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Pion photoproduction resonance couplings in the second resonance region

Richard A. Arndt, Ron L. Workman, Zhujun Li, and L. David Roper
Physics Department, Virginia Polytechnic Institute and State University, Blacksburg, Virginia 24061
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Resonance couplings for the resonant states $P_{33}(1232)$, $P_{11}(1440)$, $D_{13}(1520)$, and $S_{11}(1535)$ are extracted from a recent multipole analysis of pion photoproduction data. We calculate photoproduction residues at the resonance pole in the complex plane for these amplitudes, in addition to the resonance widths at the on-shell resonance energies.

In a recent analysis[1] of pion photoproduction data, we have obtained energy-dependent and energy-independent multipole solutions up to 1 GeV laboratory photon energy. Details of this analysis are available through the Scattering Analysis Interactive Dial-in (SAID) system. Copies of the SAID program for VAX computers and information about the dial-in can be obtained from the authors.

Here we determine resonance couplings for the $P_{33}(1232)$, $P_{11}(1440)$, $D_{13}(1520)$, and $S_{11}(1535)$ resonances. We have fitted our energy-independent multipole amplitudes with a Breit-Wigner-plus-background parametrization. In this way we determined the resonance amplitude (A_r), energy (W_0), and full width (Γ_0) (see below). Using the relations quoted in Berends and Donnachie,[2] we calculate the resonance couplings from our electric and magnetic multipoles via the relations

$$\begin{aligned} A_{l+}^{1/2} &= -\tfrac{1}{2}[(l+2)\bar{E}_{l+} + l\bar{M}_{l+}] \,, \\ A_{l+}^{3/2} &= \tfrac{1}{2}\sqrt{l(l+2)}[\bar{E}_{l+} - \bar{M}_{l+}] \,, \\ A_{(l+1)-}^{1/2} &= -\tfrac{1}{2}[l\bar{E}_{(l+1)-} - (l+2)\bar{M}_{(l+1)-}] \,, \\ A_{(l+1)-}^{3/2} &= -\tfrac{1}{2}\sqrt{l(l+2)}[\bar{E}_{(l+1)-} + \bar{M}_{(l+1)-}] \,, \end{aligned} \tag{1}$$

where the above barred multipoles are related to our quoted amplitudes[1] (E,M) evaluated at the resonance energy (W_0) through the relation

$$(\bar{E},\bar{M}) = C\left[\frac{(2j+1)\pi q_0 W_0 \Gamma_0^2}{k_0 M_N \Gamma_\pi}\right]^{1/2} \frac{(E,M)}{\hbar c} \,. \tag{2}$$

[Equations (1) have different signs and factors from similar equations used in Ref. 1.]

The factor C in Eq. (2) is $\sqrt{\frac{2}{3}}$ for isospin $\frac{3}{2}$ states and $-\sqrt{3}$ for isospin $\frac{1}{2}$ states. Here M_N is the nucleon mass, Γ_0 is the full width, and Γ_π is the πN elastic width. The variables q_0 and k_0 are, respectively, the center-of-mass pion and photon momenta at the resonance energy. The photoproduction amplitudes are assumed to have the background plus Breit-Wigner resonance form

$$A = A_I(1+iT_\pi) + A_r\left[\frac{k_0 q_0}{kq}\right]^{1/2} \frac{W_0\sqrt{\Gamma\Gamma_\gamma}}{W_0^2 - W^2 - iW_0\Gamma} \,. \tag{3}$$

The energy-dependence of Γ and Γ_γ is

$$\begin{aligned} \Gamma &= \Gamma_0\left[\frac{q}{q_0}\right]^{2l+1}\left[\frac{q_0^2+z^2}{q^2+z^2}\right]^l \\ \Gamma_\gamma &= \Gamma_0\left[\frac{k}{k_0}\right]^2\left[\frac{k_0^2+z^2}{k^2+z^2}\right] \,. \end{aligned} \tag{4}$$

The background part of the amplitude, $A_I(1+iT_\pi)$, has the same parametrization form as in our energy-dependent analysis;[1] its parameters and the parameters A_r, Γ_0, z, and W_0 were varied to fit our energy-independent amplitudes. Since the energy-independent solution satisfied Watson's theorem at low energies in our analyses[1] and we fitted that solution with the same background form plus a Breit-Wigner (BW) resonance form, Watson's theorem should be satisfied approximately at low energies for the entire amplitude and for the background and resonance parts separately.

In Table I we give our values and other determinations of the resonance couplings. The BW fit values are the last term of Eq. (3) evaluated at resonance energy (thus, they are pure imaginary). The SP values are the imaginary parts of the energy-dependent solution's amplitudes at the BW fit resonance energy. For the P_{33} and D_{13} resonances, resonance couplings derived from the Breit-Wigner parametrization are in good agreement with those obtained directly from the energy-dependent solution. For the P_{11} resonance the comparison is reasonably good. The S_{11} resonance, however, is masked in a complicated background which includes the η cusp. In this case we feel that the Breit-Wigner-plus-background form is very sensitive to the method used to parametrize the background. For the S_{11} resonance the resonance parameters are taken from Höhler.[3]

Our estimate of errors was obtained by consideration of the errors on electromagnetic amplitudes from single-energy analyses and from the scatter in single-energy results in the vicinity of the on-shell resonance energy. We believe these errors to be minimal estimates since they are based upon resonance parameters, which are assumed without errors from the π-N elastic amplitudes[4] used in our parametrization. No error estimates are made for the S_{11} resonance.

We note that our resonance couplings evaluated at

TABLE I. Resonance couplings from our energy-independent solution (SP), a Breit-Wigner fit to our energy-independent solutions (BW fit), the analysis of Crawford and Morton (Ref. 7) (CR) and quark-model predictions (Ref. 8) (FKR).

Resonance state	Reference	γp (GeV)$^{-1/2}\times 10^{-3}$ $A_{1/2}$	$A_{3/2}$	γn (GeV)$^{-1/2}\times 10^{-3}$ $A_{1/2}$	$A_{3/2}$
$P_{33}(1232)$:	SP	-133 ± 7	-244 ± 8		
$W_R=1228$ MeV	BW fit	-137	-246		
$\Gamma_\pi/\Gamma\simeq1$	CR	-145 ± 15	-263 ± 26		
$\Gamma_\pi=110$ MeV	FKR	-108	-187		
$P_{11}(1440)$:	SP	-66 ± 17		50 ± 19	
$W_R=1468$ MeV	BW fit	-64		45	
$\Gamma_\pi/\Gamma=0.63$	CR	-69 ± 18		56 ± 15	
$\Gamma_\pi=188$ MeV	FKR	27		-18	
D_{13} (1520):	SP	-25 ± 9	155 ± 6	-59 ± 14	-126 ± 15
$W_R=1513$ MeV	BW fit	-23	167	-63	-135
$\Gamma_\pi/\Gamma=0.60$	CR	-28 ± 14	156 ± 22	-56 ± 11	-144 ± 15
$\Gamma_\pi=75$ MeV	FKR	-34	109	-31	-109
$S_{11}(1535)$:	SP	78		-50	
$W_R=1527$ MeV	BW fit	50		-37	
$\Gamma_\pi/\Gamma=0.38$	CR	53 ± 15		-98 ± 26	
$\Gamma_\pi=47$ MeV	FKR	156		-108	

their respective resonance energies are in reasonable agreement with those given in previous analyses. The P_{33} resonance values from our solution[1] SP while consistent with the Particle Data Group average,[5] are considerably above those given in a recent analysis.[6] The ratio of $A^{3/2}$ to $A^{1/2}$ is, however, in agreement with the value (1.83) found in Ref. 6.

The problems encountered in separating resonance and background contributions can be circumvented if one evaluates the photon decay widths at the resonance pole position (W_p). Here the πN T-matrix (T_π) can be written as $R_\pi/(W_p-W)$, R_π being the residue. These residues have been determined in a separate analysis[4] of πN data, and are given in Table II. As we have parametrized[1] our amplitudes in the form

$$(E,M)=\alpha(1+iT_\pi)+\beta T_\pi\ , \tag{5}$$

the decay widths are determined by $(\beta+i\alpha)$ evaluated at the pole position and R_π, apart from kinematic factors. Near a pole the dimensionless amplitudes, $(\tilde{E},\tilde{M})$, have the form:

$$(\tilde{E},\tilde{M})=\frac{(qk)^{1/2}}{\hbar c}(\beta+i\alpha)\Bigg|_{W=W_p}\frac{R_\pi}{W_p-W}\ . \tag{6}$$

If the residue at the pole is assumed to be factorizable, $R=R_\pi^{1/2}R_\gamma^{1/2}$, then

$$R_\gamma=\frac{qk}{(\hbar c)^2}R_\pi(\beta+i\alpha)^2\ . \tag{7}$$

TABLE II. Contributions to photon decay widths Γ_γ (keV) obtained from the energy-dependent analysis of Ref. 1.

State			Γ_E	Γ_M	$\Gamma_{1/2}$	$\Gamma_{3/2}$
P_{33}:	$W_P=1210-i\,50$ MeV					
	$\lvert R_\pi\rvert=53$ MeV,	Pole	6	680	121	565
		Resonance	~0	610	140	470
D_{13}:	$W_P=1508-i\,62$ MeV					
	$\lvert R_\pi\rvert=40.6$ MeV,	γp Pole	382	239	13	608
		Resonance	315	210	13	512
		γn Pole	521	5	90	436
		Resonance	412	3	74	341
P_{11}:	$W_P=1357-i\,127$ MeV					
	$\lvert R_\pi\rvert=108$ MeV,	γp Pole		300	300	
		Resonance		163	163	
		γn Pole		199	199	
		Resonance		96	96	
S_{11}:	$W_P=1448-i\,99$ MeV					
	$\lvert R_\pi\rvert=54$ MeV	γp Pole	1493		1493	
		Resonance	270		270	
		γn Pole	878		878	
		Resonance	108		108	

At the pole position Eq. (2) is

$$\bar{E}_p = C\left[\frac{2(j+1)\pi q W_p R_\pi}{k M_N}\right]^{1/2} \frac{(\beta+i\alpha)}{\hbar c}\Bigg|_{W=W_p} . \quad (8)$$

The resonance photon decay widths are

$$\Gamma_\gamma = \frac{2k^2 M_N}{(2j+1)\pi M_R}\{|A^{1/2}|^2+|A^{3/2}|^2\} \equiv \Gamma_{1/2}+\Gamma_{3/2} . \quad (9)$$

The pole values of Γ_γ are obtained by evaluating Eq. (9) at the complex pole position, and are given in Table II where they are compared to the on-shell resonance decay widths obtained from resonance couplings given in Table I. No errors are stated for the pole residues or decay widths, but they may be assumed to be larger than the errors extracted at the on-shell resonance energy, due to extrapolation away from the fitted data base into the complex energy plane. In addition, the width Γ_γ can be partitioned into electric and magnetic multipole contributions via the relations

$$\begin{aligned}(|A^{1/2}|^2+|A^{3/2}|^2)_{l+} &= \tfrac{1}{4}(2j+1)[(j+\tfrac{3}{2})|\bar{E}_{l+}|^2+(j-\tfrac{1}{2})|\bar{M}_{l+}|^2] , \\ (|A^{1/2}|^2+|A^{3/2}|^2)_{(l+1)-} &= \tfrac{1}{4}(2j+1)[(j-\tfrac{1}{2})|\bar{E}_{(l+1)-}|^2+(j+\tfrac{3}{2})|\bar{M}_{(l+1)-}|^2] .\end{aligned} \quad (10)$$

At the pole position the multipole decay widths are

$$\begin{aligned}\Gamma_{E_{l+}} &= C^2(j+\tfrac{3}{2})(2j+1)|R_\gamma| , \\ \Gamma_{E_{l-}} &= C^2(j-\tfrac{1}{2})(2j+1)|R_\gamma| , \\ \Gamma_{M_{l+}} &= C^2(j-\tfrac{1}{2})(2j+1)|R_\gamma| , \\ \Gamma_{M_{l-}} &= C^2(j+\tfrac{3}{2})(2j+1)|R_\gamma| .\end{aligned} \quad (11)$$

These decay width contributions, which are proportional to $|\bar{E}|$ and $|\bar{M}|$, are also given in Table II.

[1]Richard A. Arndt, Ron L. Workman, Zhujun Li, and L. David Roper, Phys. Rev. C **42**, 1853 (1990), the preceding paper.

[2]F. A. Berends and A. Donnachie, Nucl. Phys. **B136**, 317 (1978).

[3]G. Höhler, F. Kaiser, R. Koch, and E. Pietarinen, *Handbook of pi-n scattering,* Physik Daten (Institut für Theoretische Kernphysik, Universität Karlsruhe, Germany 1979), Vol. 12-1.

[4]Richard A. Arndt, John M. Ford, and L. David Roper, solution SP89 in SAID, VPI&SU, Blacksburg, VA, 1989 (unpublished).

[5]G. P. Yost *et al.* (Particle Data Group), Phys. Lett. **204**, 1 (1988).

[6]R. Davidson, N. C. Mukhopadhyay, and R. Wittman, Phys. Rev. Lett. **56**, 804 (1986).

[7]R. L. Crawford and W. T. Morton, Nucl. Phys. **B211**, 1 (1983).

[8]R. P. Feynman, M. Kislinger, and F. Ravndal, Phys. Rev. D **3**, 2706 (1971).