Branching ratios in low-energy deuteron-induced reactions

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We consider (d,p) and (d,n) reactions on light nuclei at low energies. A simple estimate using the second-order distorted-wave Born approximation shows that Coulomb-induced predissociation of the deuteron influences the relative rate by less than 10%. This disagrees with a previous explanation of experiments involving ^{6}Li targets and invalidates speculations about such effects in "cold fusion" experiments.

I. INTRODUCTION

The influence of the Coulomb field is a familiar and dominant feature in many nuclear reactions. Apart from the usual Gamow penetration through the barrier, one of the earliest phenomenon considered was the disruption of the deuteron by the Coulomb field of the target. This was used by Oppenheimer and Phillips[1] to deduce the binding energy of the deuteron from (d,n) excitation functions measured by Lawrence *et al.*[2]

Oppenheimer and Phillips (OP) noted that the Coulomb field of the target nucleus acts only on the proton in the deuteron, not on the deuteron's center of mass. This leads to an effective polarization of the deuteron or, more precisely, a p-wave component in the deuteron's internal wave function. The degree of polarization depends sensitively on the deuteron binding energy and, OP concluded, enhances the (d,n) cross sections beyond the simple Gamow form.

Simple arguments might suggest that the OP process should also modify the relative rates of the (d,n) and (d,p) reactions on isospin zero targets. Following Coulomb disruption of the deuteron, the neutron need not penetrate the Coulomb barrier to reach the nucleus, while the proton must. Hence, one might expect that

$$\Gamma=\sigma_{(d,n)}/\sigma_{(d,p)}\leq 1 \ .$$

There are, however, many other factors that could cause Γ to differ from unity, including differing Q values, optical potentials, and Coulomb distortion of the final-state proton wave function; isospin mixing in complex nuclei is also a consideration. Such effects are expected to be smaller for lighter targets and can, in any event, be accounted for accurately through conventional distorted-wave Born approximation (DWBA) calculations.

Cecil *et al.*[3] measured the relative rates of the ^{6}Li(d,n_1) and ^{6}Li(d,p_1) reactions and claimed to have demonstrated the influence of the OP process on Γ. They found that their data agreed with conventional DWBA calculations at high bombarding energies ($E_d>500$ keV), but became systematically smaller than predictions as the bombarding energy was lowered; at $E_d=60$ keV, the shortfall was some 20%. The OP process has also been of interest in connection with recent "cold fusion" experiments as a mechanism for suppressing $d+d\rightarrow{}^3\text{He}+n$ relative to $d+d\rightarrow{}^3\text{H}+p$ (Ref. 4), as required by the experimental claims.[5] Crucial here is the magnitude of the OP effect and its variation with energy, particularly at energies below those accessible experimentally.

In this paper we estimate the influence of the OP process on the branching ratio. Apart from the calculations of Ref. 3, with which we take serious issue in the following, this has not been done quantitatively before. We find that for deuteron and ^{6}Li targets, the OP corrections to Γ are small ($<10\%$) and are largely independent of energy. Thus, we are left without a simple explanation of the data in Ref. 3 and believe that the OP process is irrelevant to low-energy $d+d$ reactions.

Our presentation is organized as follows. We begin in Sec. II by developing a simple zero-range approximation for the second-order DWBA expression for a (d,p) or (d,n) reaction. In Sec. III, we discuss the calculations of Ref. 3 in light of these expressions. In Sec. IV, we present some schematic calculations of Γ for d, ^{6}Li, and heavy targets. Throughout, our emphasis is not on doing the most sophisticated calculation possible, but rather on rough estimates of the OP correction to Γ.

II. OP CORRECTIONS TO DWBA

A. Formulation

We consider (d,p) and (d,n) reactions on an infinitely massive, structureless target nucleus of charge Z. The coordinates describing this system are the target-proton and target-neutron separations $(\mathbf{r}_p,\mathbf{r}_n)$ or, equivalently, the internucleon distance $\mathbf{r}=\mathbf{r}_p-\mathbf{r}_n$ and center-of-mass position $\mathbf{R}=(\mathbf{r}_p+\mathbf{r}_n)/2$. We take the Hamiltonian to be $(\hbar=1)$

$$H=T_p+T_n+V_p(\mathbf{r}_p)+V_n(\mathbf{r}_n)+V_{pn}(\mathbf{r}) \ . \tag{1}$$

Here, $T_{p,n}=-\nabla^2_{r_{p,n}}/2M$ are the nucleon kinetic energies (M is the nucleon mass), $V_{p,n}$ describe the interactions of the nucleons with the target, and V_{pn} is the internucleon potential. An alternative representation of the Hamiltonian is

$$H = T_r + V_{pn} + T_R + U_d(R) + (V_p + V_n - U_d) , \quad (2a)$$

$$\equiv H_r + H_R + W , \quad (2b)$$

where $T_r = -\nabla_r^2/M$ is the relative kinetic energy of the nucleons, $T_R = -\nabla_R^2/4M$ is the center-of-mass kinetic energy of the deuteron, and $U_d(\mathbf{R})$ is a distorting potential that we have added and subtracted. Clearly H_r and H_R describe the internal and center-of-mass motion of the deuteron, while W, which induces the nontrivial dynamics, can be made small by a judicious choice of U_d.

B. Second-order DWBA

The T-matrix element describing a (d,p) reaction can be written in the "post" representation as[6]

$$T_{(d,p)} = \langle \phi_B(\mathbf{r}_n)\chi_{\mathbf{k}_p}^{(-)}(\mathbf{r}_p)| V_{pn} |\Psi_{k_d}^{(+)}\rangle , \quad (3)$$

where ϕ_B is the bound-state wave function of the neutron, which solves

$$(T_n + V_n)\phi_B = -\epsilon_n \phi_B ,$$

with ϵ_n the neutron binding energy. The distorted wave $\chi_{k_p}^{(-)}(\mathbf{r}_p)$ is the solution to the one-body problem

$$(T_p + V_p)\chi_{\mathbf{k}_p}^{(-)} = E_p \chi_{\mathbf{k}_p}^{(-)} \quad (4)$$

that asymptotically approaches $e^{i\mathbf{k}_p\cdot\mathbf{r}_p}+$ "incoming spherical wave," and $\Psi_{k_d}^{(+)}$ is the exact outgoing-wave solution to the full Schrödinger equation

$$(E - H)\Psi_{\mathbf{k}_d}^{(+)} = 0 , \quad (5)$$

where $E = E_d - \epsilon_d$ and $\epsilon_d = 2.22$ MeV is the deuteron binding energy.

In the DWBA, we rewrite (5) as

$$(E - H_R - H_r)\Psi_{\mathbf{k}_d}^{(+)} = W\Psi_{\mathbf{k}_d}^{(+)} \quad (6)$$

so that to first order in W,

$$\Phi_{k_d}^{(+)} \approx \phi_d(\mathbf{r})\chi_{\mathbf{k}_d}^{(+)}(\mathbf{R}) + \frac{1}{E^+ - H_R - H_r} W\phi_d\chi_{\mathbf{k}_d}^{(+)} . \quad (7)$$

Here, ϕ_d is the internal wave function of the deuteron solving

$$(T_r + V_{pn})\phi_d(\mathbf{r}) = -\epsilon_d\phi_d(\mathbf{r}) , \quad (8)$$

$\chi_{\mathbf{k}_d}^{(+)}(\mathbf{R})$ is the distorted deuteron wave solving

$$(T_R + U_d)\chi_{\mathbf{k}_d}^{(+)}(\mathbf{R}) = E_d\chi_{\mathbf{k}_d}^{(+)}(\mathbf{R}) \quad (9)$$

with outgoing-wave boundary conditions, and E^+ indicates $\lim_{\eta\to 0^+}(E + i\eta)$. The amplitude (3) then becomes

$$T_{(d,p)} = T_{(d,p)}^{(0)} + T_{(d,p)}^{(1)} , \quad (10a)$$

$$T_{(d,p)}^{(0)} = \langle \phi_B\chi_{\mathbf{k}_p}^{(-)}| V_{pn} |\phi_d\chi_{\mathbf{k}_d}^{(+)}\rangle , \quad (10b)$$

$$T_{(d,p)}^{(1)} = \langle \phi_B\chi_{\mathbf{k}_p}^{(-)}| V_{pn}\frac{1}{E^+ - H_R - H_r} W |\phi_d\chi_{\mathbf{k}_d}^{(+)}\rangle . \quad (10c)$$

Here, $T^{(0)}$ is the usual DWBA amplitude and $T^{(1)}$ is the first-order correction. Clearly, analogous expressions can be written down for the (d,n) cross section.

We will be interested in the difference between the (d,p) and (d,n) cross sections. These will be due to both "static" effects[7] (different Q values, $V_p \neq V_n$) and dynamic effects associated with W. The relevant part of W is that due to the Coulomb interaction. In particular,

$$W = V_n + V_p - U_d \approx U_c(\mathbf{R} - \mathbf{r}/2) - U_c(\mathbf{R}) , \quad (11a)$$

$$\approx \mathbf{r}/2\cdot\nabla U_c(\mathbf{R}) , \quad (11b)$$

where U_c is the Coulomb potential generated by the target. This term is odd under isospin symmetry (i.e., under $\mathbf{r}\to -\mathbf{r}$) so that, in the absence of static effects, we may put

$$T_{(d,p)}^{(0)} = T_{(d,n)}^{(0)} ,$$
$$T_{(d,p)}^{(1)} = -T_{(d,n)}^{(1)} , \quad (12)$$

where $T_{(d,p)}^{(0)}$ is given by Eq. (10b) as

$$T_{(d,p)}^{(0)} = \int d\mathbf{R}\, d\mathbf{r}\phi_B^*(\mathbf{R} - \mathbf{r}/2)\chi_{\mathbf{k}_p}^{(-)*}(\mathbf{R} + \mathbf{r}/2) \times V_{pn}(\mathbf{r})\phi_d(\mathbf{r})\chi_{\mathbf{k}_d}^{+}(\mathbf{R}) \quad (13)$$

and

$$T_{(d,p)}^{(1)} = \tfrac{1}{2}\int d\mathbf{r}\, d\mathbf{R}\, d\mathbf{r}'\, d\mathbf{R}'\phi_B^*(\mathbf{R} - \mathbf{r}/2) \times\chi_{\mathbf{k}_p}^{(-)*}(\mathbf{R} + \mathbf{r}/2)V_{pn}(\mathbf{r}) \times G_E(\mathbf{R}\mathbf{r};\mathbf{R}'\mathbf{r}')\mathbf{r}'\cdot\nabla U_c(\mathbf{R}')\phi_d(\mathbf{r}')\chi_{\mathbf{k}_d}^{(+)}(\mathbf{R}') \quad (14)$$

with the Green's function

$$G_E(\mathbf{R}\mathbf{r};\mathbf{R}'\mathbf{r}') = \langle \mathbf{R}\mathbf{r}|\frac{1}{E^+ - H_R - H_r}|\mathbf{R}'\mathbf{r}'\rangle . \quad (15)$$

More generally, $T_{(d,p)}^{(0)} \neq T_{(d,n)}^{(0)}$ and $|T_{(d,p)}^{(1)}| \neq T_{(d,n)}^{(1)}|$ because of static effects. As the cross section is proportional to $k|T|^2$, where k is the wave number of the outgoing nucleon, we have for the ratio of cross sections

$$\Gamma = \frac{\sigma_{(d,n)}}{\sigma_{(d,p)}} = \frac{k_n}{k_p}\frac{|T_{(d,n)}^{(0)} - T_{(d,n)}^{(1)}|^2}{|T_{(d,p)}^{(0)} + T_{(d,p)}^{(1)}|^2} \approx \left[1 - 4\mathrm{Re}\frac{T^{(1)}}{T^{(0)}}\right] , \quad (16)$$

where the approximation is valid in the absence of static effects. The magnitude and energy dependence of Γ will be the focus of our study.

C. Zero-range approximation

We begin our analysis of Eqs. (13) and (14) by noting that V_{pn} is likely the shortest-range function in the integrand. Thus, to evaluate (13) we make the usual zero-range approximation

$$V_{pn}(\mathbf{r})\phi_d(\mathbf{r}) \sim D_0\delta(\mathbf{r}) , \quad (17a)$$

$$D_0 = \int d\mathbf{r}\, V_{pn}(\mathbf{r})\phi_d(\mathbf{r}) , \quad (17b)$$

so that

$$T^{(0)}_{(d,p)}=D_0\int d\mathbf{R}\phi^*_B(\mathbf{R})\chi^{(-)}_{\mathbf{k}_p}(\mathbf{R})\chi^{(+)}_{\mathbf{k}_d}(\mathbf{R}) \ . \tag{18}$$

To similarly reduce $T^{(1)}_{(d,p)}$, note that (14) can be written as

$$T^{(1)}_{(d,p)}=\int d\mathbf{R}\,d\mathbf{R}'\,d\mathbf{r}\phi^*_B(\mathbf{R}-\mathbf{r}/2)\chi^{(-)*}_{\mathbf{k}_p}(\mathbf{R}+\mathbf{r}/2)\times D_1(\mathbf{R},\mathbf{R}';\mathbf{r})\chi^{(+)}_{\mathbf{k}_d}(\mathbf{R}') \ , \tag{19a}$$

$$D_1(\mathbf{R},\mathbf{R}';\mathbf{r})=\tfrac{1}{2}\nabla U_c(\mathbf{R}')\cdot\int d\mathbf{r}'\,V_{pn}(r)G_E(\mathbf{R}\mathbf{r};\mathbf{R}'\mathbf{r}')\mathbf{r}'\phi_d(r') \ . \tag{19b}$$

From the rotational invariance of G, the $\mathbf{r}'$ integral must be proportional to $\hat{\mathbf{r}}$ and $V(r)$ makes D_1 short range in r. Thus, we can put

$$D_1(\mathbf{R},\mathbf{R}';\mathbf{r})=-2\Delta(\mathbf{R},\mathbf{R}')\hat{\mathbf{R}}'\cdot\nabla\delta(\mathbf{r}) \ , \tag{20}$$

with

$$\Delta(\mathbf{R},\mathbf{R}')=\tfrac{1}{12}U'_c(R')\int d\mathbf{r}\,d\mathbf{r}'V_{pn}(r)G_E(\mathbf{R}\mathbf{r};\mathbf{R}'\mathbf{r}')\mathbf{r}'\cdot\mathbf{r}\phi_d(r'). \tag{21}$$

Here, $U'_c(R')\equiv dU_c/dR'$. Upon inserting (20) into (19a) and performing the $\mathbf{r}$ integral by parts, we have

$$T^{(1)}_{(d,p)}=\int d\mathbf{R}\,d\mathbf{R}'[\phi^*_B(\mathbf{R})\nabla\chi^{(-)*}_{\mathbf{k}_p}(\mathbf{R})-\chi^{(-)*}_{\mathbf{k}_p}(\mathbf{R})\nabla\phi^*_B(\mathbf{R})]\cdot\Delta(\mathbf{R},\mathbf{R}')\hat{\mathbf{R}}'\chi^{(+)}_{\mathbf{k}_d}(\mathbf{R}') \ . \tag{22}$$

As we are interested in the OP process at the very lowest energies, we assume that $\chi^{(+)}_{\mathbf{k}_d}$ is pure s wave. (The contribution from p waves is estimated in Sec. II D.) Then the rotational invariance of Δ implies that the $\mathbf{R}'$ integral in (22) is proportional to $\hat{\mathbf{R}}$, and we can write

$$T^{(1)}_{(d,p)}=\int d\mathbf{R}\,d\mathbf{R}'\left[\phi^*_B(\mathbf{R})\frac{\partial}{\partial R}\chi^{(-)*}_{\mathbf{k}_p}(\mathbf{R})-\chi^{(-)*}_{\mathbf{k}_p}(\mathbf{R})\frac{\partial}{\partial R}\phi_B(\mathbf{R})\right]\times\Delta(\mathbf{R},\mathbf{R}')\hat{\mathbf{R}}\cdot\hat{\mathbf{R}}'\chi^{(+)}_{\mathbf{k}_d}(R') \ . \tag{23}$$

A more transparent form of (23) that can compare directly to (18) is

$$T^{(1)}_{(d,p)}=\int d\mathbf{R}\,d\mathbf{R}'\phi^*_B(\mathbf{R})\chi^{(-)*}_{\mathbf{k}_p}(\mathbf{R})B(\mathbf{R},\mathbf{R}')\chi^{+}_{\mathbf{k}_d}(R') \ ,$$
$$B(\mathbf{R},\mathbf{R}')=\hat{\mathbf{R}}\cdot\hat{\mathbf{R}}'\Delta(\mathbf{R},\mathbf{R}')\frac{\partial}{\partial R}\left[\ln\frac{\chi^{(-)*}_{\mathbf{k}_p}(\mathbf{R})}{\phi^*_B(\mathbf{R})}\right] \ . \tag{24}$$

D. Reduction to radial integrals

A reduction to radial form follows by putting

$$\phi_B(\mathbf{R})=f_B(R)Y_{lm}(\hat{\mathbf{R}})/R;\chi^{(-)}_{\mathbf{k}_p}(\mathbf{R})=C_l(R)Y^*_{lm}(\hat{\mathbf{R}})/R \ , \tag{25}$$

so that

$$T^{(0)}_{(d,p)}=D_0\int_0^\infty dRf_B(R)C_l(R)\chi^{(+)}_{\mathbf{k}_d}(R) \ , \tag{26}$$

and

$$T^{(1)}_{(d,p)}=\int_0^\infty dR\int_0^\infty dR'f_B(R)C_l(R)\beta(R,R')\chi^{(+)}_{\mathbf{k}_d}(R'). \tag{27}$$

where

$$\beta=\frac{R'^2}{4\pi}\int d\hat{\mathbf{R}}\,d\hat{\mathbf{R}}'\,B(\mathbf{R},\mathbf{R}')=\frac{R'^2}{48\pi}U'_c(R')\frac{\partial}{\partial R}\left[\ln\frac{C_l(R)}{f_B(R)}\right]\int d\hat{\mathbf{R}}\,d\hat{\mathbf{R}}'\,\hat{\mathbf{R}}\cdot\hat{\mathbf{R}}'\int d\mathbf{r}\,d\mathbf{r}'V_{pn}(r)G_E(\mathbf{R}\mathbf{r};\mathbf{R}'\mathbf{r}')\mathbf{r}'\cdot\mathbf{r}'\phi_d(r') \ . \tag{28}$$

The required r,r' integrals are best handled in momentum space, if we make the reasonable assumption that V_{np} vanishes in p waves (pure Serber force). Thus,

$$\int d\mathbf{r}\,d\mathbf{r}'V_{pn}(r)G_E(\mathbf{R}\mathbf{r};\mathbf{R}'\mathbf{r}')\mathbf{r}'\cdot\mathbf{r}\phi_d(r')=\int\frac{d\mathbf{q}}{(2\pi)^3}V'_{pn}(q)\phi'_d(q)G^{(R)}_{E-q^2/M}(\mathbf{R};\mathbf{R}') \ , \tag{29}$$

where

$$\begin{bmatrix}V_{pn}(q)\\ \phi_d(q)\end{bmatrix}=\int d\mathbf{r}\,e^{-i\mathbf{q}\cdot\mathbf{r}}\begin{bmatrix}V_{pn}(r)\\ \phi_d(r)\end{bmatrix}=\frac{4\pi}{q}\int_0^\infty \sin qr\begin{bmatrix}V_{pn}(r)\\ \phi_d(r)\end{bmatrix}r\,dr \ , \tag{30}$$

$$V'_{pn}(q)=dV_{pn}(q)/dq \ ,$$
$$\phi'(q)=d\phi(q)/dq \ ,$$

and the center-of-mass Green's function is

$$G^{(R)}_\omega(\mathbf{R};\mathbf{R}')=\langle\mathbf{R}|\frac{1}{\omega^+-H_R}|\mathbf{R}'\rangle \ . \tag{31}$$

The rotational invariance of $G^{(R)}$ allows the expansion

$$G^{(R)}_\omega(\mathbf{R};\mathbf{R}')=\sum_{l=0}\frac{(2l+1)}{RR'}g_{l\omega}(R,R')P_l(\hat{\mathbf{R}}\cdot\hat{\mathbf{R}}') \ , \tag{32}$$

so that

$$B(R,R')=\frac{1}{6\pi}\left[\frac{R'}{R}\right]U'(R')\frac{\partial}{\partial R}\left[\ln\frac{C_l(R)}{f_B(R)}\right]\times\int_0^\infty q^2dqV'_{pn}(q)\phi'_d(q)g_{1E-q^2/M}(R,R') . \tag{33}$$

In terms of the solutions to the radial equation for the R motion,

$$g_{1\omega}(R,R')=-\frac{M}{\pi K}F_{1\omega}(R_<)H_{1\omega}(R_>) , \tag{34}$$

where $K^2=-4M\omega$. Here, F is the solution that is regular at the origin and H is the solution that decays exponentially at large R when ω is negative (i.e., E_d is less than ϵ_d) and approaches a pure outgoing wave when ω is positive. The solutions are normalized so that their R-independent Wronskian is $F'H-H'F=K$. Thus, finally,

$$\beta(R,R')=-\frac{M}{6\pi^2}\left[\frac{R'}{R}\right]U_c'(R')\frac{\partial}{\partial R}\left[\ln\frac{C_l(R)}{f_B(R)}\right]\times\int_0^\infty\frac{q^2dq}{K}V'(q)\phi'_d(q)F_{1\omega}(R_<)H_{1\omega}(R_>) . \tag{35}$$

The incoming deuteron and outgoing wave functions are given by Coulomb wave functions. For low energies, the incoming wave function is s wave, and the outgoing nucleon has $l=l_B$:

$$\chi_d(R')=F_0(k_dR')/(k_dR')$$

and

$$C_l(R)=F_l(k_{p,n}R)/k_{p,n} ,$$

where $k_d=\sqrt{4E_dM}$, $k_{p,n}=\sqrt{2E_{p,n}M}$ the wave vector of the outgoing nucleon, p or n, and $E_{p,n}$ is its energy, $E_{p,n}=E_d+Q$.

E. P-wave contributions

To estimate OP effects on the p-wave cross section, we take the incoming deuteron wave function to be a Coulomb-distorted plane wave, whose partial wave expansion is

$$\chi^{(+)}_{\mathbf{k}_d}(R')=4\pi\sum_{lm}i^le^{i\sigma_l}\frac{F_l(k_dR')}{k_dR'}Y^*_{lm}(\hat{\mathbf{k}}_d)Y_{lm}(\hat{\mathbf{R}}') . \tag{36}$$

If we take the quantization axis to be along $\hat{\mathbf{k}}_d$ (so that only $m=0$ terms are nonvanishing) and keep only the s- and p-wave parts of the expansion, we can use the identity

$$\sigma_1=\sigma_0+\tan^{-1}\eta \tag{37}$$

to find, within irrelevant common factors,

$$T_{(d,p),m_B}=T_{(d,p)s}+\frac{i-\eta}{(1+\eta^2)^{1/2}}T_{(d,p)p,m_B} , \tag{38}$$

where m_B is the magnetic quantum number of the bound neutron. The total cross section is found by summing the cross sections for different m_B. Here, the first term comprises the s-wave contributions already calculated in Eqs. (13), and (14),

$$T_{(d,p),s}=T^{(0)}_{(d,p)}+T^{(1)}_{(d,p)} , \tag{39}$$

and the second is the subject of the present discussion,

$$T_{(d,p)p,m_B}=T^{(0)}_{(d,p)p,m_B}+T^{(1)}_{(d,p)p,m_B} . \tag{40}$$

For the zeroth-order p-wave contribution, we find, after doing the angular integrals,

$$T^{(0)}_{(d,p)p,m_B}=\tfrac{1}{3}\sqrt{(2l_B+1)(2l'+1)}\times(l_Bl'00|10)(l_Bl'm_B-m_B|10)\times\int_0^\infty dR\,f_B(R)C_l'(R)F_1(k_dR)/k_dR , \tag{41}$$

where $l'=l_B\pm1$ is the angular momentum of the outgoing proton. We find the OP correction by using the expansion (36) in Eq. (22) and considering the p-wave contribution. The $\hat{\mathbf{R}}'$ integral now yields terms proportional to both $\hat{\mathbf{R}}$ and $\hat{\mathbf{k}}_d$, so we expand the gradient as

$$\nabla=\hat{\mathbf{R}}\frac{\partial}{\partial R}+\hat{\boldsymbol{\theta}}\frac{1}{R}\frac{\partial}{\partial\theta} . \tag{42}$$

The vector $\hat{\boldsymbol{\theta}}$ can be written as

$$\hat{\boldsymbol{\theta}}=\cot\theta\hat{\mathbf{R}}-\text{cosec}\theta\hat{\mathbf{k}}_d , \tag{43}$$

of which only the $\hat{\mathbf{k}}_d$ part is needed since $\hat{\boldsymbol{\theta}}\cdot\hat{\mathbf{R}}=0$. The components of the Green's function expansion of Eq. (32) that are retained are now $g_{0\omega}$ and $g_{2\omega}$. We thus find the p-wave OP correction to the (d,p) reaction to be

$$\begin{aligned}T^{(1)}_{(d,p)p,m_B}=&-\frac{M}{6\pi^2}\sqrt{(2l_B+1)(2l'+1)}(l_Bl'00|10)(l_Bl'm_B-m_B|10)\\&\times\int\frac{dR}{R}f_BC_{l'}\frac{\partial}{\partial R}\left[\ln\frac{C_{l'}}{f_B}\right]\int q^2dqV'(q)\phi'_d(q)\int R'\,dR'U_c'(R')\frac{F_1(k_dR')}{k_dR'}[g_{0\omega}+2g_{2\omega}]\\&+\frac{M}{6\pi^2}\int\frac{dr}{r^2}f_BC_l'\int R'\,dR'U_c'(R')\int q^2dqV'(q)\phi'_d(q)\frac{F_1(k_dR')}{k_dR'}\\&\times\int d\hat{\mathbf{R}}\left[Y^*_{l_Bm_B}\frac{\partial}{\partial\theta}Y^*_{l'm'}-Y^*_{l'm'}\frac{\partial}{\partial\theta}Y^*_{l_Bm_B}\right]\text{cosec}\theta[g_{0\omega}+\tfrac{2}{3}\sqrt{4\pi/5}Y_{20}(\hat{\mathbf{R}})g_{2\omega}] .\end{aligned} \tag{44}$$

Our schematic calculations presented below show that the zeroth-order p-wave amplitude is significant ($\approx 20\%$ of the s-wave amplitude for ^{6}Li when $l'=2$, although less than 10% for other partial waves), but that the OP correction to it is still small, $\approx 10^{-2}$.

III. CRITIQUE OF PREVIOUS CALCULATIONS

Reference 3 presents two different estimates of the OP effect on Γ in $^6\text{Li}+d$ reactions. One of these is based on a shift in the effective energy at which the transfer occurs due to Coulomb polarization of the deuteron. The other, more microscopic, approach is an attempt to correct the DWBA. The first method significantly overestimated the OP effect relative to what would be required to explain the data, while the second was very small and so could not reproduce the decrease of Γ with decreasing bombarding energy. We believe that both of these estimates are flawed, independent of whether or not they explain the experiment. Our reasoning is given in the following paragraphs.

In the effective energy argument, the authors note that polarization of the deuteron a distance R from the target corresponds to a net energy gain of

$$\Delta E = -\tfrac{1}{2}\alpha \frac{Z^2 e^2}{R^4} ,$$

where $\alpha = 0.64$ fm^3 is the static polarizability of the deuteron. They then assert that "the energy available for the reaction" should be increased by $|\Delta E|$ for the (d,p) reaction and decreased by $|\Delta E|$ for the (d,n) reaction (when the proton is closer to the nucleus). To implement this idea, the authors compute an average ΔE for a given E_d using the conventional DWBA to estimate the probability that the transfer occurs at different distances. The respective DWBA cross sections are then evaluated at $E_d \pm \Delta E$ to compute Γ. Although ΔE is small, the rapid energy dependence of the cross sections means that the effect can be large; a shift of $\Delta E = 2$ keV at $E_d = 100$ keV will result in $\Gamma = 0.75$.

Apart from being unjustified by reaction theory, the difficulty with this prescription is that the polarization energy ΔE is independent of the orientation of the deuteron. Indeed, as the ground state of the deuteron has a definite parity, it cannot by "oriented' to bring the proton closer or further from the target. This can only be done by explicitly introducing odd-parity components in the relative wave function, as we have shown in the previous section. Although such admixtures are implicit in α, an explicit treatment is needed to enhance the probability that the proton is, on the average, further away from the nucleus than is the neutron. Hence, simply ascribing different energies to the (d,p) and (d,n) reactions does not account for the OP effect.

The more microscopic approach to the OP process presented in Ref. 3 is similar to that we have presented in Sec. II. However, the authors have inexplicably substituted W for V_{pn} in our Eq. (10c). [More precisely they have used the dipole approximation to W, Eq. (11b), instead of V_{pn} in Eq. (14).] This error results in a term that is *second* order in W, rather than first. Although incorrect, it does allow a simple zero-range approximation: relative p-wave motion in the deuteron is only implicit, and the net result is just a renormalization of D_0 as defined in Eq. (17). There is thus no need for the more involved derivative formulation we have introduced in Eq. (20). However, as in the polarization argument above, isospin asymmetry must be made explicit in order to have an OP effect at all.

IV. SCHEMATIC CALCULATIONS

Although it is clearly possible to perform sophisticated calculations of the OP correction we have derived (suitably generalized to arbitrary incident angular momenta), our goal in the present work is to judge *qualitatively* whether the OP process is large enough to account for the measurements of Ref. 3, and whether the OP effect plays any role at all in low-energy $d+d$ interactions. The schematic calculations of Γ that we present in this section are sufficient to provide a negative answer to both of these questions.

A. Computational details

For our estimates we take a Hulthen form for the internal wave function of the deuteron,

$$\phi_d(r) = \phi_0 \frac{e^{-\alpha r} - e^{-\beta r}}{r} , \tag{45}$$

where

$$\phi_0 = \frac{\alpha\beta(\alpha+\beta)}{2\pi\mu} , \tag{46}$$

with $\alpha = \sqrt{\epsilon_d M} = 0.231$ fm^{-1}, $\beta = 5.39\alpha = 1.247$ fm^{-1}, and $\mu = \alpha - \beta = 1.102$ fm^{-1}. The corresponding internuclear potential is

$$V_{pn}(r) = V_0 \frac{e^{-\mu r}}{1 - e^{-\mu r}} , \tag{47}$$

with

$$V_0 = -(\beta^2 - \alpha^2)/M = 62.27 \text{ MeV} .$$

Taking the Fourier transform [Eq. (29)] and differentiating with respect to the argument q, we find

$$\phi'(q) = -8\pi\phi_0 q \left[\frac{1}{(\alpha^2+q^2)^2} - \frac{1}{(\beta^2+q^2)^2} \right] \tag{48}$$

and

$$V'(q) = 32\pi\mu V_0 q \sum_{n=1}^{\infty} \frac{n}{(n^2\mu^2+q^2)^3} . \tag{49}$$

We take the bound-state wave function to be

$$f_B(r) = k_B^{1/2} e^{-k_B r} , \tag{50}$$

with $k_B = \sqrt{2\epsilon_B M}$ and $\epsilon_B = \epsilon_d + Q$ the wave number and binding energy of the captured nucleon; we have omitted here unimportant constant prefactors. The magnitude of our results (but not their energy dependence) is somewhat sensitive to the form we choose for f.

In the following, we will consider Deuterium, ^{6}Li, and

^{27}Al targets, for which we take l_B to be 0, 1, and 2, respectively. We will also multiply our values of Γ for ^{6}Li, by ratio of spectroscopic factors $S_n/S_p = 1.14$ (Ref. 3), but assume that this factor is unity for the other two targets. It should be noted that our assumption of an infinitely massive target is not unreasonable for ^{6}Li and ^{27}Al, but is clearly invalid for Deuterium. We have therefore used the $d+d$ reduced mass to describe the R motion in this latter case. Apart from this, we have made no further simplification in our numerical evaluation of Eqs. (26), (27), (35), and (44).

B. Results and discussion

Figure 1 shows the ratios $T^{(1)}_{(d,n)}/T^{(0)}_{(d,n)}$ and $T^{(1)}_{(d,p)}/T^{(0)}_{(d,p)}$ for ^{2}D, ^{6}Li, and ^{27}Al for deuteron energies up to 5 MeV. The ratios are small ($\approx 10^{-2}$) in all three cases and roughly proportional to the charge of the target nucleus.

Figure 2 shows the ratio of cross sections Γ [see Eq. (16)] for the three targets. The solid lines show this ratio without the OP correction (i.e., that due to purely static effects), while the dashed lines show the ratio *with* OP correction. The ratio is ≈ 1 for ^{2}D and ^{6}Li but ≈ 0.3 for ^{27}Al. This last is because, in contrast to ^{2}D and ^{6}Li, the Q value for the ^{27}Al$(d,n)^{28}$Si reaction ($Q = 9.36$ MeV) is larger than that for the ^{27}Al$(d,p)^{28}$Al reaction ($Q = 5.50$ MeV). However, in all cases the OP correction is remarkably constant with energy and simply reduces Γ slightly without changing its shape. In particular, there is no "turnover" at low energies as claimed for ^{2}D (Ref. 4) and observed for ^{6}Li (Ref. 3).

In support of the small OP correction indicated by our calculation, we note that the branching ratio for the $d+d$ reaction has been measured down to $E_d = 3$ keV (Ref. 8) with no obvious anomalies in the energy dependence. These data imply a zero-energy value of $\Gamma = 0.94 \pm 0.12$. Muon catalyzed $d+d$ fusion (Ref. 9), which probes a similar energy range, is also consistent with $\Gamma \approx 1$. We are not able to explain the ^{6}Li data

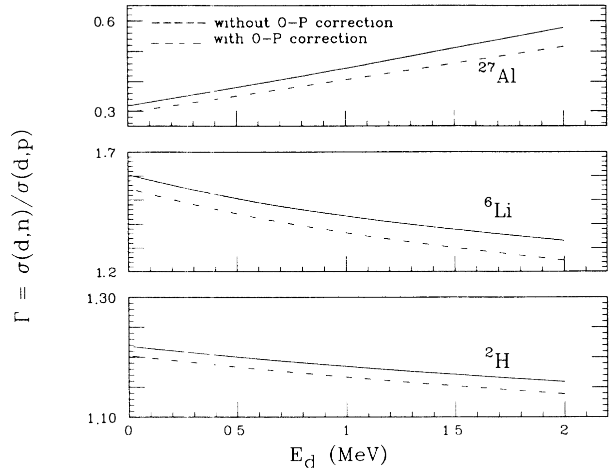

FIG. 2. $\Gamma \equiv \sigma_{(d,n)}/\sigma_{(d,p)}$ for ^{2}H, ^{6}Li, and ^{26}Al targets.

presented in Ref. 3, but note that static violations of isospin symmetry at the 15% level are not uncommon in light nuclei, and, in any event, the observed values of Γ are far too large to be relevant in discussions of "cold fusion."

Given the small OP corrections we find, one might well ask how OP were able to extract the deuteron binding energy from the data of Ref. 2. To answer this question, we show in Fig. 3 our calculated (but unnormalized and uncorrected) ^{27}Al$(d,p)^{28}$Al excitation function for $\epsilon_d = 2.4$, 2.2, and 2.0 MeV; these correspond to the original OP curves contained in Fig. 2 of Ref. 2. While the curves are distinct from each other, the differences are solely kinematic in origin (e.g., the wave numbers of the outgoing nucleons) and are *unrelated* to the Coulomb polarization of the deuteron; i.e., to the OP effect itself.

In summary, we have derived expressions for the corrections to DWBA amplitudes for low-energy (d,p) and (d,n) reactions associated with the Coulomb disruption of the deuteron. These are at variance with previous

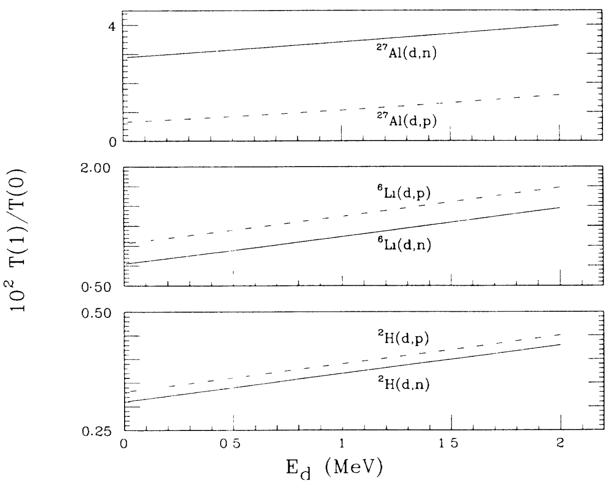

FIG. 1. $T^{(1)}/T^{(0)}$ for D(d,p)/D(d,n), ^{6}Li$(d,p)/^6$Li(d,n), and ^{27}Al$(d,p)/^{27}$Al(d,n).

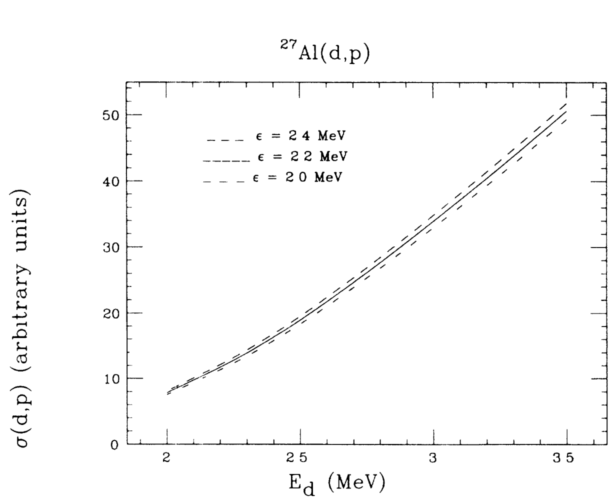

FIG. 3. ^{27}Al(d,p) excitation function for $\epsilon_d = 2.4$, 2.2, and 2.0 MeV.

work presented by Cecil *et al.* in Ref. 3. Schematic calculations show that the OP effect modifies the relative cross sections for (d,p) and (d,n) reactions at the few percent level in an energy-independent manner. These results are in agreement with data on $d+d$ interactions at low energies, but disagree with experimental results for ^{6}Li targets.

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