# Determination of the impact parameter in relativistic nucleus-nucleus collisions

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A simple method is proposed for determining the impact parameter in relativistic nucleus-nucleus collisions. Assuming a monotonous correlation between multiplicity and impact parameter, the multiplicity dependence of the measured cross section is interpreted as an impact-parameter dependence of the geometrical reaction cross section. The reliability of this method is checked to be excellent within the framework of the intranuclear cascade model. Its application to data analysis at lower and higher energies is suggested.

Relativistic nucleus-nucleus collisions are the unique tool to produce and study dense nuclear matter in the laboratory.[1-3] From straightforward geometrical considerations, theoretical models predict that the size of the dense nuclear-matter zone produced in collisions at such energies depends strongly on the impact parameter. It is thus essential to sort out the collisions according to their centrality. The impact parameter, which characterizes the initial state, is not directly measurable; it is thus necessary to find an observable strongly correlated with it. The simplest observable one can think of is the multiplicity of detected particles, which has been used extensively by all groups working in this field for presenting their results. Let us recall one of the first results from high-statistics exclusive measurements performed with an electronic detector: the ratio between the numbers of deuterons and protons was shown to depend strongly on the charged-particle multiplicity.[4] This was a proof that the value of the entropy, which is related to this ratio, cannot be extracted from inclusive measurements. Intranuclear cascade (INC) calculations[5] confirmed this hypothesis of strong correlation between impact parameter and multiplicity of emitted participant nucleons; no better correlation could be achieved even with more complicated observables. Nevertheless, this selection of the centrality by means of the multiplicity gives only a qualitative ordering of the collisions according to their impact parameter. Exclusive measurements of relativistic nucleus-nucleus collisions have been most often presented as a function of the multiplicity, without trying to get the impact parameter. For the Plastic Ball results,[3] multiplicity slices are used, with limits corresponding to increasing fractions of the maximum multiplicity. Multiplicity was already used to determine an impact-parameter range for presenting streamer chamber results[6,7] in the case of the most central collisions; the central trigger cross section was transformed into a maximum impact parameter in a geometrical picture. In this paper, this procedure is extended over the whole range of the multiplicity distribution. A simple method is proposed, which allows one to transform the qualitative estimate of the impact parameter from the multiplicity, into a quantitative one. A quantitative estimate is very convenient in order to present consistent results obtained at various energies with different targets and projectiles, and to compare experimental results and model predictions. This method has already been used as a guideline when presenting results[8,9] obtained with the Diogene detector[10] at the Saturne synchrotron in Saclay. In the present paper, the method is first described in detail. Its reliability is then demonstrated in the framework of the intranuclear cascade model.

The impact parameter $b$ of a nucleus-nucleus collision is classically defined by the distance between the straight-line trajectories of the centers of the two nuclei before their interaction. At relativistic energies the total nucleus-nucleus cross section is well approximated by the geometrical cross section: $\sigma_g = \pi(R_t + R_p)^2$, where $R_t$ and $R_p$ are the equivalent hard-sphere radii of the target and projectile nuclei, respectively. As a function of the squared impact parameter, the cross section is constant: $d\sigma_g/d(b^2) = \pi$, up to the maximum value $(R_t+R_p)^2$ of $b^2$, and it is zero beyond this value. As previously stated, the multiplicity in the final state is strongly correlated with the impact parameter $b$. More precisely, its mean value decreases monotonously as a function of $b$. To any value $m$ of the multiplicity, it is possible to associate the integral $S$ of the measured cross section $\sigma(M)$, from $m$ to infinity, $S = \sum_{M=m}^{\infty} \sigma(M)$. Because of the correlation, it is also possible to associate to $m$ the impact parameter $B$ such that $\int_0^{B^2} d(b^2) d\sigma_g/d(b^2) = S$, thus $B^2 = S/\pi$. As illustrated in Fig. 1, this procedure can be used several times to calculate the average value of $b^2$ corresponding

 

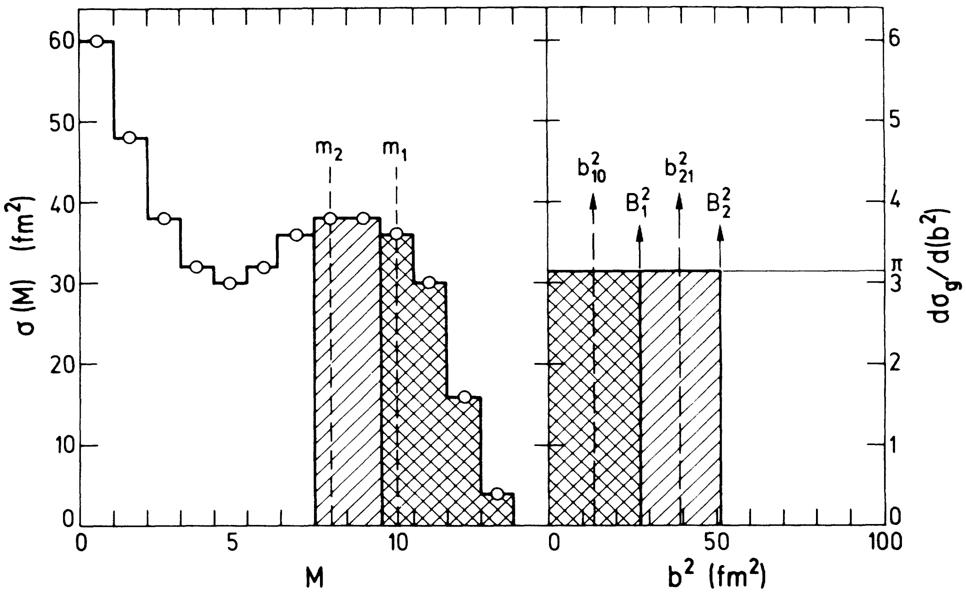


FIG. 1. Principle of the method: measured cross section versus multiplicity (left diagram) and geometrical cross section vs $b^2$, the square of the impact parameter (right diagram). The hatched areas correspond to equal integrated cross sections on both diagrams, with three multiplicity limits $m_0=\infty$, $m_1$, and $m_2$.

to any multiplicity integral $[m_i, m_j[$ as $b_{ij}^2=(B_i^2+B_j^2)/2$. This procedure is rigorous if there is no dispersion in the correlation between multiplicity and impact parameter. In practice, there will always be a finite dispersion, but the method should remain valid as long as the correlation is large enough in comparison to the dispersion, i.e., the quality factor defined by Cugnon and L'Hôte[5] is high enough.

The validity of this method is now checked in the framework of INC simulations.[11] We consider here as an example the multiplicity $M(p)$ of protons emitted within the acceptance (Table I) used for analyzing results obtained with the Diogene pictorial drift chamber.[8] Because of the angular cut at forward angles and to the threshold on transverse momentum around zero rapidity, this acceptance eliminates protons from both projectile and target spectator remnants. Since the impact parameter in the cascade model is known for each collision, it is first possible to examine in detail the distribution of the real impact parameter at any value of the multiplicity. As shown in Fig. 2 for Ne+Pb collisions at 800 MeV per nucleon, there is a strong negative correlation between $M(p)$ and the square of the impact parameter. The higher the event multiplicity, the smaller the impact parameter is on average, with a noticeable rms dispersion of the $b^2$ distribution at any value of the multiplicity. The maximum value of the dispersion is 6.5 fm$^2$, which is about 7% of the maximum squared impact parameter $(R_t+R_p)^2$. Thus the proposed method should be valid with good accuracy, except at the highest multiplicities, or smallest impact parameters, where the correlation becomes poor. If we choose as a criterion for a sufficient accuracy that the rms dispersion is smaller than half the average value of $b^2$, the method should give accurate estimates of $b^2$ as soon as $b^2$ is larger than 10 fm$^2$, i.e., about 11% of the maximum squared impact parameter. The estimated $b^2$ can be made as small as desired, by simply selecting more in the tail of the multiplicity distribution, whereas the mean value of the real $b^2$, which stays strict-

TABLE I. Proton acceptance for defining the proton multiplicity from INC simulations. $\theta$ is the polar emission angle, with respect to the beam direction, of a proton with mass $m$, rapidity $y$, and transverse momentum $p_\perp$.

| $20° < \theta < 132°$ | |
|---|---|
| $p_\perp/m > 0.36+0.72y$ | $y<0$ |
| $p_\perp/m > 0.36-0.80y$ | $y>0$ |

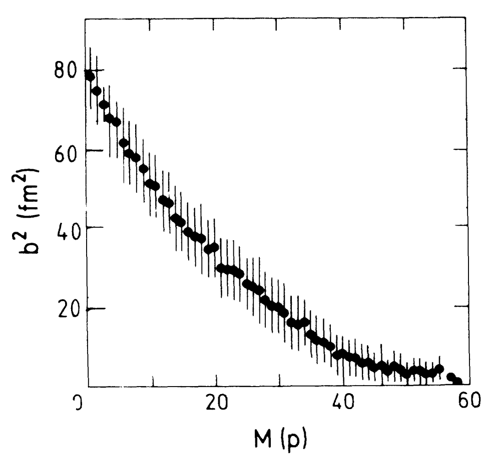


FIG. 2. Mean squared impact parameter and its rms dispersion (indicated by error bars), versus proton multiplicity, as predicted by the cascade model (Ref. 11) inside the acceptance defined in Table I, for Ne+Pb collisions at 800 MeV per nucleon.

ly positive in any multiplicity bin, is more and more sensitive to the dispersion and looses any correlation with the multiplicity (see Fig. 2). Therefore, at small impact parameters the method should underestimate the real $b^2$. At large impact parameters, the multiplicity has an absolute lower limit equal to zero, with fluctuations directed only towards positive values. Large values of the estimated $b^2$ will be reached too early, and the method should overestimate the real $b^2$.

When applying the present procedure to cascade events for impact-parameter estimation, it is possible to compare real and estimated values of the squared impact parameter, and to check quantitatively the reliability of the method within this model. This has been done for three cases (Ne+Pb collisions at 800 and 400 MeV per nucleon, and Ar+Ca collisions at 600 MeV per nucleon) and nine contiguous slices of approximately equal cross sections in the multiplicity distribution (Fig. 3). For convenience, we use the reduced impact parameter $\tilde{b}$ defined as $\tilde{b}=b/(R_t+R_p)$, with $R_t=R_0A_t^{1/3}$ and $R_0=1.12$ fm, as used in the cascade simulations. This is especially useful to compare results for different target-projectile systems. For the three cases the estimated mean squared impact parameter is very close to the mean value of the real $b^2$ in each slice. Underestimation of $b^2$ at small impact parameters, and overestimation at large impact parameters, are noticeable in the extreme slices, as expected according to the arguments above. The quality of the estimation does not depend critically on the number of slices. When this number increases, the effects of under- and overestimation extend over a larger number of slices, but over the same range of impact-parameter values. For Ne+Pb collisions at 800 MeV per nucleon, we have repeated this study with a cascade simulation that includes the formation of composite particles, and with an experimental filter that includes the detector biases concerning track reconstruction and particle identification. The shape of the multiplicity distribution is changed towards a smaller average value. However, the correlation between impact parameter and multiplicity stays almost as strong as before, and the quality of the $b^2$ estimation is the same if the number of slices is less than 10. A sensible choice for the maximum number of slices could be suggested, namely, that the ranges of the real $b^2$ distribution (mean value plus or minus the dispersion) do not overlap for contiguous slices. For Ne+Pb collisions at 800 MeV per nucleon, the overlap, which is rather large with nine slices (Fig. 3), becomes negligible when the number of slices is reduced to five.

The present method gives an impact-parameter scale for multiplicity selected data. It is not at all restricted to a given shape of the multiplicity distribution. In particular, there can be many reasons for a theoretical model not to reproduce the measured multiplicity distribution, such as the difficulty of taking into account the formation of composite particles. However, as long as the correlation between impact parameter and measured multiplicity is strong enough, this method can be used safely to estimate in the same way the impact parameter in both experiment and theory, and to compare consistently the variation, with the estimated $b^2$, of any observable. Even if the dispersion effects are not exactly the same in theory and experiment, it is better to proceed this way, in order to establish correct impact-parameter averaging for a model,[12] than to compare experimental results at a given value of the estimated $b^2$ with theoretical results obtained at the same but fixed value of the real $b^2$. When looking at the impact-parameter dependence of any observable this way, one should, however, be cautious that there may be other sources of correlation between the observable and the multiplicity than the mere impact parameter. In particular, when selecting events in the high multiplicity tail, one should be cautious that they may correspond to special classes of collisions and possibly reflect other constraints such as phase-space constraints,[13] or charge conservation as has been observed for the mean $\pi^+$ multiplicity in alpha-nucleus collisions at high proton multiplicity.[14]

The exact condition for the equality between the estimated and the real mean values of $b^2$ at any given multiplicity can be worked out. When starting from any point along the line representing the mean value of the real $b^2$ as a function of the multiplicity (Fig. 2), the integrated cross sections for smaller impact parameters (below the point) and for higher multiplicities (to the right side of the point) should be equal. The fulfillment of this condition does not depend on the shape of the multiplicity distribution but rather on the full correlation between impact parameter and multiplicity, including the dispersion. Another advantage of this method is that

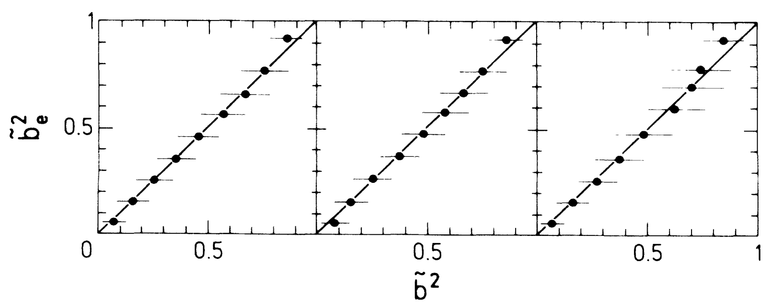


FIG. 3. Reduced squared impact parameter: mean estimated value versus mean real value, with the rms dispersion indicated by horizontal error bars, for nine slices of the multiplicity distribution, as predicted by the cascade model (Ref. 11). From left to right: Ne+Pb at 800 and 400 MeV per nucleon, and Ar+Ca at 400 MeV per nucleon.

there is absolutely no loss of information in presenting the data. From the successive estimated values of $b^2$ it is easy to go back to the original measured cross sections in the successive multiplicity slices that have to be specified. When using the reduced impact parameter, the value of $R_0$ has to be specified too. Relative errors on the cross-section measurements are simply propagated onto the $b^2$ estimations as an uncertainty in the scale. It is more convenient to use $b^2$ rather than $b$ for the following reasons: the geometrical cross section is constant as a function of $b^2$; thus choosing slices with an equal number of events leads to equally spaced values of $b^2$ estimates. However, it should never be forgotten that we are dealing with $b^2$ and not with $b$. Even for ten slices of equal cross section, the first slice corresponds to $\tilde{b}^2=0.05$ or $\tilde{b}=0.22$, a value which is far from zero; on the other hand, one should keep in mind that central collisions have zero cross section. A last comment concerns the limit of the geometrical cross section at $R_t+R_p$. It is not as sharp as assumed here because real nuclei are diffuse and nucleon-nucleon cross sections are not infinite. However, this effect should be non-negligible, and the estimated $b^2$ smaller than the real one, only for rather peripheral collisions between light nuclei, whereas our field of interest is focused on central collisions between heavy nuclei.

We have proposed a method to estimate the impact parameter of relativistic nucleus-nucleus collisions. This method is based on the assumptions that the total cross section is purely geometrical and that there is a strong correlation between multiplicity and impact parameter. Its reliability has been checked within the framework of intranuclear cascade simulations. With such an impact-parameter scale, experimental results obtained with multiplicity selection can be compared consistently to model predictions. This method could be used also in other energy domains with any variable that available theoretical models could show to be reasonably well correlated with the impact parameter of the collisions. This seems to be true at intermediate energy for the mid-rapidity charge;[15] the average parallel velocity is also being used for asymmetric systems.[16] This could also be true at ultrarelativistic energies with the most commonly used variables,[17] i.e., total multiplicity, transverse energy, or zero-degree energy.


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