

Inclusion of temperature dependence of fission barriers in statistical model calculations

J. O. Newton, D. G. Popescu, and J. R. Leigh

*Department of Nuclear Physics, Research School of Physical Sciences, Australian National University,
GPO Box 4, Canberra Australian Capital Territory 2601, Australia*

(Received 29 May 1990)

The temperature dependence of fission barriers has been interpolated from the results of recent theoretical calculations and included in a statistical model code. It is shown that the inclusion of temperature dependence causes significant changes to the values of the statistical model parameters deduced from fits to experimental data.

For many years statistical model codes have been used to fit experimental data on fission cross-sections (σ_{fis}) and pre-scission neutron multiplicities (ν_{pre}). Even though it has been long realized that fission barriers (E_f) must be temperature dependent, to the best of our knowledge only zero-temperature fission barriers, mainly calculated with macroscopic models,^{1,2} have been used in the codes. This is probably because calculations of barriers as functions of temperature T and angular momentum $J\hbar$ have only recently become available. The purpose of this paper is to indicate how the inclusion of the temperature dependence of E_f affects the results of statistical model calculations.

Garcias *et al.*³⁻⁶ have made calculations of $E_f(J, T)$ for eight nuclei covering a wide range of fissility. They have used a Thomas-Fermi model, which self-consistently incorporates the effects of rotation and temperature, with the Skyrme SkM* force. In order to gain agreement with liquid-drop barriers they have scaled the Weizsäcker coefficient for SkM*. Other recent calculations, for $J=0$ only, have been carried out with the finite-temperature Hartree-Fock method⁷ (²⁴⁰Pu) and with the extended Thomas-Fermi density variational method⁸ (²⁰⁸Pb and ²⁴⁰Pu); in both cases the SkM* force was used.

We have chosen to use only the results of Garcias *et al.* because they cover a wide range of nuclei and include the effect of angular momentum. One might therefore expect them to be consistent within themselves. We have attempted to interpolate their results in a simple way so that the $E_f(J, T)$ can be incorporated in the statistical code PACE2. However, we do not wish to imply that such interpolation is rigorously possible but merely to suggest that it is sufficient for our present purpose. We have excluded the results for the light nucleus ⁵⁹Cu, which has a fissility (0.28) well below the Businaro-Gallone point.

For temperatures up to 3–4 MeV, which cover our region of interest and above which the density of the external gas of evaporated nucleons has to be taken into account, the variation of fission barriers with temperature is well described by the expression

$$E_f(J, T) = E_f(J, 0)(1 - \alpha T^2). \quad (1)$$

This relation suggests that the fission barrier goes to

zero for $T^2 = T_{\text{cr}}^2 = 1/\alpha$, though in fact Eq. (1) may not be reliable when $E_f(J, T)$ becomes very small. The values of $E_f(J, 0)$ calculated by Garcias *et al.* are plotted versus their values of $1/\alpha = T_{\text{cr}}^2$ in Fig. 1. It can be seen that their data can be fairly well described by the line given by

$$E_f(J, 0) - 57.8 \{1 - \exp[-1.49 \times 10^{-3} T_{\text{cr}}^4(J)]\} \text{ MeV}. \quad (2)$$

Nevertheless, there remains some uncertainty in the magnitude of the values of α because the calculations of Garcias *et al.*³⁻⁶ do not agree well with those of Refs. 7 and 8.

The region in Fig. 1 of most significance for statistical model calculations is roughly that between the two vertical dashed lines. To the left of this region, where $E_f \ll B_v$, the neutron binding energy, the fission probability is so high that σ_{fis} has little dependence on the exact value of E_f . It should also be noted that the statistical model may not be reliable when $E_f \lesssim T$.⁹ To the right of this region, where $E_f \gg B_v$, the fission probability is very small and it has been shown¹⁰ that for heavy-ion induced fission of lighter nuclei, where the $J=0$ barriers are high, most of σ_{fis} arises from the region of angular momentum where $E_f(J) \approx B_v$.

We have also attempted to interpolate these results in terms of T_{cr}^2 and an effective J -dependent fissility similar to that of Blann and Komoto.¹¹ The quality of the fit was about the same as that described above.

Since the macroscopic fission barriers $E_f(J)$ of Sierk² are currently thought to give the best approximations to the real values at medium temperature and are almost invariably used for recent statistical model calculations, we have used them to give the zero-temperature values. The Sierk barriers are of course not temperature dependent but only are relevant to medium temperatures where shell effects have washed out. We have then somewhat arbitrarily corrected the Sierk barriers to finite temperature by deducing $\alpha = 1/T_{\text{cr}}^2$ from the curve in Fig. 1. Though the barriers of Garcias *et al.* greatly exceed those of Sierk for light systems, for nuclei with fissility $x \gtrsim 0.6$, which are of most interest to us, they differ only by up to ~20%. In view of the other uncertainties this difference seems of little consequence.

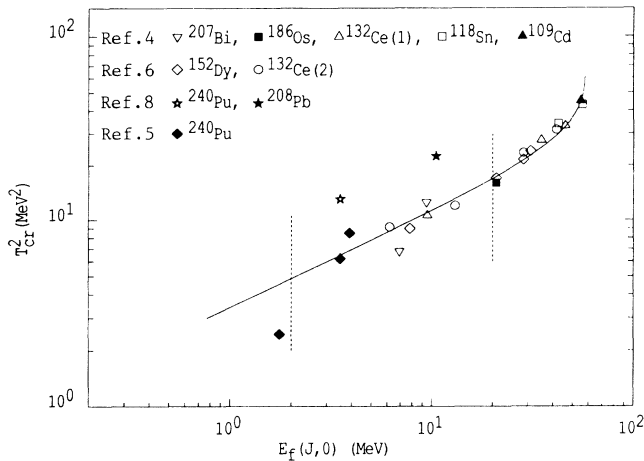


FIG. 1. Calculated values of zero-temperature fission barriers vs T_{cr}^2 for systems with differing fissilities and angular momenta. The occurrence of more than one point with the same symbol indicates that calculations were made also for $J > 0$. Apart from the two points indicated by stars, the values derive from the calculations of Garcias *et al.* The full line represents the curve of Eq. (2) (see text).

We have incorporated the relation of Eq. (2) into the Monte Carlo statistical code PACE2.¹² Calculations were first made without the temperature correction using statistical model parameters which gave excellent fits to extensive fission cross-section data,¹³ which had been fitted with the grid based code ALERT1. It was found that the PACE2 calculations gave values for σ_{fis} which were too low. However, they could be brought into good agreement with the data by increasing the ratio of level density parameters at the saddle and equilibrium points (a_f/a_v) from 1.00 to 1.03. This must reflect some differences in the codes but is not of importance for our present purpose, which is to identify the *differences* caused by the introduction of temperature-dependent fission barriers. When the temperature dependence was included in PACE2, equally good fits to the data for all systems could be obtained when the parameter a_f/a_v was changed from 1.03 to 0.97, a reduction of 6%. Two examples of fits to σ_{fis} data with $a_f/a_v = 0.97$ and with temperature dependence (full lines) and without temperature dependence (dashed lines) are shown in Fig. 2(a). If we changed a_f/a_v to 1.03 the dashed lines would move up to nearly coincide with the full lines in Fig. 2(a), whilst the latter would move up in approximately the same proportion. If all other parameters remain fixed, it is not surprising that a_f/a_v has to be reduced to fit σ_{fis} data, since the barriers are reduced by increased temperatures. Possibly surprising is that the same reduction fits systems with a wide range of fissility. The magnitude of the reduction is similar to that suggested by much simpler considerations in Ref. 13. Though we have not chosen to do this, it might also be possible to fit the data by varying other model parameters instead of or in addition to a_f/a_v . For example, the Sierk barrier might be arbitrarily reduced by a scaling factor k_f , as it is well known that there is a corre-

lated range of k_f and a_f/a_v within which equally good fits can be obtained.

Whilst the results of Garcias *et al.*³⁻⁶ agree quite well with the earlier calculations of Pi *et al.*,¹⁴ based on a finite-temperature mass formula, the agreement is less

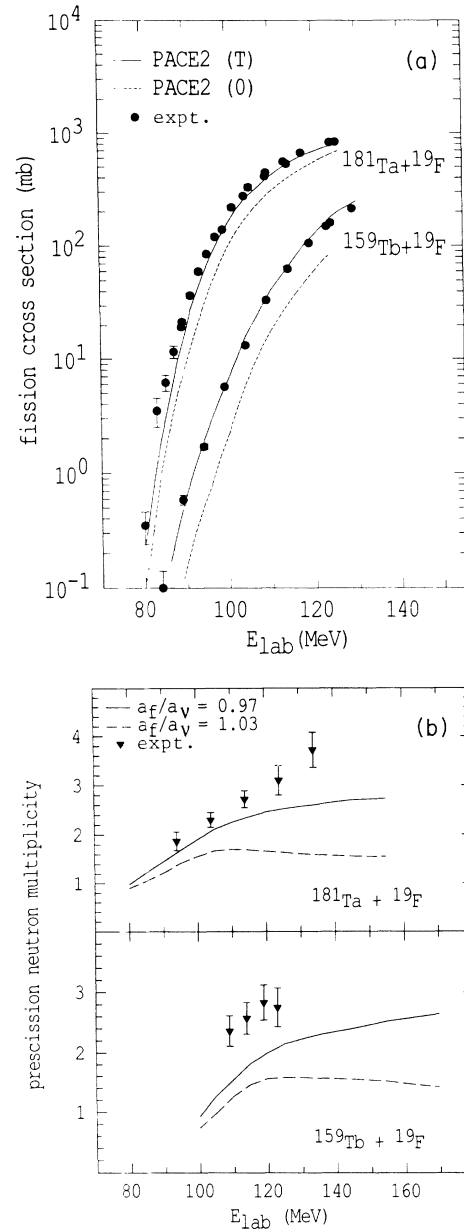


FIG. 2. (a) Fits to experimental fission cross sections for two systems with temperature-dependent fission barriers and $a_f/a_v = 0.97$ (full lines). Also shown are the results with zero-temperature fission barriers, the other statistical model parameters remaining the same (dashed lines). (b) Precission neutron multiplicities versus laboratory bombarding energies for two systems. The points with error bars are experimental data. The continuous lines are calculated with temperature-dependent fission barriers and $a_f/a_v = 0.97$, whilst the long-dashed lines are calculated with zero-temperature barriers and $a_f/a_v = 1.03$. Both cases results in equally good fits to fission cross-section data.

good with the recent ($J=0$) calculations of Guet *et al.*⁸ for ^{240}Pu and ^{208}Pb . Their results are also shown in Fig. 1 (as stars) and would give a value for α in Eq. (1) about half of that for Garcias *et al.* This, if correct, would imply a reduction in a_f/a_v of about 3% rather than 6%.

The quantity v_{pre} is known¹⁵ to depend sensitively on a_f/a_v and to be insensitive to any scaling of the fission barrier. A reduction of a_f/a_v by 6% has a significant effect on the calculated value of v_{pre} as shown in Fig. 2(b).

This work is part of a continuing effort to improve statistical model calculations so that eventually the values of the parameters derived from fits to experimental data have real physical meaning. The present calculation shows that inclusion of temperature dependent fission barriers has a significant effect. It should therefore be included in statistical model codes. However, it is also clear that more theoretical work needs to be done to reliably establish the absolute magnitude of the effect.

Other effects that must also be included in codes are

the finite relaxation time (transient time) to build up equilibrium in the fission degree of freedom at the saddle point and particle emission during the saddle-to-scission transition; both of these result from nuclear "viscosity." One consequence of the viscosity is apparently to require large values of $a_f/a_v \sim 1.1$.²⁰ Inclusion of the temperature dependence of E_f might reduce these. Improvements in level density formulations, particle transmission coefficients, and fusion angular momentum distributions also need to be made and require further experiments and basic theoretical work. The parameter a_f/a_v is commonly taken to have a value very close to unity but this is not necessarily correct and the few theoretical calculations¹⁶⁻¹⁹ are in conflict. For a more extensive review of these questions, see Ref. 21. Much more work remains to be done before reliable physical conclusions can be drawn from statistical model calculations; this paper is a small, but necessary step towards that goal.

-
- ¹S. Cohen, F. Plasil, and W. J. Swiatecki, *Ann. Phys. (N.Y.)* **82**, 557 (1974).
- ²A. J. Sierk, *Phys. Rev. C* **33**, 2039 (1986).
- ³F. Garcias, M. Barranco, J. Nemeth, C. Ngô, and X. Viñas, *Nucl. Phys. A* **495**, 169c (1989).
- ⁴F. Garcias, M. Barranco, J. Nemeth, C. Ngô, and X. Viñas, *Symposium on Nuclear Dynamics and Nuclear Disassembly* (World Scientific, Singapore, 1989), p. 274.
- ⁵F. Garcias, M. Barranco, H. S. Wio, C. Ngô, and J. Nemeth, *Phys. Rev. C* **40**, 1522 (1989).
- ⁶F. Garcias, M. Barranco, A. Faessler, and N. Ohtsuka, *Z. Phys. A* **336**, 31 (1990).
- ⁷J. Bartel and P. Quentin, *Phys. Lett. B* **152**, 29 (1985).
- ⁸C. Guet, E. Strumberger, and M. Brack, *Phys. Lett. B* **205**, 427 (1988).
- ⁹H. A. Kramers, *Physika* **7**, 284 (1940).
- ¹⁰R. J. Charity, J. R. Leigh, J. J. M. Bokhorst, A. Chatterjee, G. S. Foote, D. J. Hinde, J. O. Newton, S. Ogaza, and D. Ward, *Nucl. Phys. A* **457**, 441 (1986).
- ¹¹M. Blann and T. T. Komoto, *Phys. Rev. C* **26**, 472 (1982).
- ¹²A. Gavron, *Phys. Rev. C* **21**, 230 (1980).
- ¹³J. O. Newton, D. J. Hinde, R. J. Charity, J. R. Leigh, J. J. M. Bokhorst, A. Chatterjee, G. S. Foote, and S. Ogaza, *Nucl. Phys. A* **483**, 126 (1988).
- ¹⁴M. Pi, X. Viñas, and M. Barranco, *Phys. Rev. C* **26**, 733 (1982).
- ¹⁵D. Ward, R. J. Charity, D. J. Hinde, J. R. Leigh, and J. O. Newton, *Nucl. Phys. A* **403**, 189 (1983).
- ¹⁶C. J. Bishop, I. Halpern, R. W. Shaw, Jr., and R. Vandebosch, *Nucl. Phys. A* **198**, 161 (1972).
- ¹⁷P. A. Gottschalk and T. Lederberger, *Nucl. Phys. A* **278**, 16 (1977).
- ¹⁸N. Cârjan, H. Delagrange, and A. Fleury, *Phys. Rev. C* **19**, 2267 (1979).
- ¹⁹J. Tõke and W. J. Swiatecki, *Nucl. Phys. A* **372**, 141 (1981).
- ²⁰P. Grangé, S. Hassani, H. Weidenmüller, A. Gavron, J. R. Nix, and A. J. Sierk, *Phys. Rev. C* **34**, 209 (1986).
- ²¹J. O. Newton, *Fiz. Elem. Chastits At. Yadra* **21**, 821 (1990) [*Sov. J. Part. Nucl.* (in press)].