Schematic model for nuclear molecules as doorway states for fusion

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An elementary simple model for nuclear molecules is used to describe the molecular spectrum of the ^{12}C-^{12}C system. Through that model the molecular potential is determined. Without further parameters the total fusion cross section around and below the barrier is calculated with good results indicating a correlation between the molecular spectrum and fusion. It is concluded that nuclear molecules may possibly be the doorway states for fusion. The simplicity of the model used allows a deeper schematic insight of the mechanism of fusion.

I. INTRODUCTION

Extensive studies of subbarrier fusion of heavy ions revealed a discrepancy of the total fusion cross section to calculations of a simple quantum tunneling through the barrier.[1] The observed cross section in heavy systems is often higher by orders of magnitude and one would like to understand the reason for this. The first step is to look for a possible mechanism of fusion. In Ref. 2 indications were found that, for the ^{16}O-^{16}O system, the doorway state for fusion may be a nuclear molecule. If this is assumed to be correct the reaction mechanism could be as follows: Once the two nuclei form a nuclear molecule, the nuclei are trapped and, if time is sufficiently large, other processes, connected to compound nucleus formation, will lead to fusion of the system. This sounds plausible because a collective channel, as is the case for the above-mentioned process, normally has a higher probability to be realized.

To keep track we first consider a simple model that is easy to understand for nuclear molecules.[3] Furthermore, some assumptions are made of how this nuclear molecule is formed and how it forms a compound nucleus. We try to keep these assumptions very simple. This paper is organized as follows: In Sec. II we summarize the main features of an elementary, analytic model for nuclear molecules.[3] As one example we fit the molecular spectrum of the ^{12}C-^{12}C system, which will be used throughout the paper. We show that the molecular part determines the molecular potential, which in Sec. III will be used to calculate the total fusion cross section of the ^{12}C-^{12}C system *without further parameters*. The results in Sec. III are in good qualitative agreement with experiment and suggest that nuclear molecules may be the doorway states for fusion. In Sec. IV we present the conclusions and discuss the possible future application to heavier systems.

As one result the model presented demonstrates that in order to be consistent, ^{12}C has to have a large static deformation which agrees with Ref. 4 and assuming a spherical nucleus will lead to unphysical results for energies around the barrier. Furthermore, the model will indicate a common signature of the elastic cross section for nuclear molecules and the total fusion cross section around and below the barrier. In all steps the model exhibits its very elementary, also oversimplified, property which nevertheless might give a deeper insight into the fusion process.

The reason why we consider in this contribution only the ^{12}C-^{12}C system lies in the broad range of data available, both in the nuclear molecule and fusion sector. The intention is to learn how fusion may take place and to apply the model later on to heavy systems, where it will be more applicable, as we will see below.

II. AN ELEMENTARY MODEL FOR NUCLEAR MOLECULES

In Ref. 3 an elementary, analytic model for nuclear molecules was developed, whose main results will be summarized: The model is a geometrical one and assumes well-deformed nuclei that because of the reduced Coulomb repulsion, touch each other with their "nose" (see Fig. 1). In the case of ^{12}C-^{12}C system the difference in energy of the nose-nose orientation to the belly-belly orientation is roughly 6 MeV, assuming a constant charge distribution with $R_0 = 1.26\ A^{1/3}$,[1] (R_0 is the radius of the spherical nucleus) and an oblate deformation of $\beta = 0.66$.[4] This large deformation permits the applicability of the model.

Once the molecule is formed the system can undergo a variety of collective excitations as "butterfly" and "belly dancer" modes, rotation of the whole system, and β, γ vibrations of the individual nuclei. In Fig. 2 we show the spectrum of a ^{12}C nucleus. Note that it has a typical spectrum of a rotor. Assuming a $I(I+1)$ rule for the ground-state band the 4^+ state would only lie a little bit higher than the measured one.[5] Because of this, a geometrical description of ^{12}C must be justified, though it is

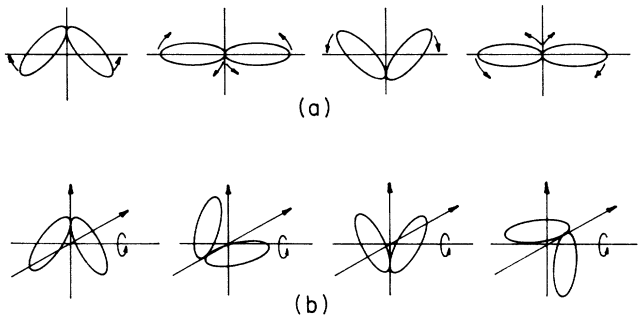

FIG. 1. (a) "Butterfly" and (b) "belly dancer" modes of a nuclear molecule in the nose-to-nose orientationn. The arrows indicate the collective motion at a given instant (Ref. 3).

considered to be one of the limiting cases. Figure 2 also shows that the first excited β-vibrational band is at 7.65 MeV above the ground state[5] which will turn out to be well above the Coulomb barrier, i.e., we can choose a simplified version of the nuclear molecule model excluding β and γ vibrations.

Note that the first excited state in ^{12}C at 4.44 MeV and with $J^P=2^+$ (P represents parity) is interpreted as a *rotational* state of an individual nucleus, therefore it *cannot* appear in the ^{12}C-^{12}C molecule of our model. The rotation of the ^{12}C nuclei is converted into a butterfly mode and a complete individual rotation is forbidden due to sticking together. The fact that in the spectrum of the molecule appears a molecular state at rather the same energy[6] is considered to be a coincidence and its nature will be explained below.

Excluding β and γ vibrations the spectrum of the molecule is to zero order given by[3]

$$E=\frac{\hbar^2}{2\mu r_0^2}[I(I+1)-K^2]+E_\epsilon(|K|+2n_\epsilon)+E_r n_r+E_0 \tag{1}$$

with $I=0,2,4,\ldots$ if $K=0$ and $I=K,K+1,K+2,\ldots$ if $K\neq 0$, and $K=$even.

In Eq. (1) r_0 is the distance of the two center of charges, $\mu=6m$ is the reduced mass ($mc^2=938$ MeV), I is the angular momentum, K the projection of I onto the intrinsic molecular axis (this axis is along the connection line of the two center of masses), n_ϵ the oscillation number of the butterfly mode, n_r the oscillation number of the relative vibration, E_ϵ gives the strength of the butterfly mode, E_r the strength of the relative vibration, and E_0 gives the energy of the ground state.

Note that the energy formula is nearly equivalent to the one of the rotation-vibration model.[7] This model was also used by Cindro[8] who approximated the nuclear model by a strongly deformed nucleus. The relative vibration is then equivalent to the β vibration and the γ mode is related to asymmetric modes, like the butterfly mode. There is, however, an essential difference of a factor $\frac{1}{2}$ in $E_\epsilon|K|$. This led to a not very satisfactory agreement of that simple model to experiment. We will see that with a new interpretation of the molecular states the agreement to experiment will be good.

The parameters $\hbar^2/2\mu r_0^2$, E_ϵ, E_r, and E_0 are fitted as follows (see Fig. 3): First we identify the lowest 4^+, 6^+, and 8^+ state as a member of the ground-state band whose 0^+ and 2^+ members are assumed not to be seen because of lying well below the barrier. Parting from the lowest 4^+ state and using the slope of the ground-state band, which gives $\hbar^2/2\mu r_0^2=0.11$ MeV, the position E_0 of the 0^+ ground state can be deduced at an energy of about 2.06 MeV. In order to fit E_ϵ we note that for $(I,K,n_\epsilon,n_r)=(2,2,0,0)$ and $(0,0,1,0)$, these two states are nearly degenerate. Due to the rotational part in (1) the

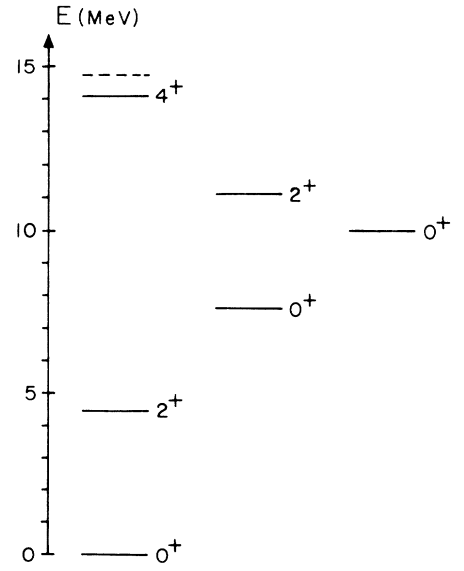

FIG. 2. Experimental spectrum of ^{12}C (Ref. 5). The dashed line gives the position of the 4^+ state, if a $I(I+1)$ rule is assumed and fixing the energy difference of the 2^+ state to the ground state.

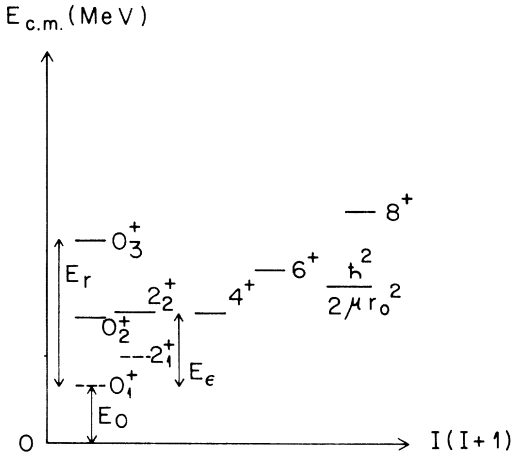

FIG. 3. Illustration of the fitting procedure of the molecular spectrum (detailed discussion in text). The states shown are only of an illustrative nature though their relative positions are borrowed from the molecular spectrum of ^{12}C-^{12}C.

2^+ state is slightly higher by $\hbar^2/\mu r_0^2$, which is very small. Such a nearly degenerate pair exists at the energy of 4.25 MeV for the 0^+ and at 4.62 MeV for the 2^+.[6] To them we assign therefore the quantum numbers $(K,n_\epsilon,n_r)=(0,1,0)$ and (2,0,0), respectively. Taking into account the energy difference to the ground state we get $E_\epsilon=1.13$ MeV. The next 0^+ state is interpreted as the first excited relative vibrational 0^+ state. It is at an energy of 5.80 MeV and its quantum numbers will be $(K,n_\epsilon,n_r)=(0,0,1)$. From the energy difference $E(0^+)-E_0$ of this 0^+ state we fit $E_r=3.74$ MeV using (1). The procedure of fitting is again illustrated in Fig. 3.

The calculated spectrum is given in Fig. 4 compared to the experimental results[6] and the calculation of Ref. 8. One immediately notes that the density of the states can be roughly reproduced while in Ref. 8 the density is too low. The model predicts a third 0^+ state at the energy 6.62 MeV, compared to the experimental value of 6.30 MeV, which cannot be reproduced by the model of Ref. 8. The main characteristics of the experimental spectrum can be reproduced by our model. This is important because later on we assume that fusion can take place only if there is a molecular state present. There will appear more states due to β and γ vibrations above 7.65 MeV$+E_0$ which are omitted here. This might lead at higher energies to a too small cross section.

Restricting to the relative motion we can deduce the molecular potential: the relative potential is given by a parabola $(C_r/2)(r-r_0)^2$ (r_0 is the position of the minimum), whose stiffness is determined through $E_r=\hbar\sqrt{C_r/\mu}$ and r_0 can be deduced through $\hbar^2/2\mu r_0^2$ giving $r_0\simeq 5.6$ fm. The position of the potential minimum is determined via $V_0=2.06-\frac{1}{2}E_r$. In the region where the Coulomb potential and the oscillator are of the same energy we interpolate the molecular potential using a combination of an attractive Wood-Saxon and a Coulomb type potential,

$$V=-\frac{V_1}{1+e^{(r-R)/a}}+\begin{cases}V_c/r^x, & r\approx r_{\text{barrier}}\\ Z^2e^2/r, & r\gtrsim r_{\text{barrier}}+2\text{ fm}\end{cases} \tag{2}$$

where $Z=6$, $R=7$ fm, $a=0.64$ fm and $V_c=36.69$ MeV, $x=0.86$, and $V_1=9.064$ MeV are adjusted in such a way that in the region $6\lesssim r\lesssim 11$ the potential V_c/r^x reproduces the Coulomb potential of an oblate disk with $\beta=0.66$. The deduced potential is given in Fig. 5. Note that (2) is the potential when the two nuclei touch each other with their nose and are not inclined with respect to each other.

The barrier turns out to be now at roughly 5.16 MeV instead of 6.21 MeV for spherical ^{12}C nuclei. It is important to mention that the oscillator part is fixed by the spectrum of the nuclear molecule and that it is consistent with the large deformation of the ^{12}C nuclei. For a sharp mass distribution the two ^{12}C nuclei touch each other along their major axes at $R=7$ fm. A Coulomb barrier at 6.21 fm would require too stiff oscillator potential and leads immediately within our model to a contradiction to the spectrum of the nuclear model. The consistency of our model gives us therefore a lot of confidence to proceed further.

The lack of the elementary model of nuclear molecules is the infinite lifetime and therefore zero width of the states due to the oscillator potential, i.e., all states are bound. As outlined in Ref. 3 this lack can be circumvent-

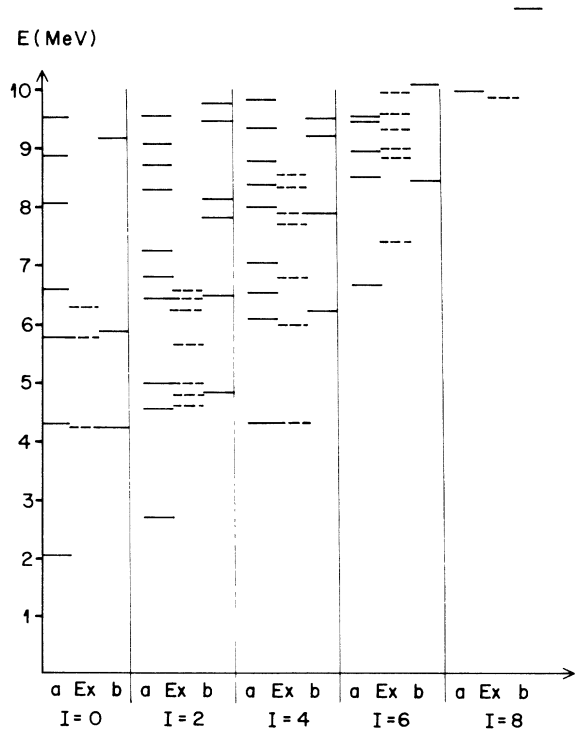

FIG. 4. Spectrum of the ^{12}C-^{12}C nuclear molecule. The experimental data are taken from Ref. 6, *a* is our calculation and *b* that of Ref. 8.

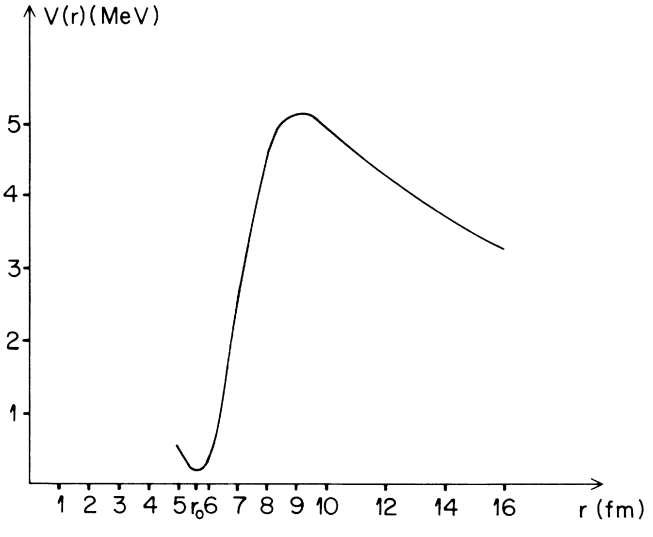

FIG. 5. The molecular potential is deduced from fitting the molecular spectrum. Note that the barrier is at 5.16 MeV instead of 6.21 MeV of Ref. 1 assuming spherical deformed ^{12}C nuclei.

ed assigning to each state a width using the Hill-Wheeler approximation.[9] (An exact treatment would shift simultaneously the states in energy. Due to the large width near the barrier this can be hopefully neglected). The partial width in the exit channel of each state is determined via[1,9]

$$\Gamma_i = \frac{\hbar\omega_r}{2\pi} T_i \tag{3a}$$

where

$$T_i = \frac{1}{1+\exp[(2\pi/\hbar\omega_b)(V_I - E_i)]} \tag{3b}$$

is the transition probability through the barrier, $\hbar\omega_r = E_r$, b refers to the barrier

$$\hbar\omega_b \simeq \left| -\frac{\hbar^2}{\mu}\frac{d^2V_b}{dr^2}\right|_{r=r_b}^{1/2}, \tag{3c}$$

E_i is the energy of the state and

$$V_I = V_b + \frac{\hbar^2}{2\mu r_b^2} I(I+1) \tag{3d}$$

with $V_b = 5.16$ MeV as the barrier energy. The position of the barrier is at $r_b \simeq 9$ fm. In 3(c) we approximated $\hbar\omega_b$ by using instead of V_I the value of V_b. We will now proceed to use all of this information to determine within our model *without any further parameter* the fusion cross section below and around the barrier.

III. THE TOTAL FUSION CROSS SECTION AND RESULTS FOR ^{12}C-^{12}C

As outlined in Refs. 1 and 2 the total fusion cross section is given by

$$\sigma_{\text{fus}} = \frac{2\pi}{k^2}\sum_I (2I+1) T_I(E_{\text{c.m.}}) . \tag{4}$$

Here $I = 0,2,4,...$ is the angular momentum of the partial wave and k is the asymptotic wave number with $\pi/k^2 = 10.83/E_{\text{c.m.}}$ for the ^{12}C-^{12}C system. The $E_{\text{c.m.}}$ is the center of mass energy which is equal to the energy of the beam. $T_I(E_{\text{c.m.}})$ is the probability to form a compound nucleus at the partial angular momentum I.

To calculate $T_I(E_{\text{c.m.}})$ we encounter a principal problem: The model of nuclear molecules, presented in Sec. II, is completely separated from its entrance and exit channel. In order to describe the fusion reaction process one has to superimpose assumptions for the formation and decay process. This will imply discrepancies in the quantitative description of the fusion cross section, though hopefully the detailed structure can still be explained.

The total probability $T_I(E_{\text{c.m.}})$ we divide in factors, one related to the entrance channel, i.e., how to form the intermediate molecular state, and another to the exit channel, i.e., how to form the compound nucleus. The entrance channel is composed of three ingredients: The first one is associated with a geometrical aspect which refers to the relative orientation of the two nuclei and the molecular modes. The second one is related to the penetration probability through the barrier and the third one is determined by the existence of a molecular state in the molecular potential. The exit channel corresponds to the formation of a compound nucleus parting from a molecular state.

Thus we write the total probability $T_I(E_{\text{c.m.}})$ as

$$T_I(E_{\text{c.m.}}) = T_{\text{comp}}(E_{\text{c.m.}}) \left[\sum_i T^I_{\text{mol}}(i) T^I_{\text{pen}}(i) T^I_{\text{or}}(i)\right] \tag{5}$$

where "or" refers to the orientation effect, "pen" refers to the penetration through the barrier, and "mol" to the existence of a state in the potential pocket. The argument "i" is associated to the molecular states with partial angular momentum I. The probability of compound nucleus formation from the molecular state is described by $T_{\text{comp}}(E_{\text{c.m.}})$. The factors in (5) are correlated, as we describe later. However, the above division of probabilities in a simple product is artificial and in addition seems not to exhibit the resonant Breit-Wigner distribution as described in ordinary reaction theory. But the resonant structure is approximately given by $T^I_{\text{mol}}(i)$ [see Eq. (8)]. In other words, behind Eq. (5) is a semiclassical picture where the different steps of the reaction process are independently considered and the resonant structure is introduced via a Gaussian instead of a Breit-Wigner form.

In what follows we describe the different factors in detail.

A. Orientation effect in the entrance channel

In the entrance channel the relative orientation is distributed uniformly. As was pointed out in Ref. 1 the Coulomb repulsion between the nuclei tend to form a oblateness of the nuclei but the attractive nuclear interaction nearly cancels this effect. The part of the entrance wave function, describing the relative orientation, can therefore be approximated by $\psi_e = 1/\sqrt{2}$ (we only consider the ϑ angle, because in the nuclear molecule we will have only an explicit ϑ dependence). The relative orientation of the nuclei *in the molecule* is described by the wave function of butterfly mode, which in Ref. 3 was denoted by $\chi_{2K,n_\epsilon}(\epsilon)$ with $-\infty < \epsilon < \infty$. The variable ϵ was defined by $\sin\vartheta = \epsilon$ for $\vartheta \ll 1$. For the application to $^{238}_{92}$U-$^{238}_{92}$U this was justified: the function $\chi_{2K,n_\epsilon}(\epsilon)$ is proportional to $e^{-(\lambda/2)\epsilon^2}$ with $\lambda^2 = 6B\beta_0^2 C_\epsilon/\hbar^2$. Here B is the collective mass, β_0 the ground-state deformation, and C_ϵ the potential parameter of the butterfly mode. For the $^{238}_{92}$U system this parameter turns out to be very large so we have a wave function strongly peaked around $\epsilon \simeq \vartheta = 0$. To assume a range of $-\infty < \epsilon < +\infty$ is therefore justified. But for the ^{12}C-^{12}C molecule λ turns out to be 1.52, which is very small. We are then confronted with two problems. (i) The wave function $\chi_{2K,n_\epsilon}(\epsilon = \sin\vartheta)$ is significantly different from zero at $\vartheta = \pm\pi/2$. (ii) The system has a symmetry $\vartheta \to \pi - \vartheta$. This was not necessary to take into account for ^{238}U-^{238}U, because the potential in $\epsilon = \sin\vartheta$ for $\vartheta \ll 1$ could be approximated by a harmonic oscillator which was well separated by its image part at $\vartheta = \pi$.

Nevertheless we are convinced that χ_{2K,n_ϵ} gives a good approximation of the wave function. In order to get around the above problems we first define χ_{2K,n_ϵ} only in the domain $0 \le \vartheta < \pi$ and then normalize it. Due to $\epsilon = \sin\vartheta$ it already exhibits the symmetry (ii).

The overlap of the entrance state to the molecular one is now calculated by

$$T^I_{\text{or}}(i) = \left| \frac{1}{\sqrt{2}} \int_0^\pi d\vartheta \sin\vartheta \chi'_{2K,n_\epsilon}(\epsilon = \sin\vartheta) \right|^2 = |\langle \Psi_e | \chi' \rangle|^2 , \quad (6)$$

where the prime indicates the new normalized function $\chi_{2K,n_\epsilon}(\epsilon)$ in the interval $0 \le \vartheta \le \pi$. The function Ψ_e refers to the entrance channel where the orientation function is normalized by $1/\sqrt{2}$. The argument "i" stands for the state number "i" classified by (I,K,n_ϵ,n_r).

B. Penetration probability

For that we use the Hill-Wheeler procedure.[9] There the penetration probability is given by

$$T^I_{\text{pen}}(E_{\text{c.m.}}) = \frac{1}{1+\exp[(2\pi/\hbar\omega_b)(V_I - E_{\text{c.m}})]} , \quad (7)$$

where $E_{\text{c.m.}}$ is the center of mass energy and V_I is taken from (3d), $\hbar\omega_b$ from (3c).

To get V_I we took the potential of Fig. 5 which is in reality a cut through the global potential at the nose-to-nose orientation. We assumed (see discussion in Sec. II) that this orientation gives the main contribution to the formation of nuclear molecules because in other orientations the barrier rises to higher energies due to the Coulomb repulsion.

C. About the probability to find a molecular state

In our model there can only be an occupation of the molecular state, if on the other side of the barrier exists for the incident energy a state to be occupied. To this state we associate a total width $\Gamma_{\text{tot}}(i)$, described in detail below. The probability to occupy the molecular state, after having passed the barrier, is then approximated by a Gaussian distribution

$$T^I_{\text{mol}}(i) = e^{-(E_{\text{c m}} - E_i)^2/\Gamma^2_{\text{tot}}(i)} . \quad (8)$$

The total width $\Gamma_{\text{tot}}(i)$ is given by the sum of the partial width of different decaying channels. We only consider two, i.e., the elastic and the compound channel. The elastic decaying width Γ_i is given by (3a), i.e., is related to the penetration of the barrier. For the compound width Γ_{comp} we proceed as follows: In Ref. 10 (pp. 20 and 21) for a resonance at $E_{\text{c.m.}} = 11.4$ MeV, which we identify in our model state, a calculated compound width of 70–150 keV is reported. We decided to take the upper limit, i.e., 150 keV. The system ^{12}C-^{12}C can be considered as an excited state of 24 Mg, where $E_{\text{c.m.}} = 0$ corresponds to 13.93 MeV in the 24 Mg nucleus.[6] The probability to form a compound nucleus is usually $\sim\sqrt{E}$, with $E = 13.93 + E_{\text{c.m.}}$. Adjusting Γ_{comp} to $E_{\text{c.m.}} = 11.4$ MeV we get

$$\Gamma_{\text{comp}} = 0.150 \frac{\sqrt{13.93 + E_{\text{c.m.}}}}{\sqrt{25.33}} , \quad (9)$$

where $E_{\text{c.m.}}$ and Γ_{comp} are given in MeV. The total width of state is i then given by

$$\Gamma_{\text{tot}}(i) = \Gamma_{\text{comp}} + \Gamma_i . \quad (10)$$

D. Probability of forming a compound nucleus

We use expression (8) for Γ_{comp} and

$$T_{\text{comp}}(E_{\text{c.m.}}) = \frac{1}{\hbar}\Gamma_{\text{comp}} \quad (11)$$

Having determined all relevant factors in our model, the probability to obtain fusion in the channel of the partial angular momentum I is given by Eq. (5). We now can proceed to discuss the results obtained for the ^{12}C-^{12}C system.

For ^{12}C we can expect two counter producing processes. First, the transition probability for fusion is *increased* going from spherical ^{12}C nuclei to deformed ones, i.e., lowering the barrier. Second, due to the low density of molecular states below and around the barrier the transition probability for fusion will be *decreased*. We will see that in average those two effects cancel approximately around the barrier so that it is not surprising that assuming spherical ^{12}C nuclei the fusion data can be, on average, reproduced quite well around the barrier by a simple penetration model.[1]

Above the barrier we will expect an increasing deviation from the experimental data, because other processes, e.g., excitation of β and γ vibrations, can occur. Above the energy $E_{\text{c.m.}} = 7.65$ MeV$+E_0$ the β vibration becomes important, so one should take into account not only the β vibration but also mixtures with the other modes. The number of states increases rapidly to higher energies.[3] Due to this effect, which adds to the energy (1) a term $E_\beta n_\beta$ that we do not take into account here, a too small value of the total fusion cross section can be expected starting from around 9.7 MeV in the center of mass.

In Figs. 6 and 7 a comparison of the calculated to the experimental fusion cross section is given. In Fig. 6 we plotted the ratio of σ_{fus} to

$$\sigma_{\text{tr}} = \frac{2\pi}{k^2} \sum_I (2I+1) T_{\text{tr},I}(E_{\text{c.m.}}) , \quad (12)$$

where $T_{\text{tr},I}(E_{\text{c.m.}})$ is the simple transmission coefficient using the potential of Fig. 5. The $T_{\text{tr},I}(E_{\text{c.m.}})$ has the same form as in (3b) with E_i substituted by $E_{\text{c.m.}}$. Data are taken from Refs. 1 and 11–13.

Around 4 MeV there is a steep rise in the fusion cross section, reproduced by the model. There follows a broad bump between 4 and 5 MeV and a peaked structure around 5 MeV in experiment. Also this is roughly reproduced by the model, especially the position, though the oscillations are overemphasized. After 5 MeV the general qualitative rise is reproduced though the theoretical

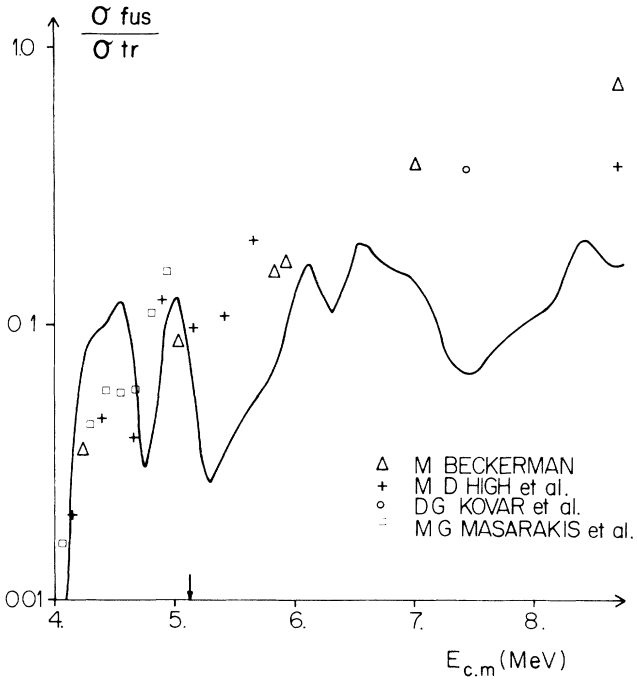

FIG. 6. Ratio of the total fusion cross section σ_{fus} for ^{12}C-^{12}C to a simple barrier penetration σ_{tr}. For the simple barrier penetration the potential of Fig. 4 was used. The experimental data are taken from Refs. 1, 11, 12, and 13. The arrow at 5.16 MeV indicates the position of the barrier of the potential of Fig. 5.

value is too low and the overemphasized oscillation persists.

This is also the case in Fig. 7, where the absolute fusion correction above 7 MeV is plotted. As we already pointed out we do not expect a good agreement high above the barrier ($\simeq 5.16$ MeV), but still the general rise and, if the average between the oscillations is taken, the step structure is reproduced. The theoretical values are off by a factor of roughly 2.

In Fig. 8 the nuclear structure factor

$$\tilde{S}=\sigma_{\text{fus}}E_{\text{c.m.}}e^{+(87.2/\sqrt{E_{\text{c.m}}}+0.46E_{\text{c.m}})}, \tag{13}$$

for the ^{12}C-^{12}C system, as defined in Ref. 13, is given. Also there the oscillations between 4–6 MeV are reproduced well. The description is similar to the one of Fig. 6. [The values of Ref. 11 were multiplied by $\exp(0.46E_{\text{c.m.}})$ where another definition of the nuclear structure factor was used.]

It is interesting how the oscillations can be related to the molecular structure: The first theoretical resonance of Figs. 6 and 8 is identified with the sum of the states $(I,K,n_\epsilon,n_r)=(0,0,1,0)$, (4,0,0,0) and (2,2,0,0), where the first two form the bump in the first broad resonance. It is worth noting that the bump in the first resonance would be a peak if the factor $T^I_{\text{or}}(i)$ were not taken into account. The second resonance is identified with the state (2,0,1,0). The next bumps in Figs. 6 and 8 above 6 MeV are identified with groups of resonances in Fig. 4, where a drop in the density of states can be guessed at around 7.5 MeV. This produces the minimum at this energy in the theoretical calculation. To summarize, *the oscillation*

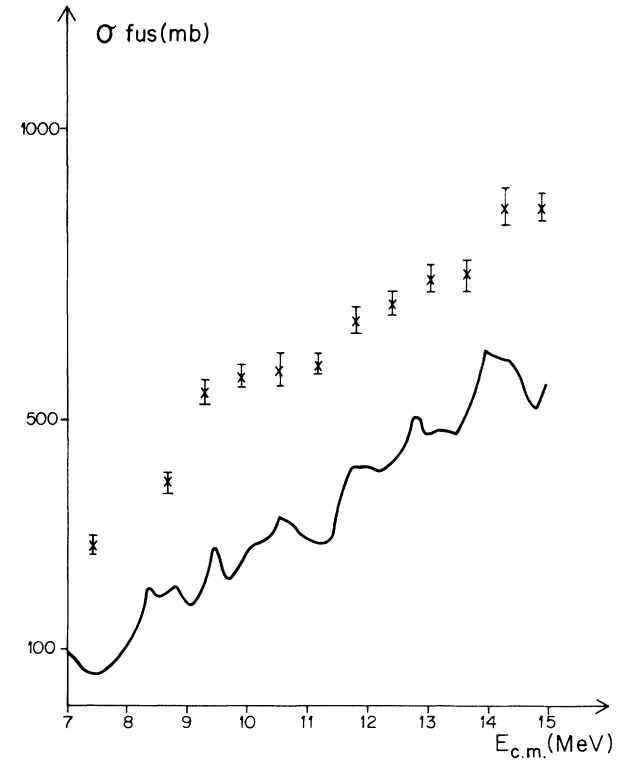

FIG. 7. Total fusion cross section of ^{12}C-^{12}C in absolute values. The data are taken from Ref. 12.

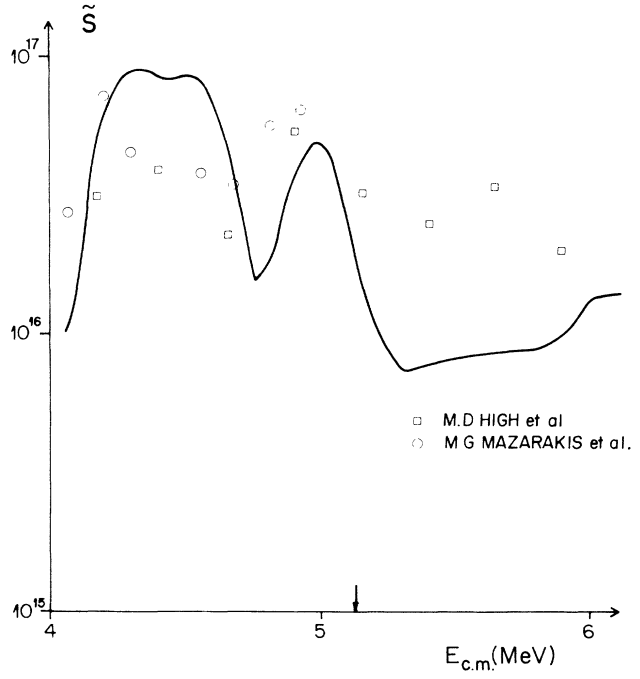

FIG. 8. The nuclear structure factor $\tilde{S}$ (definition in text) as a function of the center-of-mass energy $E_{\text{c.m}}$. The solid line refers to theory and the circles and squares are experimental data.

structure is a product of the molecular structure in the molecular potential. When the density of states increases (diminishes) the fusion cross section also increases (diminishes).

As one can see the simple model works satisfactorily, proving to us that the main characteristics of the fusion process might be described by the model. The results convinced the authors that the nuclear molecules are probably the doorway states for fusion around and below the barrier. Furthermore, we have here a clear signature of a correlation of apparently two distinct physical processes: the spectrum of the nuclear molecule and the total fusion cross section.

IV. CONCLUSIONS AND OUTLOOK

The authors have presented a simple mechanism for fusion around and below the barrier. Before the system fuses a nuclear molecule is formed as a doorway state. An elementary transparent model for nuclear molecules[3] was used to describe the spectrum of ^{12}C-^{12}C, to determine the molecular potential, and finally to calculate *without further parameters* the total fusion cross section. To be consistent with the molecular spectrum, the ^{12}C nucleus has to be strongly deformed, in agreement with Ref. 4. Inspite of its simplicity the model can surprisingly well reproduce the data both in the molecular and fusion sector showing a clear correlation between them. In the fusion sector the data are reproduced qualitatively in structure even above the barrier up to ~ 15 MeV in the center of mass. This led the authors to the following conclusions.

(a) Our model represents an elementary and transparent way to describe the dynamics of fusion schematically.

(b) Fusion probably takes place through forming a nuclear molecule as a doorway state and confirms the conclusion of Kondo *et al.*,[2] for the ^{16}O-^{16}O case.

(c) Following the discussion of the results in Sec. III we can see that fusion increases if the level density can be increased.

The level density can be increased taking heavier systems with a low β and γ vibrational energy of the individual nucleus. As shown in Ref. 3 in the case of ^{238}U the level density can be increased tremendously. This might also explain why the fusion cross section is sometimes by orders of magnitudes higher than in a simple barrier penetration model.[1] Our plan is to investigate systematically heavier systems[14] where the simple model of nuclear molecules is more justified. We are then, however, confronted with a particular problem: in heavier systems the resonant structure of the molecular part is either very weak or absent. This, we think, is due to the high level density which, together with the large width of each state, does not permit a resolution of resonances. One has to get the parameters from somewhere else. One possibility is to do microscopic studies[15,16] in order to calculate the parameters E_r and E_ϵ.

We are confident that also in heavier systems the model might work because it has worked quite well for the ^{12}C-^{12}C system in two apparent distinct physical processes, the molecular spectrum and the fusion cross section. It reproduced well the molecular spectrum and even details in the total fusion cross section. It further showed a clean consistency, e.g., with a large deformation of a single ^{12}C nucleus.

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