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Consistent description of intruder states

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The coexistence of normal and intruder states is described within the framework of the proton-neutron interacting boson model. It is shown that the excitation energy of the intruding configuration and the structure of both the normal and intruding configurations can be described without large differences between the strength of the neutron-proton quadrupole force used for either. Detailed comparison is made for ^{114}Cd.

I. INTRODUCTION

Since the discovery of intruder excitations in atomic nuclei, many attempts to describe the coexistence of the "deformed" intruder states with the normal "spherical" configuration have been undertaken.[1] The intruder states, as observed in or near to single-closed shell nuclei mainly result from particle-hole excitations across the closed shell. In doubly even nuclei with a certain neutron or proton excess, i.e., the Sn and Pb nuclei or the $N=82$ nuclei, they are formed by 2p-2h configurations (for single-closed shell nuclei) or 2p-4h (4p-2h) configurations, in addition to the normal 2h (2p) states, for nuclei with two valence holes (particles) outside the closed shell. On the experimental side extensive measurements have studied the behavior of intruder excitations throughout large chains of isotopes and/or isotones. These measurements have shown that the excitation energy of the intruder states has a minimum near midshell and that the excitation energy $E_{\rm intr}(0^+)$ in doubly even nuclei can become so low that the intruder state becomes the first excited state, such as in the Pb isotopes.[2] This behavior has been explained qualitatively as resulting mainly from the attractive neutron-proton quadrupole force.[3] The question we want to answer is whether it is possible to describe, in a consistent way, the correct energy of the intruder states and the structure of the normal (spherical) states as well as their mixing with the intruding states (Secs. II and III). As a test we study ^{114}Cd in detail in Sec. IV.

II. CONFIGURATION MIXING IN THE INTERACTING BOSON MODEL

The excitation energy of the lowest intruder state described as a 2p-2h configuration in even-even nuclei is described by the expression[3]

$$E_{\rm intr}(0^+)=2(\varepsilon_{\rm p}-\varepsilon_{\rm h})-\Delta E_{\rm pairing}+\Delta E_M+\Delta E_{\rm coll} . \quad (1)$$

The first term denotes the unperturbed energy of a 2p-2h excitation. The second term describes the gain in pairing energy coming from the extra pair correlation energy among the particle and hole pair. The monopole correction ΔE_M describes the shift in unperturbed energy for the given 2p-2h configuration with changing neutron number N (for proton 2p-2h intruder excitations) or changing proton number Z (for neutron 2p-2h intruder excitations) as caused mainly by the residual proton-neutron interaction. The last term $\Delta E_{\rm coll}$ takes account of the extra binding energy gained due to the increase of quadrupole collectivity and deformation when the number of interacting protons and neutrons becomes larger.

In order to evaluate the latter collective contribution to the excitation energy, $\Delta E_{\rm coll}$, we use the interacting boson model (IBM-2) approach to describe intruder configurations near closed shells.[4,5] Then, for instance, the normal configuration is described as an interacting system of N_ν neutron bosons and one N_π proton boson describing the 2h states with respect to the closed proton shell. The intruder configuration is described by N_ν neutron bosons and $N_\pi+2$ proton bosons describing the 2p-4h states. The IBM-2 Hamiltonian for both configurations is diagonalized separately and an energy Δ is added to the intruder configuration that accounts for the energy difference between the regular and the 2p-4h type of intruding configurations. This energy is given by the three first terms of Eq. (1), so

$$\Delta\equiv 2(\varepsilon_{\rm p}-\varepsilon_{\rm h})-\Delta E_{\rm pairing}+\Delta E_M . \quad (2)$$

Finally, the lowest states in both configurations are admixed by[4]

$$H_{\rm mix}=\alpha(s_\pi^\dagger s_\pi^\dagger+s_\pi s_\pi)^{(0)}+\beta(d_\pi^\dagger d_\pi^\dagger+\tilde{d}_\pi\tilde{d}_\pi)^{(0)} . \quad (3)$$

The excited energy spectrum of the mixed configurations is then obtained which takes into account the collective contribution $\Delta E_{\rm coll}$ to $E_{\rm intr}(0^+)$. This term is dominated by the attractive neutron-proton quadrupole-quadrupole interaction $\kappa Q_\nu Q_\pi$. The lowering of the intruder state near midshell is then described by the dependence of its matrix element on the number of valence nucleons. This matrix element is approximately given by the expression

$$\Delta E_{\rm coll}\approx 2\kappa\Delta N_\pi N_\nu . \quad (4)$$

TABLE I. The parameters used in the calculation of ^{114}Cd. All units are in MeV, except χ_π and χ_ν which are dimensionless. $FS=0.06$, $FK=0.12$, $\alpha=\beta=0.08$, and $\Delta=4.00$.

N_π	ε	κ	χ_ν	χ_π	$c_{0\nu}$	$c_{2\nu}$	$c_{0\pi}$	$c_{2\pi}$
1	0.83	−0.14	−0.9	−0.05	−0.2	−0.05	0.0	0.0
3	0.30	−0.15	−0.5	0.75	−0.2	−0.05	0.0	0.2

For proton 2p-2h intruder excitations which induce, in the IBM-2 approach two extra pairs, i.e., $\Delta N_\pi=2$, the main mass dependence of the intruder excitation is given by the quadrupole energy gain which varies as $\Delta E_{\text{coll}} \approx 4\kappa N_\nu$.

III. CONSISTENT DESCRIPTION OF INTRUDER STATES

The quantitative verification of the influence of the neutron-proton quadrupole force up to now has been somewhat doubtful, in that the strength of the quadrupole force κ describing the intruder configuration was determined in order to reproduce the intruder energy $E_{\text{intr}}(0^+)$. This results in rather large different values for κ(normal) and κ(intruder) when describing the normal and the intruder configuration, for instance $\kappa(n)=-0.14$ MeV and $\kappa(i)=-0.19$ MeV in Ref. 5. This is not what one would expect since, as indicated above, the matrix element of the quadrupole force has a strong and specific dependence on the number of interacting nucleons and it should be such that the additional gain in binding energy when creating an intruder 2p-2h configuration is coming from the specific N_π and N_ν dependence in ΔE_{coll} [Eq.

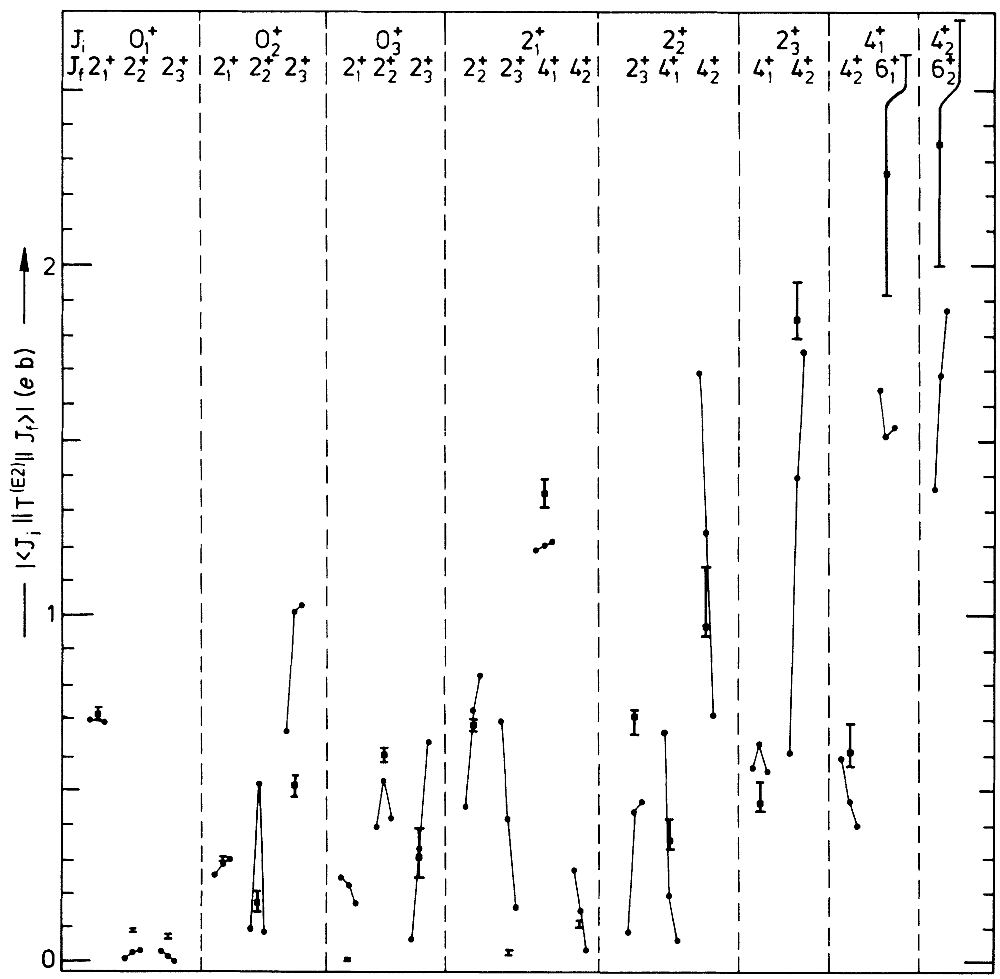

FIG. 1. The experimental $E2$ reduced matrix elements (Ref. 7) (indicated as the full squares with error bars as given in each case) compared with the theoretical calculations. The theoretical results, for three slightly different values of ε_d ($N_\pi=3$) equal to 0.28 MeV (left-hand point), 0.30 MeV (middle point), and 0.32 MeV (right-hand point) are given and connected with a full line. Transitions starting from a given initial state are separated by the vertical dashed lines.

(4)] and is not forced by choosing different κ values. By now taking κ as also strongly dependent on this number of nucleons, part of the argument is lost. Secondly, as shown by Casten, many experimental quantities seem to depend only on the product N_pN_n,[6] indicating a rather constant value for κ within a shell.

The purpose of our study is to show that when one uses expression (1), which is derived from shell-model arguments together with a consistent determination of the IBM-2 parameters for the normal and intruder configuration, a good description of the mixing of both configurations can be obtained. The consistency we claim is based on the fact that the IBM-2 parameters when going from one nucleus to another within a shell are approximately known on the basis of microscopic studies and phenomenological studies of chains of isotopes. This shows that some parameters, such as κ, are nearly constant in a shell, while others, such as χ_ν, χ_π, and ε, are rapidly changing quantities.

IV. APPLICATION TO ^{114}Cd

Since the nucleus ^{114}Cd has been investigated by many authors and an almost complete set of $E2$ reduced matrix elements for the low-lying normal and intruder states became available recently[7] we have concentrated, in particular, on this nucleus. In ^{114}Cd the normal configuration is described as an interacting system of $N_\nu=8$ neutron bosons and one $N_\pi=1$ proton boson with respect to the $Z=50$ shell. The intruder configuration is described by $N_\nu=8$ neutron bosons and $N_\pi=3$ proton bosons. The large set of parameters as occurring in such a mixing calculation gives great freedom in the choice of the parameters. We determine these parameters as follows. For $H_{\text{IBM-2}}(N_\pi=1)$ the parameters of Ref. 5 are taken with the exception of the values of χ_ν and χ_π as will be discussed below. The values for the $N_\pi=3$ Hamiltonian are determined as follows. First, expression (2) is used to determine the value of Δ. The value of the 2p-2h unperturbed single-particle energy, $2(\varepsilon_p-\varepsilon_h)=8.780$ MeV, is determined from the experimental one-proton separation energies[8] using the prescription of Ref. 9. The pairing correction is obtained from the experimental one- and two-proton separation energies[8] using the prescription of Ref. 3 and yields $\Delta E_{\text{pairing}}=4.366$ MeV. Finally, the

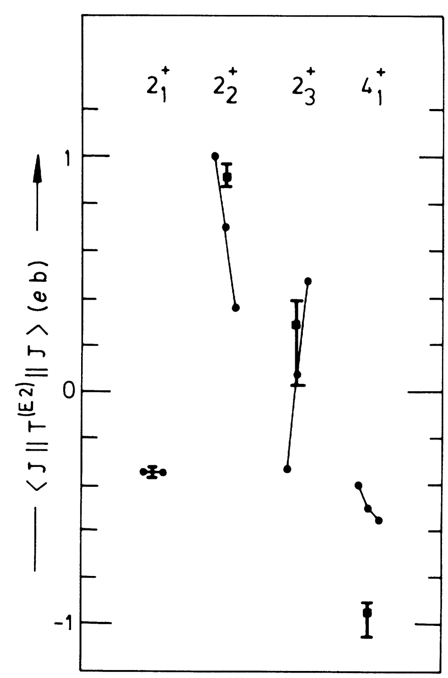

FIG. 2. See the caption to Fig. 1, but now for the experimental diagonal $E2$ reduced matrix elements.

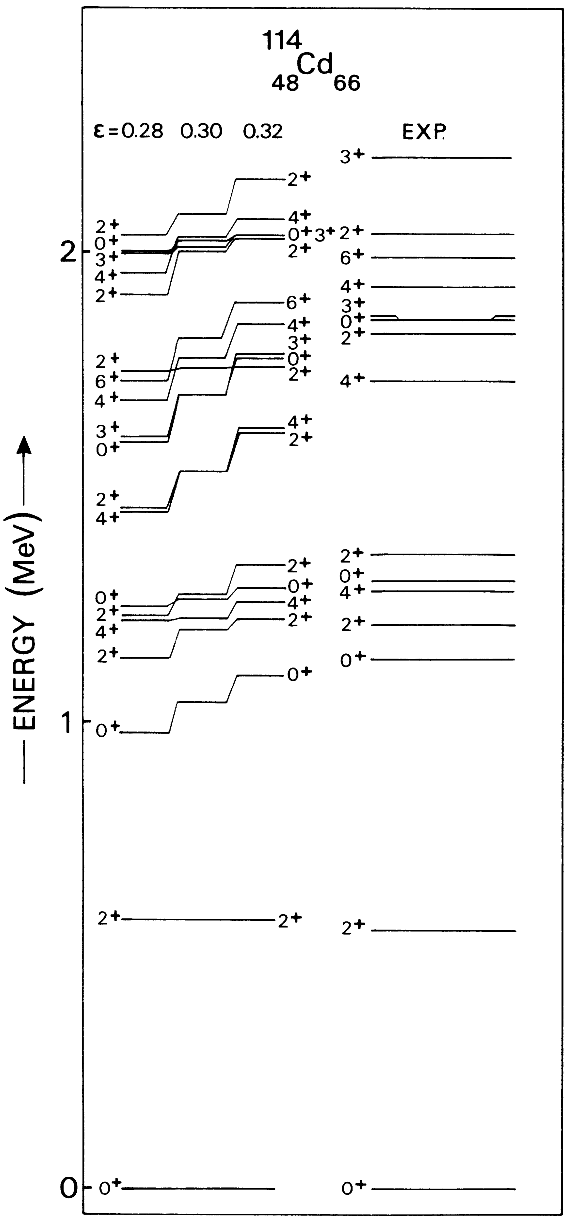

FIG. 3. Comparison between the calculated energy levels in ^{114}Cd (left part of the figure) for ε_d ($N_\pi=3$) equal to 0.28 MeV (left-hand side), 0.30 MeV (middle), and 0.32 MeV (right-hand side) and the experimental data (right part of the figure).

monopole correction is obtained from Ref. 10 and equals $\Delta E_M = -0.412$ MeV. So we obtain the value $\Delta = 2(\varepsilon j - \varepsilon j') - \Delta E_{\text{pairing}} + \Delta E_M = 4.0$ MeV. Here a difference with the previous descriptions[5,11,12] occurs where a rough estimation of $\Delta \approx 5$ MeV was used. This is already an indication that the additional binding provided by an *ad hoc* increase in κ in previous calculations will, in our calculation, come out quite naturally. The last correction in expression (1) is due to the *p-n* quadrupole collectivity and will be given by the extra binding energy of the $N_\pi = 3$ configuration with respect to the $N_\pi = 1$ configuration. This contribution is essentially dominated by the neutron-proton quadrupole interaction energy, although the other interaction terms also contribute. Having determined Δ in a precise way, we now start from the parameter set as obtained in Ref. 5. In order to move the intruder state to the right energy the value of κ has to be lowered from -0.19 MeV to -0.15 MeV. This is *exactly* the value obtained for the Ru isotopes, with six proton holes outside the $Z = 50$ core, by Van Isacker and Puddu[13] and is indeed very close to the value of -0.14 MeV as used for the $N_\pi = 1$ configuration. Finally we have adjusted the *d*-boson energy ε_d in order to reproduce the $B(E2)$ values as discussed below.

As mentioned before we modified the values of χ_ν and χ_π with respect to those used in Ref. 5 for the normal as well as for the intruder configuration. This is motivated by the recently measured quadrupole moments of the three lowest 2^+ states: $Q(2_i^+) = -0.27$ *e* b $(+0.01, -0.02)$, $+0.69$ *e* b $(+0.03, -0.04)$, $+0.22$ *e* b $(+0.08, -0.20)$.[7] The large and positive quadrupole moment of the second excited 2^+ state is in contradiction to all previous calculations where a negative quadrupole moment of -0.31 *e* b was obtained. Since the quadrupole moments in IBM-2 are mainly determined by the sum $\chi_\nu + \chi_\pi$, the experimental values of $Q(2_1^+)$ and $Q(2_2^+)$ are used to determine the sum $\chi_\nu + \chi_\pi$ as -0.95 for $N_\pi = 1$ and 0.25 for $N_\pi = 3$. This was done before mixing the two configurations. Furthermore, using the value for $\chi_\nu = -0.90$, obtained by Sambataro for ^{114}Cd,[14] for the normal configuration and $\chi_\nu = -0.50$, obtained by Van Isacker and Puddu for ^{110}Ru,[13] for the intruder configuration, we obtain the values $\chi_\pi = -0.05$ for the normal and $\chi_\pi = 0.75$ for the intruder states. Here, we should mention that the resulting spectra and electromagnetic properties are not very sensitive to the fact that χ_π is different in the $N_\pi = 1$ and $N_\pi = 3$ system, as long as the above sums are fulfilled. They are both somewhat larger than the values of Refs. 14 and 13, namely, $\chi_\pi = -0.2$ and 0.4. However, in the calculation of ^{110}Ru no transitions were fitted and Fahlander *et al.*[7] measured a somewhat smaller quadrupole moment for the first excited state in ^{114}Cd than the one used by Sambataro to fit χ_π.

In order to calculate the *E*2 transitions for the mixed states we use the quadrupole transition operator as defined in Ref. 5:

$$T^{(E2)} = e_1(Q_\pi + Q_\nu)_1 + e_3(Q_\pi + Q_\nu)_3 \,, \tag{5}$$

where $(Q_\pi + Q_\nu)_i$ $(i = 1, 3)$ is the quadrupole operator acting in each subsystem. The effective charge e_3 was taken to be 0.103 *e* b, which is the value obtained in the description of the Ru isotopes.[13] The charge e_1 was fitted in order to reproduce the $B(E2; 0_1^+ \to 2_1^+)$ giving $e_1 = 0.086$ *e* b. The ratio $e_3/e_1 = 1.2$ obtained in this way is smaller than the one of 1.6 used in Ref. 5.

Using $\alpha = \beta = 0.08$ MeV as obtained in Ref. 5, we finally adjust the boson energy ε_d for the intruder configuration taking into account experimental energies and the $B(E2)$ values. It emerged that we have to lower ε_d from the value of 0.6 MeV, given for ^{110}Ru in Ref. 13,

TABLE II. The mixed wave functions as obtained from the present calculations. For each state the wave function is given for ε_d $(N_\pi = 3)$ equal to 0.28 MeV (first row), 0.30 MeV (second row), and 0.32 MeV (third row). The notation $|i\rangle, |\bar{i}\rangle$ denotes the *i*th state with J^π in the configuration spaces $N_\pi = 1$ and $N_\pi = 3$, respectively.

	$N_\pi = 1$				$N_\pi = 3$			
J_i^π	$\lvert 1\rangle$	$\lvert 2\rangle$	$\lvert 3\rangle$	$\lvert 4\rangle$	$\lvert\bar{1}\rangle$	$\lvert\bar{2}\rangle$	$\lvert\bar{3}\rangle$	$\lvert\bar{4}\rangle$
0_2^+	−0.073	0.370	−0.034	−0.046	0.924	0.027	0.001	0.005
	−0.067	0.491	−0.033	−0.044	0.866	0.035	0.001	0.008
	−0.058	0.644	−0.029	−0.039	0.760	0.045	0.001	0.011
0_3^+	−0.021	−0.919	−0.042	−0.019	0.367	−0.127	−0.035	−0.024
	−0.030	−0.864	−0.046	−0.027	0.488	−0.102	−0.027	−0.022
	−0.041	−0.759	−0.054	−0.036	0.641	−0.081	−0.021	−0.019
2_2^+	−0.126	−0.376	−0.115	−0.037	0.899	0.144	−0.004	−0.019
	0.070	0.747	0.076	0.021	−0.605	−0.254	−0.007	0.033
	−0.028	−0.915	−0.032	−0.007	−0.306	−0.257	−0.013	0.037
2_3^+	−0.077	0.796	−0.089	−0.036	0.382	−0.451	−0.025	0.036
	−0.112	0.559	−0.149	−0.050	0.765	−0.253	−0.023	0.022
	−0.117	0.283	−0.196	−0.059	−0.921	0.126	0.020	−0.010
4_1^+	0.883	−0.010	0.032	0.002	0.459	−0.083	0.009	−0.033
	0.935	−0.006	0.024	0.002	0.344	−0.077	0.009	−0.033
	0.959	−0.004	0.022	0.001	0.272	−0.070	0.009	−0.032

to 0.3 MeV. The parameters so determined are listed in Table I. The reduction of ε_d with a factor of 2 is similar to that which obtained in a description of backbending in the Dy isotopes.[15] When varying around $\varepsilon_d=0.3$ MeV all features of the quintuplet of states around 1.2 MeV can be reproduced although not simultaneously for the same value of ε_d. This is due to the fact that all five states are mixing at the same time. To show this we give in Figs. 1 and 2 the experimental $E2$ matrix elements[7] and the theoretical values for $\varepsilon_d=0.28$, 0.30, and 0.32 MeV, respectively. In Fig. 3 we also show the corresponding energy spectra obtained with these three values. Of the three values the last one best reproduces the excitation energies. In general, the $E2$ matrix elements are described in a qualitative way. They still show quite a strong dependence on the admixtures of the normal and intruder configuration as well on admixtures between the states of both configurations among themselves. This is illustrated in Table II where we show the wave functions for the states belonging to the quintuplet around 1.2 MeV. Since in ^{114}Cd this mixing happens at the same time for the second and third 0^+ state, the second, third, and fourth 2^+ state, and the first and second 4^+ state, an exact reproduction of all the $E2$ matrix elements is hardly achievable. Finally, we mention that in our calculation we can make the ratio

$$B(E2;0_3^+\rightarrow 2_2^+)/B(E2;0_3^+\rightarrow 2_1^+)$$

go to infinity due to an exact cancellation of the normal and intruder contribution making the denominator equal to zero. The very large experimental value of 40 000 for this ratio is, in this sense, an indication of the mixing as pointed out in Ref. 11, albeit in a more schematic way.

The values of α and β, as used above, are still somewhat arbitrary concerning their microscopic understanding. In order to show their influence we have calculated the spectrum and $E2$ matrix elements using the values $\alpha=-0.25$ MeV and $\beta=-0.16$ MeV as obtained from the shell-model arguments used in Ref. 5. The larger mixing matrix element deteriorates the fit of the energy spacings, due to the fact that levels with the same J^π repel each other more. On the other hand, the $E2$ matrix elements are reproduced rather well by this calculation, as shown in Fig. 4. In this figure, we also give the results for the simple vibrational mixing calculation[7] and the calculation of Ref. 5. Finally, for completeness, we compare in Table III the theoretical and known experimental $E2$ matrix elements for the decay of the higher-lying 0_4^+, 2_4^+, and 2_5^+ states.

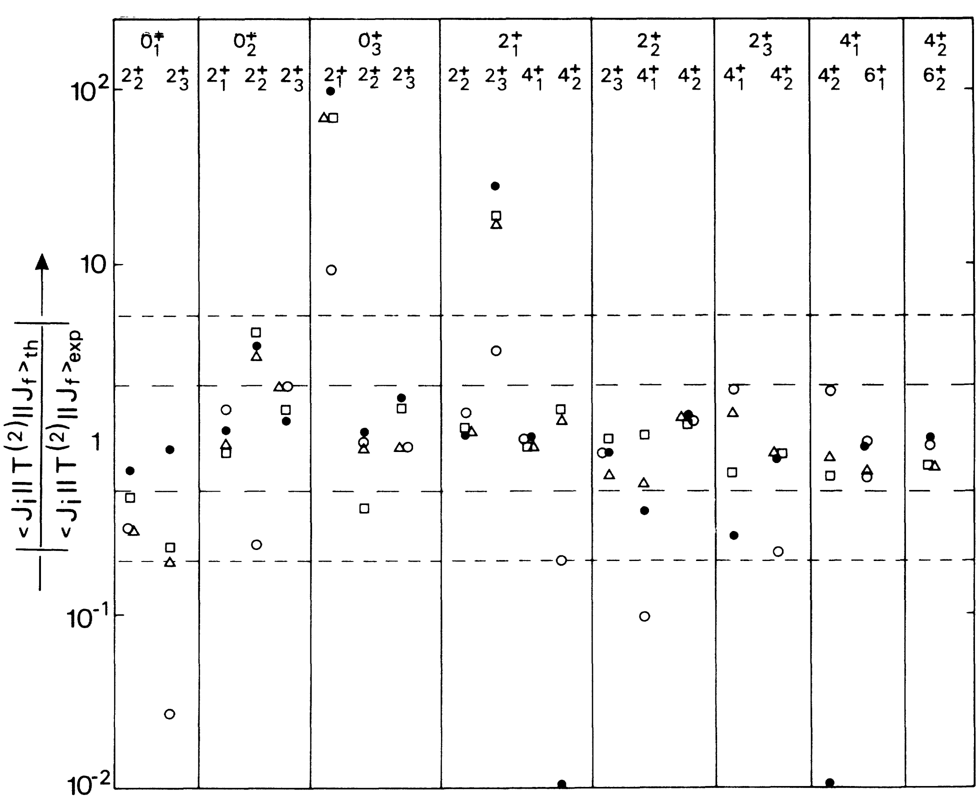

FIG. 4. The ratio between the calculated and experimental $E2$ reduced matrix element for different theoretical calculations. The empty circles are the present mixing calculations with $\alpha=-0.25$ MeV and $\beta=-0.16$ MeV (○), the triangles the present mixing calculations using the parameters of Table I (△), the squares the previous mixing calculation (□), the full circles the results of the simple vibrational mixing calculation (●); both are taken from Ref. 7. The two dashed lines correspond to a deviation with a factor of 2 and with a factor of 5, respectively.

TABLE III. Comparison between the theoretical and known experimental $E2$ matrix elements for the decay of the higher-lying 0_4^+, 2_4^+, and 2_5^+ states in ^{114}Cd.

J_i^+ J_f^+	$\lvert\langle J_i^\pi \Vert E2 \Vert J_f^\pi \rangle\rvert$ (e b) Theory	Experiment
$0_4^+ \to 2_1^+$	0.023	0.09[a]
2_3^+	0.259	0.26[a]
$2_4^+ \to 0_1^+$	0.0067	0.056
0_2^+	0.171	0.86
0_3^+	0.047	0.87
2_1^+	0.119	0.08
2_2^+	0.47	0.28[b]
2_3^+	1.197	1.33
$2_5^+ \to 0_1^+$	0.030	0.042
0_3^+	0.59	0.61
0_4^+	0.0	0.61[a]
2_1^+	0.0105	0.33[b]
2_2^+	0.205	0.28[b]

[a]The experimental matrix elements are uncertain because of the influence from uncertain signs of interference terms.
[b]The experimental matrix elements are obtained assuming no $M1$ admixture following Ref. 7.

V. CONCLUSION

We have shown that by using a description, expressed by (1), in order to determine the relative excitation energy of intruder configurations with respect to the normal configuration, one can obtain a consistent value for the quadrupole strength κ in both the normal and intruder state. After reducing the d-boson energy for the intruder state by a factor of 2, as done in Ref. 15, a good description of the energy spectra is obtained. In order to calculate the $E2$ transition matrix elements we used the measured quadrupole moments in ^{114}Cd to determine the quadrupole operator and determined the effective charges from the $B(E2;0_1^+ \to 2_1^+)$ values in ^{114}Cd and the Ru isotopes. From this, a good description of the mixing of the lowest intruder states with the normal deformed states can be obtained. We have not discussed the higher-lying levels around 2.5 MeV, since we expect these states to eventually show 4p-6h contributions. It would be interesting to see whether an approach along the lines of Ref. 15 would be able to explain the states at higher energy. Calculations for the whole region of even-even Cd nuclei ($98 \le A \le 130$) are in progress.

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[1]K. Heyde, P. Van Isacker, M. Waroquier, J. L. Wood, and R. A. Meyer, Phys. Rep. **102**, 291 (1983), and references therein.
[2]P. Van Duppen, E. Coenen, K. Deneffe, M. Huyse, and J. L. Wood, Phys. Rev. C **35**, 1961 (1987).
[3]K. Heyde, J. Jolie, J. Moreau, J. Ryckebush, M. Waroquier, P. Van Duppen, M. Huyse, and J. L. Wood, Nucl. Phys. **A466**, 189 (1987).
[4]P. Duval and B. R. Barrett, Phys. Lett. **100B**, 223 (1981).
[5]K. Heyde, P. Van Isacker, M. Waroquier, G. Wenes, and M. Sambataro, Phys. Rev. C **25**, 3160 (1982).
[6]R. F. Casten, Phys. Lett. **152B**, 145 (1985); Phys. Rev. Lett. **54**, 1991 (1985); Nucl. Phys. **A443**, 1 (1985).
[7]C. Fahlander, A. Bäcklin, L. Hasselgren, A. Kavka, V. Mittal, L. E. Svensson, B. Varnestig, D. Cline, B. Kotlinski, H. Grein, E. Grosse, R. Kulessa, C. Michel, W. Spreng, H. J. Wollersheim, and J. Stachel, Nucl. Phys. **A485**, 327 (1988).
[8]A. H. Wapstra and G. Audi, Nucl. Phys. **A432**, 1 (1985).
[9]K. Heyde, J. Jolie, J. Moreau, J. Ryckebush, M. Waroquier, and J. L. Wood, Phys. Lett. B **176**, 255 (1986).
[10]K. Heyde, P. Van Isacker, R. F. Casten, and J. L. Wood, Phys. Lett. **155B**, 303 (1985).
[11]A. Aprahamian, D. S. Brenner, R. F. Casten, R. L. Gill, A. Piotrowski, and K. Heyde, Phys. Lett. **140B**, 22 (1984).
[12]A. Mheemeed, K. Schreckenbach, G. Barreau, H. R. Faust, H. G. Börner, R. Brissot, P. Hungerford, H. H. Schmidt, H. J. Scheerer, T. von Egidy, K. Heyde, J. L. Wood, P. Van Isacker, M. Waroquier, G. Wenes, and M. L. Stelts, Nucl. Phys. **A412**, 113 (1984).
[13]P. Van Isacker and G. Puddu, Nucl. Phys. **A348**, 124 (1980).
[14]M. Sambataro, Nucl. Phys. **A380**, 365 (1982).
[15]K. Heyde, J. Jolie, P. Van Isacker, J. Moreau, and M. Waroquier, Phys. Rev. C **29**, 1428 (1984).