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Microscopic study of $p+\alpha$ bremsstrahlung

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Bremsstrahlung double cross sections for $p+\alpha$ collisions are computed by using the wave functions obtained from a single-configuration resonating-group calculation. The photon-emission operator employed is translationally invariant and the center-of-mass motion is strictly eliminated from the transition amplitude. Analytical expressions for the reduced matrix elements are derived without invoking the long-wavelength approximation. With no adjustable parameters, it is found that there is a very good agreement between calculated and experimental values. Effects of the $p+\alpha$, $\frac{3}{2}^-$ resonance show up distinctly in the cross-section curve when this resonance occurs in the final channel. On the other hand, the presence of the very broad $\frac{1}{2}^-$ level becomes detectable from the bremsstrahlung cross-section result only when there is a proper choice of the geometrical arrangement of the particle detectors.

I. INTRODUCTION

Recently, we performed a microscopic study of ^{3}He$+\alpha$ bremsstrahlung[1] (hereafter referred to as LTK) by using wave functions obtained from a single-configuration resonating-group calculation.[2] In this study, a careful formulation of the problem was carried out, in which we not only derived analytical expressions for both the direct and the exchange parts of the reduced matrix elements without invoking the oft-used long-wavelength approximation, but also strictly eliminated the total center-of-mass motion from the transition amplitude. The result which we obtained for the bremsstrahlung double cross section turned out to be indeed quite reasonable. At the ^{3}He incident energy of 7.4 MeV, the value calculated in the case where the ^{3}He and the α particle are detected at laboratory angles of 37° in the coplanar geometry is equal to 8.3 μb/sr^2, which agrees satisfactorily with the only existing measured value[3] of 12.6±3.4 μb/sr^2. Considering the fact that there are *no* adjustable parameters in our calculation, we feel that this favorable comparison points rather convincingly to the usefulness of employing resonating-group wave functions in describing the features of electromagnetic transitions between scattering states. This is gratifying, since successful conclusions have also been reached in our previous investigations, employing formulations of similar kinds, of form factors of ^{7}Li,[4] the ^{3}He$+\alpha$ radiative-capture reaction,[5] and other electromagnetic- and weak-transition problems in the seven-nucleon systems.[6]

In this investigation, we continue the bremsstrahlung study by considering the case of $p+\alpha$ collision. This is a more interesting problem from the experimental viewpoint, since there are more data points available[7] and the accuracy of the measured result is considerably higher than that in the ^{3}He$+\alpha$ case. On the theoretical side, the situation is also quite favorable. For a large range of energy up to about 18 MeV in the c.m. system, there are no open reaction channels. In addition, the composite nucleus involved, i.e., the α particle, has a low compressibility. Both of these facts indicate that a single-configuration $p+\alpha$ resonating-group calculation should be quite adequate in the low-energy region. Indeed, it was found that such a calculation[8] does yield phase-shift results in very good agreement with value obtained from a detailed analysis[9] of the $p+\alpha$ scattering data. Therefore, the resultant wave functions can be confidently used to calculate bremsstrahlung cross sections, and a subsequent comparison with experiment can yield information concerning the adequacy of the resonating-group nonlocal potential in describing the off-shell behavior of the $p+\alpha$ interaction.

In the next section, we present a brief discussion of the formulation of the $p+\alpha$ bremsstrahlung problem. The resonating-group phase shifts will be analyzed by employing a one-level resonance formula, the purpose being to determine the location and the width of the broad $^2P_{1/2}$ level in the low-excitation region. Results for the bremsstrahlung double cross sections are given in Sec. III, where a comparison with experimental data is also made. Finally, in Sec. IV, we summarize the findings of this investigation and make some concluding remarks.

II. BRIEF DISCUSSION OF THE FORMULATION

Bremsstrahlung cross sections can be calculated by using formulas given in Refs. 1, 4, 5, which were derived for the general case of an $A+B$ system, with A and B being s-shell clusters. In our present study of the $p+\alpha$ system, where A and B represent the α particle with $N_A=4$ and the proton with $N_B=1$, respectively, all that one needs to do is to replace Eq. (33) in (LTK) by the equation

$$\frac{1}{\sqrt{N!}}\langle \phi_A\phi_B G_f(\mathbf{R}')Z(\mathbf{R}_{\mathrm{c.m.}})|H_e(\mathbf{k}_\gamma,\varepsilon^*_\mu)|\mathcal{A}\{\phi_A\phi_B G_i(\mathbf{R}')\}Z(\mathbf{R}_{\mathrm{c.m.}})\rangle = Z_A F_0^b + Z_B F_0^c + F_1^b + (F_1^{a1}+F_1^{a2}) , \quad (1)$$

and to make some trivial changes in, e.g., Eq. (50) of LTK, and so on.

For our discussion below, we need kinematical relations given by Eqs. (15) and (16) of LTK (see also Ref. 10). These relations are

$$E_\gamma = E_p\left[1-\frac{4\sin^2\theta_\alpha+\sin^2\theta_p}{4\sin^2(\theta_\alpha+\theta_p)}\right] , \quad (2)$$

$$E_f = E_i - E_\gamma , \quad (3)$$

where E_p is the incident proton energy in the laboratory system, E_γ denotes the photon energy, and E_i and E_f represent the relative energies in the c.m. frame for the $p+\alpha$ system in the initial and final channels, respectively. For the experimental setup,[7] a coplanar-asymmetric arrangement for the detectors was used, with the protons observed at a laboratory angle of 70° (i.e., $\theta_B=70°$) and the α particles at an opposite angle of 30° (i.e., $\theta_A=30°$) with respect to the beam axis. With such an angle combination, E_γ has a rather large value equal to $0.515E_p$, indicating that the bremsstrahlung process does effectively probe the off-shell structure of the $p+\alpha$ interaction.

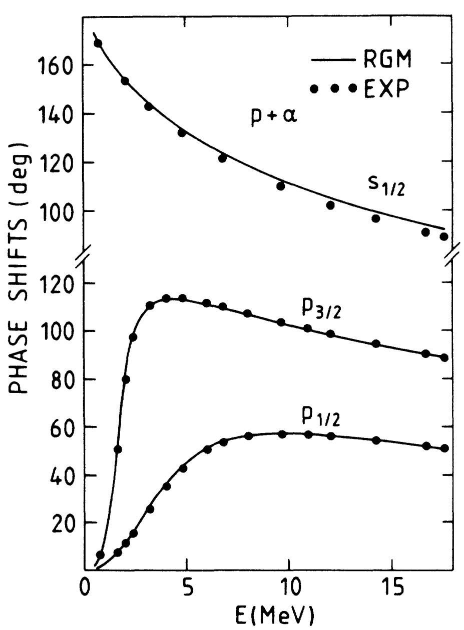

FIG. 1. Phase shifts for $p+\alpha$ scattering obtained with potential set III of Ref. 8. Data points (●) represent the empirical phase shifts given in Ref. 9.

Microscopic wave functions used for the computation of the $p+\alpha$ bremsstrahlung double cross sections are those obtained from a single-configuration resonating-group calculation employing potential set III of Ref. 8. Since this calculation has already been discussed at length previously, we refer the readers to Ref. 8 for details. We show here in Fig. 1 only a comparison between the calculated and empirical[9] values for the S- and P-wave phase shifts, where one notes that the agreement is very satisfactory.

In Fig. 1, the ${}^2P_{1/2}$ phase shifts show the presence of a resonance level with a large width. To determine the properties of this broad level, we have analyzed the phase-shift result with a single-level resonance formula. The hard-sphere radius R is chosen to be 5 fm. This particular value is assumed, because the resultant resonance energies of the ${}^2P_{3/2}$ states in the $n+\alpha$ and $p+\alpha$ systems turn out to be 0.90 MeV and 1.86 MeV, respectively, which are close to the empirical values of 0.89 MeV and 1.97 MeV.[11] Using this value of R, we find that the resonance energy and width of the $p+\alpha$, ${}^2P_{1/2}$ level are equal to 4.55 MeV and 5.2 MeV, respectively. These values will be used in the following to discuss the features of the bremsstrahlung cross-section results.

III. RESULTS

Experiments[7] for $p+\alpha$ bremsstrahlung were performed for incident proton energies between 6.9 and 12 MeV, with the angle combination $(\theta_\alpha, \theta_p)=(30°,70°)$ as mentioned in the preceding section. In Table I, we tabulate the detailed results of our calculation at $E_p=9.0$ MeV, with the main purpose being to investigate the convergence behavior of the double cross section $d^2\sigma/d\Omega_p d\Omega_\alpha$ with respect to the maximum values of the $p+\alpha$ intercluster relative orbital angular momenta l_i and l_f used in the calculation. The multipoles that we have included are $E1$, $E2$, and $M1$. As can be seen from this table, all the results show convergence already at $l_i(\max)=l_f(\max)=3$. As expected, $E1$ is the dominant transition. The $E2$ contribution is also fairly significant, but is about an order of magnitude smaller. On the other

TABLE I. Double cross section $d^2\sigma/d\Omega_p d\Omega_\alpha$ in μb/sr^2 for $E_p=9.0$ MeV and $(\theta_\alpha,\theta_p)=(30°,70°)$.

	$d^2\sigma/d\Omega_p d\Omega_\alpha$ (μb/sr^2)		
$l_{i,f}$(max)	$E1$	$E1+E2$	$E1+E2+M1$
1	1.65	1.79	1.66
2	26.38	27.76	27.65
3	26.28	28.47	28.33
4	26.28	28.47	28.33
5	26.33	28.53	28.39
6	26.32	28.51	28.37
7	26.31	28.49	28.35
8	26.31	28.50	28.36

hand, the $M1$ transition seems to yield little contribution. In addition, we have investigated the contributions from $E3$ to $M2$, and found that these transitions are entirely unimportant. Therefore, throughout the rest of this section, we shall show results which are calculated with $l_i(\max)=l_f(\max)=8$, and with $E1+E2+M1$ transitions.

We show in Fig. 2 calculated double cross sections for $(\theta_\alpha,\theta_p)=(30°,70°)$ in the E_p range from 1.5 to 24 MeV. For convenience in discussion, the energy scale E_f, with $E_f=0.285E_p$ for this particular angle combination, is also shown. Here we note that there are the following interesting features:

(i) There is a small peak at $E_p=2.3$ MeV or $E_i=1.84$ MeV. This barely visible peak is associated with the presence of the $^2P_{3/2}$ or $\frac{3}{2}^-$ resonance in the initial channel. By examining our detailed numerical results for the reduced matrix elements, we find that the important transitions responsible for this peak structure are $\frac{3}{2}^- \rightarrow \frac{5}{2}^+$ and $\frac{3}{2}^- \rightarrow \frac{3}{2}^+$.

(ii) A large peak occurs at $E_p=7.5$ MeV or $E_f=2.14$ MeV. The latter value is close to the resonance energy of 1.86 MeV for the $\frac{3}{2}^-$ level. Therefore, we can conclude again that it is related to the presence of the $p+\alpha$, $\frac{3}{2}^-$

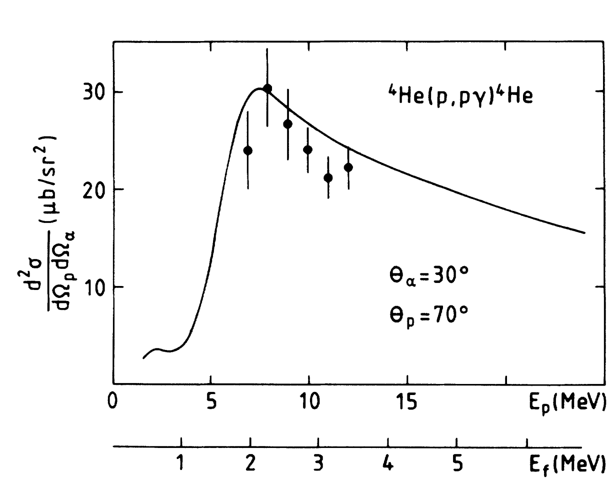

FIG. 2. Double cross section $d^2\sigma/d\Omega_p d\Omega_\alpha$ at $(\theta_\alpha,\theta_p)=(30°,70°)$ as a function of E_p. Data points (●) are those of Ref. 7.

resonance, but this time in the final channel. The dominant transitions in this case are $\frac{5}{2}^+ \rightarrow \frac{3}{2}^-$ and $\frac{3}{2}^+ \rightarrow \frac{3}{2}^-$.

(iii) There are no discernible effects arising from the $^2P_{1/2}$ or $\frac{1}{2}^-$ resonance in either the initial or the final channel. This is clearly a consequence of the fact that the energy width of the $\frac{1}{2}^-$ level ($\Gamma=5.2$ MeV) is much larger than that of the $\frac{3}{2}^-$ level ($\Gamma=1.5$ MeV). Therefore, our calculation shows that measuring bremsstrahlung cross sections may not be a sensitive way to detect the presence of very broad levels.

Similar to the ^{3}He$+\alpha$ case discussed in LTK, we observe here also that the bremsstrahlung double cross-section peak arising from a resonance in the final channel is much more prominent that that arising from the same resonance in the initial channel. The reason for this is as follows. In the $p+\alpha$ case, E_f for the initial-channel peak is equal to 0.65 MeV, which is much smaller than the value of E_i equal to 6.0 MeV for the final-channel peak. This suggests that the large difference in these peak structures can be explained by invoking arguments based on Coulomb-plus-centrifugal-barrier effects, in the way as has been discussed in LTK.

A comparison between the calculated and experimental[7] bremsstrahlung double cross sections at $(\theta_\alpha,\theta_p)=(30°,70°)$ is also shown in Fig. 2. Here one finds that the agreement is quite remarkable. The calculated cross sections overpredict by only 5%, a slight discrepancy which probably comes from our use of wave functions obtained from a single-configuration resonating-group calculation that does not take into account the specific distortion of the α cluster. Although it is expected that, because of the extreme stability of the α particle, the specific distortion effect will not have a large influence, we do feel that this small discrepancy of 5% in magnitude can be removed if we adopt even better wave functions from a multiconfiguration resonating-group formulation which includes not only the $p+\alpha$ cluster configuration, but also $d+{}^3$He and $p+\alpha^*$ cluster configurations.[12,13]

To see how bremsstrahlung double cross sections depend on the geometrical arrangement of the detectors, we have also calculated with an arbitrary choice of $(\theta_\alpha,\theta_p)=(50°,30°)$. For this particular angle combination, E_γ and E_f are equal to $0.33E_p$ and $0.47E_p$, respectively. The result is shown in Fig. 3, where one finds that the cross-section curve exhibits a distinct peak at $E_p=4.3$ MeV or $E_f=2.0$ MeV. This peak arises from the presence of the $\frac{3}{2}^-$ resonance in the final channel. In contrast, the effect of the $\frac{3}{2}^-$ resonance in the initial channel can no longer be seen with this angle combination, because the curve is sharply rising in the energy region where this resonance in the initial channel occurs. The most interesting observation is, however, that there appears now a shoulder in the cross-section curve. This shoulder is centered at E_f around 5 MeV, indicating that it arises from the presence of the $\frac{1}{2}^-$ level in the final channel. It covers a wide range of energies, which obviously reflects the fact that the $\frac{1}{2}^-$ level has a large energy width. Thus, we do find that, with a judicious choice of the angle combination, even the effects of a broad level

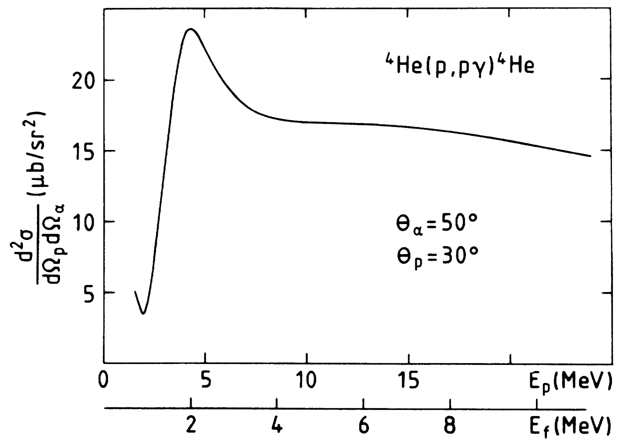

FIG. 3. Double cross section $d^2\sigma/d\Omega_p d\Omega_\alpha$ at $(\theta_\alpha,\theta_p)=(50°,30°)$ as a function of E_p.

can be detected, although not too dramatically. Therefore, for future bremsstrahlung measurements, it seems important to us that the experiment should be performed with more than one angle combination, because such an effort should be helpful towards extracting more accurate nuclear-structure information.

IV. CONCLUSION

In this investigation, we have computed the $p+\alpha$ bremsstrahlung double cross sections by using wave functions obtained from a single-configuration resonating-group calculation. With *no* adjustable parameters, it is found that the calculated and measured results agree very well with each other. The only minor discrepancy is that the calculated values are about 5% too large, which can be attributed to the fact that the specific distortion effect of the α cluster has not been taken into account. This latter effect, expected to be small because of the extreme stability of the α particle, can be properly considered by including not only the $p+\alpha$ but also other cluster configurations in the resonating-group formulation.

Effects of the $p+\alpha$, $\frac{3}{2}^-$ resonance, which has a comparatively narrow width of about 1.5 MeV, show up distinctly in the cross-section curve when this resonance occurs in the final channel. On the other hand, when this resonance occurs in the initial channel, there do appear some discernable features, but in a much less dramatic way and also not at all angle combinations of the particle detectors. Similarly, we find that a broad level in the final channel, such as the $\frac{1}{2}^-$ level with a large width of about 5 MeV, can show up in the cross-section curve, if the angle combination in the experimental arrangement is properly chosen.

The success of our present investigation indicates that the $p+\alpha$ effective interaction obtained in a resonating-group calculation does have the correct off-shell behavior. This interaction has, however, a complicated nonlocal structure and is difficult to apply directly to problems such as a three-body nucleon + nucleon + α calculation. As a consequence, considerable effort has been spent to construct local potentials which are nearly phase equivalent to the intercluster interaction obtained from the resonating-group calculation. For example, Horiuchi[14] has proposed a semiclassical treatment of the nonlocal potential by using the Wentzel-Kramers-Brillouin (WKB) approximation. The resultant $p+\alpha$ deep local potential[15] contains one 1s Pauli-forbidden state, but does produce quite well the resonating-group phase shifts. The off-shell behavior of such a constructed local potential has, however, not been tested with respect to the resonating-group results. Our opinion is that a bremsstrahlung test should be carried out to determine the correctness of this local potential. It has also been proposed recently[16,17] that the procedure of supersymmetric transformation can be used to construct an exactly phase-equivalent shallow local potential which contains no Pauli-forbidden state from a deep local potential. Bremsstrahlung calculations should also be performed here in order to test against the standards set by the resonating-group theory.

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