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Semistatistical model for $^{12}C+^{12}C$ reaction cross sections below the Coulomb barrier

Sohail A. Khan
Department of Physics, Islamia University, Bahawalpur, Pakistan

William P. Beres
Department of Physics, Wayne State University, Detroit, Michigan 48202
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Sub-Coulomb cross sections for $^{12}C+^{12}C \rightarrow \alpha+^{20}Ne$, $^{12}C+^{12}C \rightarrow p+^{23}Na$, and $^{12}C+^{12}C \rightarrow n+^{23}Mg$ are calculated in a semistatistical framework that allows a comparison to experiment in terms of the general energy dependence and magnitude. Intermediate structure previously produced in the generalized doorway model via calculations of the structure factor is not evident in the present approach because of the large energy dependence of the reaction cross sections. The compound nucleus is described in a generalized doorway model where the doorways are obtained by coupling the shape resonances in $^{12}C+^{12}C$ to the single and mutual excitation channels. The transmission coefficients for decay are determined in a modified form of the Hauser-Feshbach method. Agreement is good in terms of energy dependence and magnitude even though the experimental cross sections vary by about 4 orders of magnitude between 3 and 5 MeV c.m.

Sub-Coulomb reaction cross-section calculations in $^{12}C+^{12}C$ are important in order to keep pace with the accumulated experimental data. Such calculations would serve as a check on the validity of the theory and model employed, and allow for extrapolations to astrophysical energies that can be taken with confidence. Most of the reaction cross-section data is summarized in a review article by Barnes *et al.*[1]

In an earlier paper[2] we presented a nuclear projection model for heavy-ion reactions in the sub-Coulomb region. Generalized doorways were introduced as the set of resulting states obtained by coupling the shape resonances to the single and mutual excitation channels. The energies and continuum widths of the resonances below and near the Coulomb barrier were obtained for $^{12}C+^{12}C$ and found to be consistent with observation. In another instance[3] the generalized doorways were used to calculate the modified structure factor down to about 3 MeV c.m., and an extrapolation to astrophysical energies was made. In Ref. 3 we factored out the large energy dependence of the reaction cross sections below the Coulomb barrier and obtained an intermediate structure that agreed well with experiment. Experimental cross sections vary over about 4 orders of magnitude,[4] and in the present paper our emphasis is in producing the general energy dependence and magnitude of the $^{12}C+^{12}C$ cross sections at the expense of the previously verified intermediate structure. To do this we employ a semistatistical approach that preserves the concept of generalized doorways in the entrance channel. The cross sections to important exit channels are determined from a nuclear-structure description of compound-nucleus formation and the eventual statistical decay to the exit channels. The latter contributions are obtained using a modified version of the well-known Hauser-Feshbach[4] method.

One can describe the situation as shown in Fig. 1 in terms of the generalized doorways D and states q one step more complicated than the D states. The incoming reduced mass particle can enter the compound nucleus either (i) directly when the incoming energy is far from a doorway energy, or (ii) via a doorway if the incident energy is close to a doorway energy. The formation of a compound nucleus is enhanced in situation (ii) and the D states that are formed initially can either decay directly, or they can go to other states q, forming the compound nucleus. The latter states can then decay to the various exit channels in the usual statistical manner. If we assume that each of those channels in which the doorway state can decay directly with high probability is much less observed in comparison to all other states together, then the cross section $\sigma_{\beta\beta'}$ for decay to the final channel β' from the initial channel β can be written as

$$\sigma_{\beta\beta'} = \sum_{D,J,\pi} \sigma^{D,J,\pi}_{\beta,\mathrm{comp}} \frac{T^{J,\pi}_{\beta'}}{\sum_{\beta''} T^{J,\pi}_{\beta''}} . \tag{1}$$

In Eq. (1) $\sigma^{D,J,\pi}_{\beta,\mathrm{comp}}$ is the cross section for compound-

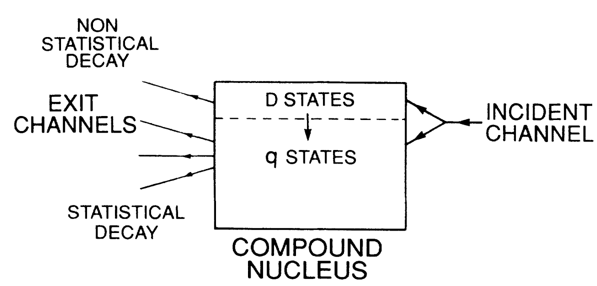

FIG. 1. Schematic of the generalized doorway model for $^{12}C+^{12}C$ reactions. The generalized doorways are labeled D and the more complicated states are labeled q. See text for details.

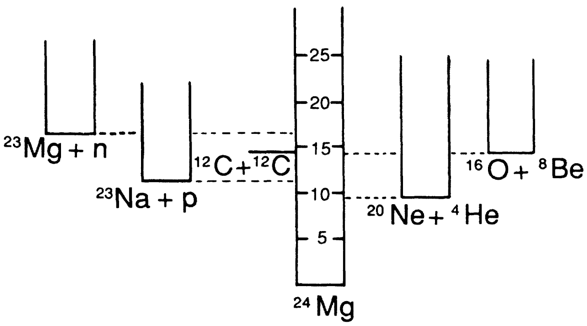

FIG. 2. Channels available to the compound nucleus ^{24}Mg at an excitation energy of about 21 MeV. The various states in each channel are given in Table I.

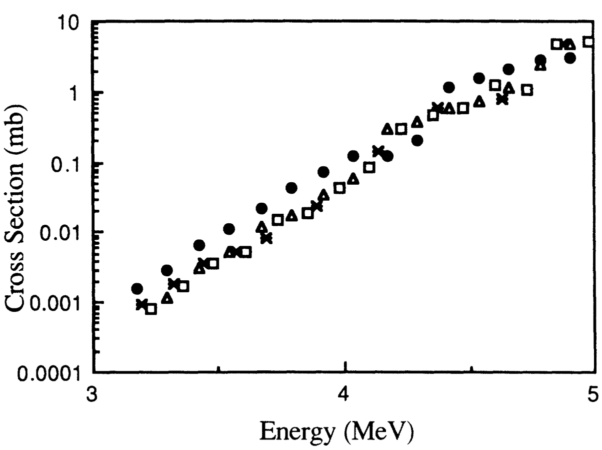

FIG. 3. Cross sections (mb) versus energy (MeV) c.m. for theory (represented by ●) and experiment for the ^{2}Ne+α channel [$\beta = {}^{12}$C+^{12}C and $\beta' = {}^{20}$Ne+α in Eq. (1)]. The experimental references are as follows: △ is for Mazarakis and Stephens (Ref. 9); □ is for Patterson, Winkler, and Zaidens (Ref. 10); ✳ is for High and Cujek (Ref. 11).

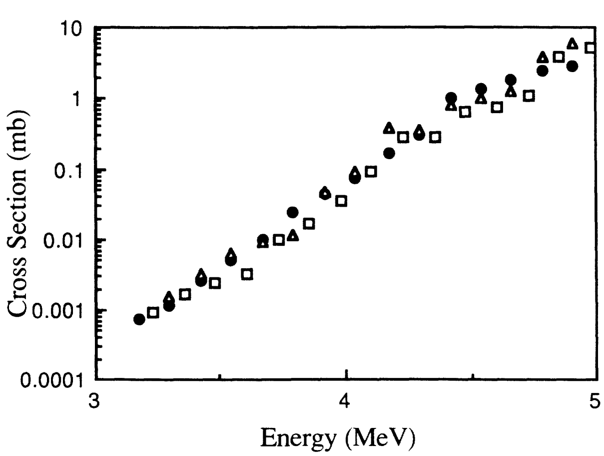

FIG. 4. Cross sections (mb) versus energy (MeV) c.m. for theory (represented by ●) and experiment for the ^{23}Na+p channel [$\beta = {}^{12}$C+^{12}C and $\beta' = {}^{23}$Na+p in Eq. (1)]. The experimental references are as follows: Δ is for Mazarakis and Stephens (Ref. 9); □ is for Patterson, Winkler, and Zaidens (Ref. 10).

TABLE I. The levels employed in the present calculations. The quantity E^* refers to the energy level of the residual nucleus after decay of the compound nucleus ^{24}Mg. The labels J and π denote the spin and parity of the level.

Residual state E^* MeV	J^π	Residual state E^* MeV	J^π
^{20}Ne		^{23}Na	
Ground	0^+	5.80	$\frac{3}{2}^+$
1.63	2^+	5.80	$\frac{5}{2}^-$
4.25	4^+	5.80	$\frac{7}{2}^+$
4.97	2^-	5.80	$\frac{3}{2}^-$
5.63	3^-	5.80	$\frac{5}{2}^+$
5.80	1^-	5.80	$\frac{7}{2}^-$
6.75	0^+	6.20	$\frac{1}{2}^-$
7.02	4^-	6.20	$\frac{9}{2}^+$
7.19	3^-	6.20	$\frac{1}{2}^+$
7.23	0^+	6.20	$\frac{9}{2}^-$
7.46	2^+	6.80	$\frac{3}{2}^+$
7.86	2^+	6.80	$\frac{5}{2}^-$
8.52	5^-	6.80	$\frac{7}{2}^+$
8.60	0^+	6.80	$\frac{3}{2}^-$
8.70	1^-	6.80	$\frac{5}{2}^+$
8.80	2^+	6.80	$\frac{7}{2}^-$
8.84	6^+	7.60	$\frac{3}{2}^-$
8.91	1^-	7.60	$\frac{5}{2}^+$
9.10	4^+	7.60	$\frac{7}{2}^-$
9.18	3^-	7.60	$\frac{3}{2}^+$
9.58	2^+	7.60	$\frac{5}{2}^-$
^{23}Na		7.60	$\frac{7}{2}^+$
Ground	$\frac{3}{2}^+$	7.70	$\frac{1}{2}^+$
0.44	$\frac{5}{2}^+$	7.70	$\frac{9}{2}^-$
2.08	$\frac{7}{2}^+$	7.70	$\frac{11}{2}^+$
2.39	$\frac{1}{2}^+$	7.70	$\frac{1}{2}^-$
2.64	$\frac{5}{2}^+$	7.70	$\frac{9}{2}^+$
2.70	$\frac{3}{2}^+$	7.70	$\frac{11}{2}^-$
2.98	$\frac{9}{2}^+$	^{23}Mg	
3.68	$\frac{3}{2}^-$	Ground	$\frac{3}{2}^+$
3.85	$\frac{5}{2}^-$	0.44	$\frac{5}{2}^+$
3.92	$\frac{3}{2}^+$	2.08	$\frac{7}{2}^+$
4.30	$\frac{3}{2}^-$	2.39	$\frac{1}{2}^+$
4.30	$\frac{5}{2}^+$	2.64	$\frac{5}{2}^+$
4.30	$\frac{7}{2}^-$	2.70	$\frac{3}{2}^+$
4.30	$\frac{3}{2}^+$	2.98	$\frac{9}{2}^+$
4.30	$\frac{5}{2}^-$	^{12}C	
4.30	$\frac{7}{2}^+$	Ground	0^+
4.50	$\frac{1}{2}^+$	4.43	2^+
4.50	$\frac{1}{2}^-$	^{8}Be	
5.50	$\frac{1}{2}^-$	Ground	0^+
5.50	$\frac{1}{2}^+$	2.90	2^+

TABLE II. Optical-model potentials used in obtaining the transmission coefficients for different exit channels. The optical potential is given

$$V(r,E)=\frac{(-V_0+C_0E)}{1+\exp\left[\frac{r-R}{a_R}\right]}-\frac{W}{1+\exp\left[\frac{r-R}{a_1}\right]}-(W_g+C_1E)\exp\left[\frac{(r-R)^2}{b^2}\right]+V_c ,$$

where $b=0.98$. In all cases except the ^{20}Ne$+\alpha$ channel, $a_1=a_R$. For ^{20}Ne, $a_1=0.523$ fm. In each case the Coulomb potential was taken to be that due to a uniformly charged sphere. The parameters in the table come from Refs. 7 and 8.

Exit channel	V_0 MeV	R fm	W MeV	a_R fm	W_g MeV	C_0	C_1
^{20}Ne$+\alpha$	82.63	4.7	6.34	0.565	0	0	0
^{23}Na$+p$	55	3.56	0	0.5	4.0	0.5	0.5
^{23}Mg$+n$	51	3.56	0	0.5	4.0	0.5	0.5
^{16}O$+^{8}$Be	50	5.77	4.0	0.4	0	0	0

nucleus formation of state J^{π} at energy E_D in the initial channel, $T_{\beta''}^{J,\pi}$ is the statistical transmission coefficient for the exit channel of interest, and β'' indicates all possible exit channels. The cross section $\sigma_{\beta,\mathrm{comp}}^{D,J,\pi}$ is given by

$$\sigma_{\beta,\mathrm{comp}}^{D,J,\pi}=\frac{\pi}{k^2}(2J+1)T_{\beta}^{D,J,\pi} , \quad (2)$$

where k is the wave number. The transmission coefficients in forming the compound nucleus $T_{\beta}^{D,J,\pi}$ are evaluated in the generalized doorway scheme, whereas, the transmission coefficients for statistical decay are obtained using methods based on Ref. 4.

For the case of ^{12}C$+^{12}$C collisions the cross sections $\sigma_{\beta,\mathrm{comp}}^{D,J,\pi}$ have been calculated in Ref. 3 using the potential of Kondo *et al.*[5] The various exit channels which are open to the ^{24}Mg compound nucleus at an excitation energy of 21 MeV are displayed in Fig. 2. The most important channels are the following.

(i) $\alpha+^{20}$Ne with a Q value of 4.62 MeV. Experimental work on this exit channel has yielded important assignments of spins for the resonances observed in the ^{12}C$+^{12}$C reaction.[6]

(ii) $p+^{23}$Na with a Q value of 2.24 MeV. This channel has a yield which is comparable to the $\alpha+^{20}$Ne channel yield at very low energies, and is less than the $\alpha+^{20}$Ne yield at high energies.

(iii) $n+^{23}$Mg with a Q value of -2.62 MeV. This channel has a yield observed to be much less than that of (i) and (ii).

The total reaction cross section that has been deduced experimentally below the Coulomb barrier is based on an analysis of the three exit channels mentioned above. While the ^{16}O$+^{8}$Be and the elastic and inelastic ^{12}C$+^{12}$C channels give negligible cross sections at the low energies of interest here, they were, nevertheless, included in our calculation of the denominator in Eq. (1). Hauser-Feshbach predictions for the cross sections to different exit channels have shown good general agreement[7] over the energy interval from 10.15 to 12.8 MeV c.m. However, these calculations do not produce intermediate structure since *both* the entrance *and* exit channels are treated in a statistical manner because of the large density of states in the compound nucleus.

Table I gives the excited states that were included in the different residual nuclei in our calculations. Altogether there are 21 levels of ^{20}Ne, 48 levels of ^{23}Na, and 7 levels of ^{23}Mg that can possibly contribute in the energy range of interest. The number of contributing excited states will diminish as the available energy decreases. The transmission coefficients for compound-nucleus decay were calculated from an optical model using parameters derived from fits to the elastic-scattering data. The optical-model parameters are based on Refs. 7 and 8 and are given in Table II. The ^{12}C$+^{12}$C parameters are not listed in the table and are taken from our work in Ref. 2.

The results are shown in Figs. 3, 4, and 5 which contain the comparison between theory and experiment[9–12] for $\alpha+^{20}$Ne, $p+^{23}$Na, and $n+^{23}$Mg, respectively. The experimental cross-section measurements have percen-

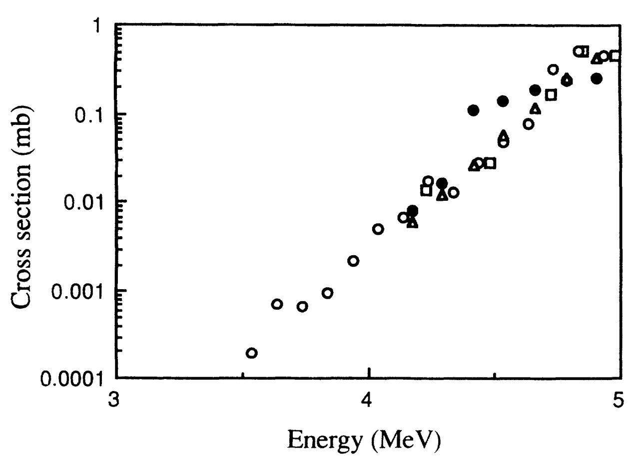

FIG. 5. Cross sections (mb) versus energy (MeV) c.m. for theory (represented by ●) and experiment for the ^{23}Mg$+n$ channel [$\beta=^{12}$C$+^{12}$C and $\beta'=^{23}$Mg$+n$ in Eq. (1)]. The experimental references are as follows: △ is for Mazarakis and Stephens (Ref. 9); □ is for Patterson, Winkler, and Zaidens (Ref. 10); ○ is for Dayras, Switkowski, and Woosley (Ref. 12). The theoretical calculations were only performed down to about 4.2 MeV since we became aware of the measurements of Dayras *et al.* only after our work had been completed.

tage errors that vary between about 6–20 % in the energy region of interest. Computer uncertainties at such extremely low cross sections restricted our calculations to above about 3.0 MeV. The agreement with experiment is good in terms of energy dependence and magnitude in all three cases. We note especially that the agreement is present even though the experimental cross sections vary by about 4 orders of magnitude between about 3 and 5 MeV c.m. An intermediate structure is not evident because of the large energy dependence of the reaction cross sections below the Coulomb barrier. However, in analyzing the structure factor as in Ref. 3 it is clear that the generalized doorway model indeed produces an intermediate structure below the barrier.

We conclude that the generalized doorway model for the $^{12}C+^{12}C$ compound nucleus coupled with statistical decay to various exit channels provides a good general description of the energy dependence and magnitude of the observed decay cross sections.

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