37), with $\Delta(b)$ in the form of Eq. (39), has also been derived by Ahmad[7] starting from the introduction of completenesses of intermediate states. In that case, however, the subsequent simplifications have been obtained by neglecting all the contributions which would imply higher-order correlation functions. The same result can be obtained from the more general correlation-function expansion considered by Alkhazov *et al.*[5] when high-order correlation functions are approximated in terms of one-body densities and second-order correlation functions.

As a final consideration we remind the reader that, for the nucleon-nucleus elastic scattering, the uncorrelated-nucleus approximation is equivalent to the frozen-target hypothesis. The above analysis shows that, for the same process, taking into account the correlations up to the second order is equivalent to assuming that all the excited states are coupled to the ground state but not between them.

D. Multichannel case with a ladder coupling scheme

Let us consider now the case of a system of levels under the assumption that the level i can be coupled only to the levels $i\pm1$, and that all the coupling elements are equal [cf. Fig. 3(b)]. A situation of this kind can occur, for example, in the case of a harmonic vibrational spectrum aside from angular-momentum coefficients. The simplest case is associated with $L=0$ pair-transfer processes between members of a pairing excitation spectrum.[23]

According to the above schematization, the coupling-matrix elements are assumed to have the form

$$\mathbf{D}_{i,i\pm1}(b)=\nu(b),\quad \mathbf{D}_{ii}(b)\approx\mathbf{D}_{00}=\lambda(b)\ , \tag{41}$$

i.e., only the principal diagonal and the two adjacent diagonals have nonzero elements. It is easy to evaluate the powers of the matrix $\mathbf{D}$ and to give recursion relations for any number of levels and for any order of scattering, so that algebraic expressions for the scattering amplitudes of the type of Eq. (34) or (35) can be written. For the sufficiently interesting problem in which the ground state is coupled to two excited states according to scheme (41), one gets a closed form for Γ_{i0} in terms of hyperbolic functions. For the elastic and the two inelastic channels, one obtains

$$\Gamma_{00}(b)=1-[1-\lambda(b)]^{A_TA_P}-\frac{1}{2}\sum_{M=2(\text{even})}^{A_TA_P}\binom{A_TA_P}{M}[1-\lambda(b)]^{A_TA_P-M}[\sqrt{2}\nu(b)]^M$$
$$\approx 1-\tfrac{1}{2}e^{-A_TA_P\lambda(b)}\{\cosh[A_TA_P\sqrt{2}\nu(b)]+1\}\ , \tag{42}$$

$$\Gamma_{10}(b)=\frac{1}{\sqrt{2}}\sum_{M=1(\text{odd})}^{A_TA_P}\binom{A_TA_P}{M}[1-\lambda(b)]^{A_TA_P-M}[\sqrt{2}\nu(b)]^M$$
$$\approx\frac{1}{\sqrt{2}}e^{-A_TA_P\lambda(b)}\sinh[A_TA_P\sqrt{2}\nu(b)]\ , \tag{43}$$

$$\Gamma_{20}(b)=\frac{1}{2}\sum_{M=2(\text{even})}^{A_TA_P}\binom{A_TA_P}{M}[1-\lambda(b)]^{A_TA_P-M}[\sqrt{2}\nu(b)]^M$$
$$\approx\tfrac{1}{2}e^{-A_TA_P\lambda(b)}\{\cosh[A_TA_P\sqrt{2}\nu(b)]-1\}\ . \tag{44}$$

V. FURTHER APPLICATIONS: MUTUAL EXCITATION AND CHARGE EXCHANGE

In the above sections, two features which one faces in considering actual nuclear reactions have not been discussed: the possibility that the projectile is also excited and the problems arising in connection with angular momentum, spin, and isospin of the nuclei.

It is not an easy task to take into account these features in general terms, since each specific reaction presents peculiar characters. We shall examine in the perturbative approximation the mutual excitation of two nuclei and charge-exchange reactions. This will evidence, on the one hand, the flexibility of the formalism to describe specific reactions; on the other hand, these examples stress the fact that each case needs its own appropriate techniques.

Let us briefly recall some results from Refs. 13 and 14 concerning the inelastic scattering. In the perturbative approximation the inelastic scattering with excitation of the target from the 0^+ ground state to the state Φ_{LM}^T is described by the amplitude [cf. Eq. (31)]

$$F_{LM,0}(\bar{\Delta})=\frac{ik}{2\pi}A_TA_P\int d^2b\,e^{i\bar{\Delta}\cdot\bar{b}}\mu_{LM}(\bar{b})[1-\lambda(b)]^{A_TA_P-1}\ , \tag{45}$$

where $\lambda(b)$ is given by Eq. (24) and

$$\mu_{LM}(\bar{b})=\langle \Phi^T_{LM}\Phi^P_0|\gamma|\Phi^T_0\Phi^P_0\rangle \ . \tag{46}$$

By introducing the target transition density

$$\rho^T_{LM}(\bar{r})=\rho^T_L(r)Y_{LM}(\hat{r})$$

and by defining the momentum space transition density

$$\hat{\rho}^T_L(q)=4\pi\int dr\, r^2 j_L(qr)\rho^T_L(r) \ , \tag{47}$$

one gets, for the eikonalized form factor, the expression

$$\mu_{LM}(\bar{b})=e^{iM\Phi_b}\mu_{LM}(b)$$
$$=\frac{e^{iM\Phi_b}}{ik_{NN}}B_{LM}\int_0^\infty dq\, q\hat{\rho}^P_0(q)f_{NN}(q)\hat{\rho}^T_L(q)J_M(qb) \ , \tag{48}$$

where B_{LM} is a geometrical factor vanishing for $L+M$ odd:

$$B_{LM}=(-1)^M\left[\frac{2L+1}{4\pi}\right]^{1/2}$$
$$\times\frac{[(L-M)!(L+M)!]^{1/2}}{(L-M)!!(L+M)!!}\left[\frac{1+(-1)^{L+M}}{2}\right] \ . \tag{49}$$

This leads to the final expression for the inelastic cross section

$$(d\sigma/d\omega)_L=\sum_{M=-L}^{L}\left|k\int_0^\infty db\, bJ_M(qb)\mu_{LM}(b)[1-\lambda(b)]^{A_TA_P-1}\right|^2$$
$$\approx\sum_{M=-L}^{L}\left|k\int_0^\infty db\, bJ_M(qb)\mu_{LM}(b)e^{-\lambda(b)A_TA_P}\right|^2 \ . \tag{50}$$

A. Mutual excitation

The formalism briefly recalled above can be naturally extended in order to describe a wider class of reactions of surface character. As typical examples, we will shortly discuss the cases of mutual excitation and charge-exchange reactions. Let us first consider mutual excitation processes, which have been usually described within a multistep distorted-wave Born approximation (DWBA).[24] These processes correspond to the events which leave both reaction partners in an excited state. From the point of view of the reaction mechanism, we have assumed in our approach that any reaction process can be described in terms of a sequence of nucleon-nucleon collisions of either elastic or inelastic character. The simplest scenario for a mutual excitation process is therefore to imagine a series of elastic collisions together with two inelastic collisions which are responsible for the subsequent excitation of the two nuclei (paths *a* and *b* in Fig. 4). This description, even if we are summing the elastic-scattering collisions to all possible orders, corresponds to the conventional two-step processes. In an alternative picture (path *c* in Fig. 4), we can assume that it is a single nucleon-nucleon collision which simultaneously leads to the excitation of both nuclei and, in the standard language, this corresponds to a one-step process.

The scattering amplitudes associated with these two processes are easily evaluated along the multichannel formalism. In this case the completeness to be inserted between each microscopic collision is truncated to

$$|\Phi^T_0\Phi^P_0\rangle\langle\Phi^T_0\Phi^P_0|+\sum_M|\Phi^T_{LM}\Phi^P_0\rangle\langle\Phi^T_{LM}\Phi^P_0|+\sum_{M'}|\Phi^T_0\Phi^P_{L'M'}\rangle\langle\Phi^T_0\Phi^P_{L'M'}|+\sum_{MM'}|\Phi^T_{LM}\Phi^P_{L'M'}\rangle\langle\Phi^T_{LM}\Phi^P_{L'M'}| \tag{51}$$

(we assumed that both nuclei have $J^\pi=0^+$ in their ground states). This leads to the expression for the scattering amplitude

$$F_{LM,L'M'}(\bar{\Delta})=F^{\text{two-step}}_{LM,L'M'}(\bar{\Delta})+F^{\text{one-step}}_{LM,L'M'}(\bar{\Delta}) \ , \tag{52}$$

where the two contributions are given by

$$F^{\text{two-step}}_{LM,L'M'}(\bar{\Delta})=\frac{ik}{2\pi}A_TA_P\int d^2b\, e^{i\bar{\Delta}\cdot\bar{b}}\mu^T_{LM}(\bar{b})\mu^P_{L'M'}(\bar{b})[1-\lambda(b)]^{A_TA_P-2}$$
$$\approx\frac{ik}{2\pi}A_TA_P\int d^2b\, e^{i\bar{\Delta}\cdot\bar{b}}\mu^T_{LM}(\bar{b})\mu^P_{L'M'}(\bar{b})e^{-A_TA_P\lambda(b)} \tag{53}$$

and

$$F^{\text{one-step}}_{LM,L'M'}(\bar{\Delta})=\frac{ik}{2\pi}A_TA_P\int d^2b\, e^{i\bar{\Delta}\cdot\bar{b}}\tau_{LM,L'M'}(\bar{b})[1-\lambda(b)]^{A_TA_P-1}$$
$$\approx\frac{ik}{2\pi}A_TA_P\int d^2b\, e^{i\bar{\Delta}\cdot\bar{b}}\tau_{LM,L'M'}(\bar{b})e^{-A_TA_P\lambda(b)} \ . \tag{54}$$

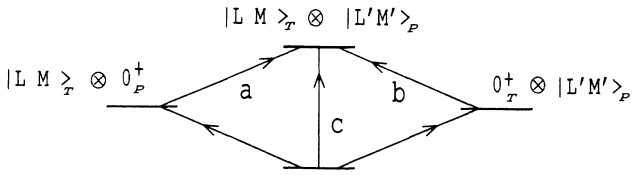


FIG. 4. Coupling scheme involved in the description of a mutual-excitation process.

In these expressions the inelastic matrix elements $\mu^T_{LM}(\bar{b})$ and $\mu^P_{L'M'}(\bar{b})$ are associated with the inelastic excitation of either the target or the projectile. They are given in terms of the folding of the NN scattering amplitude with the density ρ_0 of one of the colliding nuclei and the transition density ρ_L of the other, i.e.,

$$\mu^T_{LM}(\bar{b})=\langle\Phi^T_{LM}\Phi^P_0|\gamma|\Phi^T_0\Phi^P_0\rangle \approx\langle\Phi^T_{LM}\Phi^P_{L'M'}|\gamma|\Phi^T_0\Phi^P_{L'M'}\rangle = \frac{e^{iM\phi_b}}{ik_{NN}}B_{LM}\int_0^\infty dq\, q\hat{\rho}^P_0(q)f_{NN}(q)\hat{\rho}^T_L(q)J_M(qb) \tag{55}$$

and

$$\mu^P_{L'M'}(\bar{b})=\langle\Phi^T_0\Phi^P_{L'M'}|\gamma|\Phi^T_0\Phi^P_0\rangle \approx\langle\Phi^T_{LM}\Phi^P_{L'M'}|\gamma|\Phi^T_{LM}\Phi^P_0\rangle = \frac{e^{iM'\phi_b}}{ik_{NN}}B_{L'M'}\int_0^\infty dq\, q\hat{\rho}^P_{L'}(q)f_{NN}(q) \times\hat{\rho}^T_0(q)J_{M'}(qb)\ . \tag{56}$$

[Note that we have made the same approximation here for μ that we made for λ in Eq. (24).] The matrix element $\tau_{LM,L'M'}(\bar{b})$ associated with the simultaneous excitation of the modes is instead expressed as the folding of the NN amplitudes with both transition densities in the target and in the projectile by the form

$$\tau_{LM,L'M'}(\bar{b})=\langle\Phi^T_{LM}\Phi^P_{L'M'}|\gamma|\Phi^T_0\Phi^P_0\rangle = \frac{e^{i(M+M')\phi_b}}{ik_{NN}}B_{LM}B_{L'M'} \times\int_0^\infty dq\, q\hat{\rho}^P_{L'}(q)f_{NN}(q) \times\hat{\rho}^T_L(q)J_{M+M'}(qb)\ . \tag{57}$$

The relative role of the two processes is mainly governed by the bombarding energy through the collision time, the dependence being essentially linear for the one-step process and quadratic for the two-step process. The direct excitation is therefore expected to become more important in the high-energy regime.

B. Charge-exchange reactions

Charge-exchange processes have been extensively used as a tool for the study of nuclear properties in the spin-isospin degrees of freedom; for the standard description of the reaction mechanism see, e.g., Ref. 25. At variance with the standard inelastic processes where the angular momentum associated with the vibration can be taken from the reservoir of the ion-ion relative motion, conservation of charge implies here the mutual excitation of both colliding nuclei. For this reason the possibility of two-step processes as previously discussed are ruled out. Other kinds of two-step processes are, on the other hand, allowed, such as a stripping reaction followed by a pickup reaction, although their importance is expected to decrease at high energies.[25] In any case, the inclusion of transfer reactions in the Glauber approach is not straightforward. We will therefore discuss only the direct one-step charge-exchange reaction mechanism.

The expression of the scattering amplitude for the charge-exchange process will be much like the one for the one-step amplitude for mutual excitation. There is, however, one relevant difference. Although not explicitly stated in the previous examples, we have been dealing with processes ruled by the isoscalar component of the NN scattering amplitude, both in elastic and inelastic collision. Now we have to use, in connection with the NN scattering amplitude, just the part which is responsible for the specific transition. As an example, for the Gamow-Teller transition, characterized by $\Delta S=\Delta T=1$, at any order n of scattering we will consider $n-1$ elastic collisions associated with the isoscalar part of the microscopic amplitude, and one induced by the isovector spin-flip component f^{st}_{NN}.

In a general case, the final expression for the transition amplitude will assume the form

$$F^{\text{charge-exchange}}_{AB,ab}(\Delta)=ikA_TA_P\sum_{L_{\rm tr}M_{\rm tr}KM}\sum_{J_TM_TJ_PM_P}\sum_{lml'm'L}\sum_{stt_z}C^{J_TM_TJ_PM_PL_{\rm tr}M_{\rm tr}}_{ll'LKstt_z}(AB,ab) \times e^{-iM_{\rm tr}\phi_\Delta}\int b\,db\,R^{PT}_{M_{\rm tr}}(b)J_{M_{\rm tr}}(\Delta b)e^{-A_TA_P\lambda(b)}\ , \tag{58}$$

where

$$R^{PT}_{M_{tr}}(b)=\frac{1}{ik_{NN}}B_{lm}B_{l'm'}B_{KM}\times\int_0^\infty dq\, q\hat{\rho}^P_{J_P lst}(q)f^{st}_{NN}(q)\times\hat{\rho}^T_{J_T l'st}(q)J_{(m+m'+M)}(qb) \quad (59)$$

in terms of the corresponding transition densities $\hat{\rho}^P_{J_P lst}(q)$ and $\hat{\rho}^T_{J_T l'st}(q)$. In Eq. (58) we introduced a geometrical factor C which takes into account, in a reaction $A(a,b)B$, for the couplings between the different angular momentum; spin and isospin components are summed on all possible s and t spin and isospin transferred. J_T and J_P are the angular-momentum transferred in the target and the projectile, respectively ($\bar{L}_{tr}=\bar{J}_T+\bar{J}_P$), and ϕ_Δ accounts for the angular dependence of the momentum transferred in the plane orthogonal to the incidence direction.

VI. CONCLUSIONS AND PERSPECTIVES

The method developed in this work can be viewed as a generalization of the optical-limit approach[3,4] to the description of multichannel scattering processes between heavy ions at intermediate energies. The basic assumption is that all the sequences of scattering of the same order give the same contribution to the scattering amplitude. In this way one avoids the difficulties arising from the cumbersome combinatorial features of the multiple-scattering theories. In several interesting cases the resulting simpler series can be summed to give analytical expressions. As a by-product, the center-of-mass correlation problems are also avoided.

The applications of the method to the study of elastic and inelastic scattering have been successfully carried out in previous works (see Ref. 14 and references quoted therein). Owing to the richness of experimental studies of charge-exchange processes between heavy ions at intermediate energies, it looks particularly promising to apply the above procedure to the analysis of these reactions. Finally, since, in the eikonal approximation, it is easy to obtain the optical potential by inversion of the elastic-scattering amplitude,[12] it can be interesting to study the high-energy behavior of the polarization optical potential in both perturbative and strong-coupling situations.

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