Validity of on-shell distortion factors for $NN \to \pi d$

B. Blankleider
Paul Scherrer Institute, PSI, CH-5232 Villigen PSI, Switzerland

A. Švarc
Ruđer Bošković Institute, OOUR FEP, Zagreb, Croatia, Yugoslavia
and Paul Scherrer Institute, PSI, CH-5232 Villigen PSI, Switzerland
(Received 2 October 1989)

We test the validity of calculating the process $NN \to \pi d$ in lowest order, with initial- and final-state interactions being included via square roots of s matrices for the respective elastic processes (so-called Sopkovich approximation). This is done by constructing a coupled-channels model with effective two-body potentials between the channels NN, πd, and $N\Delta$. These potentials are taken to be separable, and are fitted to partial-wave amplitudes coming from a three-body calculation. We obtain reasonable fits in those partial waves where coupling to the $N\Delta$ channel is strongest; in particular, the dominant $J=2^+$ channel is well described. Within such a two-body potential model, it is known that the Sopkovich approximation should be valid for small, short-range transition potentials in the limit of high energies. Using this coupled-channels model, we find that the Sopkovich approximation in the 2^+ channel works well only above 200-MeV total center-of-mass kinetic energy. Comparison with an approximation which assumes only a small transition potential leads us to observe that (i) the 2^+ $NN \to \pi d$ transition potential is indeed small everywhere except at the Δ resonance peak (160 MeV), and (ii) away from this peak, the Sopkovich approximation breaks down below 200 MeV mainly due to the neglect of off-shell effects. For the smaller $J=2^-$, 3^-, and 4^+ partial waves, we find the Sopkovich approximation to work at least as well as for the 2^+. No conclusions are made about the $J=0^+$ and 1^- partial waves; these obtain large contributions from s-state πN rescattering which is not easily included in our model.

I. INTRODUCTION

One of the main challenges of intermediate-energy physics has been to understand nucleon-nucleon (NN) scattering above pion production threshold. This is reflected in the many studies of the $NN \leftrightarrow \pi d$ reaction.[1–13] Not only does this reaction form the simplest inelastic channel for NN scattering, but it is also responsible for the absorptive part of $\pi d \to \pi d$. There have been essentially two lines of approach in studying $NN \to \pi d$. In the first, coupled-channel equations are solved for the πNN system.[1–6] Some of these calculations couple all the processes $\pi d \to \pi d$, $NN \to \pi d$, and $NN \to NN$ in a unitarity way.[2–6] Despite the careful treatment of three-body unitarity, these approaches are as yet limited to using three-dimensional integral equations, and therefore cannot take into account the full complexity of a relativistic theory. In the second approach,[7–13] the emphasis has been to perform detailed calculations of the Born diagrams illustrated in Fig. 1. The direct production diagram, Fig. 1(a), describes mainly pion production near threshold, while production via the Δ, Fig. 1(b), dominates pion production over a wide energy region around the Δ resonance. An important feature of such calculations is that they are able to be done fully relativistically.[9–13] On the other hand, the handling of initial- (ISI) and final-state interactions (FSI) in such approaches is problematical. This situation is of course not unique to $NN \to \pi d$. Both in nuclear and high energy physics, it has been common to take into account ISI and FSI by multiplying the calculated transition Born amplitude by factors related to the corresponding elastic scattering phase shift.[14–17] These are *on-shell* prescriptions which do not take into account the possible off-shell contributions in the integral connecting the Born term with initial and final states. Generally speaking, within nonrelativistic potential models, these prescriptions can be shown to be accurate for small, short-range Born terms in the limit of high energies.[17] Because the final prescription is expressible purely in terms of s matrices, it is hoped that such proofs can be generalized to relativistic field theories where potentials may not have a rigorous meaning. One such prescription, the so-called Sopkovich formula,[14] has been used by Locher and Švarc[9] in their relativistic calculation of the diagrams of Fig. 1. The goal of this paper is to assess the validity of using the Sopkovich formula in the particular case of the $NN \to \pi d$ process.

In the Sopkovich prescription for $NN \to \pi d$, the sum of the diagrams in Fig. 1 is considered as the Born term $B_{\pi d,NN}$. The full $NN \to \pi d$ t matrix, $T_{\pi d,NN}$, is then approximated as

$$T_{\pi d,NN} \approx S^{1/2}_{\pi d,\pi d} B_{\pi d,NN} S^{1/2}_{NN,NN} , \tag{1.1}$$

where square roots of elastic-scattering s matrices play the role of on-shell distortion factors. Although this looks formally like a statement of Watson's theorem[18] for the case of $NN \to \pi d$, Eq. (1.1) is meant to apply even

FIG. 1. Born diagrams for the $NN \rightarrow \pi d$ process: (a) shows the direct pion production diagram, while (b) depicts production via the Δ resonance.

when there are strong inelasticities in all the amplitudes. In order to check the domain of validity of such an approximation, we have constructed a coupled channels model of the processes $NN \rightarrow NN$, $NN \rightarrow \pi d$, and $\pi d \rightarrow \pi d$, where the channels are made up of the two-body states NN, πd, and $N\Delta$. All particles in these channels are treated as elementary, so that the breakup of the deuteron, or the decay of the Δ is not explicitly included. This might seem a drastic simplification, but it is in keeping with the usual proof of the Sopkovich prescription.[17] Moreover, much of this neglect of open three-body channels can be effectively compensated in our model by adjusting transition potentials to fit experimental data.

Clearly, such a model is most appropriate in those partial waves which couple strongly to $N\Delta$ states. This is the case for the dominant $J=2^+$ partial wave. Indeed its dominance is understood to arise from the coupling to an $N\Delta$ in relative s state. As shown by more microscopic three-body models,[4] coupling to $N\Delta$ states is also particularly important in the $J=2^-$, 3^-, and 4^+ partial waves. On the other hand, $J=0^+$ and 1^- partial waves receive large contributions from s-wave πN rescattering, and in this case our simple three-channel model is not expected to be applicable.

In this paper we concentrate our discussion on the dominant $J=2^+$ partial wave channel. For the smaller 2^-, 3^-, and 4^+ partial waves, our results indicate that the Sopkovich prescription works at least as well as $J=2^+$, and will therefore be discussed in less detail here.

The partial-wave transition potentials between the three channels $v_{ij}(i,j \equiv NN, N\Delta, \pi d)$ are taken to be real, and of rank-one separable Yamaguchi form (however we set $v_{\pi d, \pi d}=0$ as prescribed by three-body theory). The model is therefore exactly solvable, in principle algebraically, for all the possible off-shell amplitudes in question. The parameters of the model, consisting of eight potential strengths and ranges, are used to fit data for the physical processes $NN \rightarrow NN$, $NN \rightarrow \pi d$, and $\pi d \rightarrow \pi d$ in the energy range $0 \le E_k \le 300$ MeV where E_k is the total center-of-mass kinetic energy (the equivalent beam laboratory kinetic energies can be read off from Fig. 3). Because of the difficulty in obtaining a complete set of experimentally determined partial wave amplitudes, we have fitted to amplitudes predicted by the few-body calculation of Afnan and Blankleider[4] (AB). This should be sufficient to fix our potential strengths and ranges in order to approximately describe true experimental data.

With our transition potentials thus determined we are explicitly able to compare the exact $NN \rightarrow \pi d$ t-matrix $T_{\pi d, NN}$ with the Sopkovich modified Born term of Eq. (1.1). We find that for the $J=2^+$ partial wave, the Sopkovich prescription works well only for $E_k > 200$ MeV. In particular, it is inaccurate below, and close to the Δ resonance position. In order to see which condition for the validity of the Sopkovich prescription is not met below 200 MeV, we introduce a new approximation, Eq. (3.6), which only assumes that the Born term is small. Although of similar form to Eq. (1.1), the new approximation includes the full off-shell contributions. Except right at the Δ peak, we find that this approximation works very well across the whole resonance energy region. This suggest that although $B_{\pi d, NN}$ is mostly small enough to satisfy one of the conditions for the Sopkovich prescription, off-shell effects are too large to be neglected for energies below 200 MeV.

The details of the three-channel separable potential model, the choice of the input data and details of the fitting procedure are given in Sec. II. The numerical results are discussed in Sec. III and a summary is given in Sec. IV.

II. THE COUPLED-CHANNELS MODEL

In an analysis of absorptive processes, Durand and Chiu[17] (DC) considered the case where any number of two-body channels are coupled together through a real symmetric potential matrix v_{ij}. By a formal elimination of channels, it is possible to reexpress this problem in terms of just two channels, coupled together through complex potentials. For this effective two-channel problem, DC were able to show that, under appropriate conditions, the partial wave transition t matrix, T_{ij} can be approximated as

$$T_{ij} \approx e^{i\delta_i} B_{ij} e^{i\delta_j}, \quad i,j=1,2 , \tag{2.1}$$

where B_{ij} is the Born term for the transition, δ_i and δ_j are phase shifts for the elastic channels i and j, respectively. We shall call Eq. (2.1) the Sopkovich approximation or prescription, as it was originally derived by him[14] using a method based on the Glauber high-energy approximation. As shown by DC, the conditions under which this relation is true are essentially that the Born term is small and of short range, and that the energies are high enough.

Because we would like to quantify these conditions for the case of $NN \rightarrow \pi d$, we basically implement the procedure of DC numerically for the three channels NN, πd, and $N\Delta$. There are of course other open channels possible, but it is well known that, at intermediate energies, the πNN system is dominated by the $N\Delta$ channel in the ${}^5S_2$ partial wave.

For the purpose of this study, it is therefore appropriate to perform a detailed examination of the above three-channel model in the $J=2^+$ partial wave. As we shall discuss later, our three-channel model can also be used for $J=2^-$, 3^-, and 4^+ partial waves; however, the $J=2^+$ channel turns out to be more restrictive than these with respect to the applicability of the Sopkovich prescription.

We shall also keep only the lowest coupled orbital angular momentum contribution within each channel. We thus end up with just three partial wave channels: $(i=1)$

an NN state in the $^1D_2$ partial wave, ($i=2$) a πd state in the $^3P_2$ partial wave, and ($i=3$) the $N\Delta$ $^5S_2$ state. The coupled channel equations for the processes $NN\to NN$, $NN\to\pi d$, and $\pi d\to\pi d$, can now be written in operator form as

$$T_{ij}=v_{ij}+\sum_{k=1}^{3} v_{ik}G_kT_{kj},\quad i,j=1,2,3\ , \tag{2.2}$$

where T_{ij} and v_{ij} are the partial wave transition t matrix and potential, respectively, and G_k is the Green's function in channel k. We solve Eqs. (2.2) in momentum space where they form a coupled set of Lippmann-Schwinger equations. We use relativistic kinematics for all particles, and our amplitudes are defined as in Goldberger and Watson.[19] For simplicity we assume rank-1 separable forms for the potentials

$$v_{ij}=|g_i\rangle\lambda_{ij}\langle g_j|\ , \tag{2.3}$$

where we chose Yamaguchi expressions for the form factors

$$g_i(k)=\frac{C_ik^{l_i}}{(\beta_i^2+k^2)^{n_i}}\ . \tag{2.4}$$

In Eq. (2.4), k is the center of mass momentum, l_i is the channel orbital angular momentum, C_i and β_i are the strength and range which we treat as free parameters. The coupling strength λ_{ij} is set to ±1 for $i=j$ and treated as a free parameter for $i\neq j$. The power index n_i is used to set the form of the potentials. In addition, in order to agree with three-body models of πd elastic scattering, we set v_{22} to zero. In our model, all particles are treated as elementary, having the following masses: $m_\pi=139.57$ MeV, $m_d=1875.6$ MeV, $m_N=938.93$ MeV, and $m_\Delta=1232.8$ MeV. Thus the vertex functions of both the deuteron and the Δ are not explicitly included in our model. Although seemingly a drastic approximation, it is compensated by the fact that ours is a phenomenological model where effective potentials are varied to fit data. The advantage of having strictly two-particle channels is that the problem is thereby expressed directly within the same framework as that of the theoretical work of DC. This facilitates a better understanding of our numerical results. We also note that two-body unitarity is maintained exactly in our calculations.

There are eight free parameters in our model: the separable form factor strengths and ranges C_{NN}, β_{NN}, $C_{\pi d}$, $\beta_{\pi d}$, $C_{N\Delta}$, $\beta_{N\Delta}$, and the cross-channel coupling strengths $\lambda_{NN,\pi d}$ and $\lambda_{NN,N\Delta}$. We note that the transition strength $\lambda_{\pi d,N\Delta}$ is not a free parameter for us; instead, it is fixed to be ±1. This stems from the fact that, as in few-body models, we take $v_{\pi d,\pi d}=0$. This leaves the strength $C_{\pi d}$ free to determine the true strength of the $N\Delta\to\pi d$ potential.

The eight parameters were used to fit to the partial wave amplitudes coming from the few-body calculation of Afnan and Blankleider.[4] Only the on-shell amplitudes for the physical reactions $NN\to NN$, $NN\to\pi d$, and $\pi d\to\pi d$ were fitted. The fits were performed in the energy range $0\le E_k\le 300$ MeV, where E_k is the total center of mass kinetic energy defined in terms of the total energy E as

$$E_k=E-m_{\pi^+}-2m_N\ . \tag{2.5}$$

Of course, fitting to experimentally determined amplitudes for the physical processes would have been preferable, but as yet there do not appear to be any reliable amplitude analyses for π-d elastic scattering above the Δ resonance region. At this stage we prefer to use theoretically generated data which at least are consistent with two- and three-body unitarity. We point out, however, that it is in any case doubtful that our main conclusions would change even if such experimental data were available. Even an approximate data set should determine our potential strengths and ranges well enough to see if the conditions for the Sopkovich approximation are met.

The parameter fitting procedure is a standard one. It has been done using the MINUIT D506 (CERN-LIBRARY) program running on a VAX 8650 computer. The parameters of our best fit to the $J=2^+$ amplitude data are given in Table I.

In order to test the Sopkovich prescription, the coupled three-channel problem has to be reduced to a two-channel problem with effective complex potentials. In our model this can be done by eliminating the $N\Delta$ channel from the set of equations (2.2). Because we use separable interactions, this can be done algebraically, with the final result being

$$T_{ij}=V_{ij}+\sum_{k=1}^{2} V_{ik}G_kT_{kj},\quad i,j=1,2\ , \tag{2.6}$$

where the effective potentials are given by

$$V_{ij}=|g_i\rangle\left[\lambda_{ij}+\lambda_{i3}\frac{\langle g_3|G_3|g_3\rangle}{1-\lambda_{33}\langle g_3|G_3|g_3\rangle}\lambda_{3j}\right]\langle g_j|\ . \tag{2.7}$$

The meaning of this result can be better displayed by expanding the denominator in Eq. (2.7) into a power series. In terms of the separable potentials given in Eq. (2.3), this

TABLE I. Parameters of the coupled channels model, defined in Eqs. (2.3) and (2.4), corresponding to the fit shown in Fig. 3.

Channel i	C_i ($\mathrm{fm}^{l_i-2n_i+1}$)	β_i (fm^{-1})	n_i	$\lambda_{i,NN}$	$\lambda_{i,\pi d}$	$\lambda_{i,N\Delta}$
NN ($^1D_2$)	2.56	2.92	2	-1	10^{-7}	2.62
πd ($^3P_2$)	5.61	1.24	2	10^{-7}	0	-1
$N\Delta$ ($^5S_2$)	2.10	1.50	2	2.62	-1	1

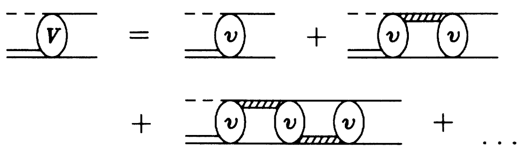

FIG. 2. Illustration of the series equation (2.8) for the $NN \rightarrow \pi d$ potential V_{21}.

series is

$$V_{ij} = v_{ij} + v_{i3} G_3 v_{3j} + v_{i3} G_3 V_{33} G_3 v_{3j} + \cdots \ . \quad (2.8)$$

As displayed graphically in Fig. 2 for the $NN \rightarrow \pi d$ case, the potential V_{ij} contains all the N-Δ rescattering contributions. The usefulness of the Sopkovich approximation now depends crucially on the rapid convergence of this series. In particular, it would be desirable if only the first two terms need be retained. These terms would then correspond to the ones in Fig. 1, calculated in more microscopic approaches. For the purposes of testing the Sopkovich prescription, we therefore define the Born term as

$$B_{ij} = |g_i\rangle \lambda_{ij} \langle g_j| + |g_i\rangle \lambda_{i3} \langle g_3 | G_3 | g_3 \rangle \lambda_{3j} \langle g_j| \ . \quad (2.9)$$

We note that this also agrees with the way a (modified) Sopkovich formula has been used by Locher and Švarc.[9]

Having specified our model, our procedure then involves comparing the Sopkovich formula for $NN \rightarrow \pi d$, Eqs. (2.1) and (2.9), with the full solution of the coupled equations (2.2).

III. RESULTS AND DISCUSSION

The parameters of our best fit to the $J = 2^+$ amplitudes of AB are given in Table I. It is interesting to observe that the $\lambda_{NN,\pi d}$ potential strength comes out of the fitting procedure as extremely small. That the p-wave pion production process is dominated by coupling to an $N\Delta$ intermediate state, thus comes out naturally from our fit. This agrees with observations coming from more microscopic models.[9]

In Fig. 3 we compare the obtained t matrices (filled circles) for the processes $NN \rightarrow NN$, $NN \rightarrow \pi d$, and $\pi d \rightarrow \pi d$ with the corresponding AB model predictions which have served as our input data (open circles). The agreement is quite good for $NN \rightarrow \pi d$ and $\pi - d$ elastic scattering, while it is of somewhat poorer quality for $N - N$ elastic scattering at energies above the Δ resonance. On the other hand, the AB model itself does not give a good account of the true experimental NN data at the higher energies. In this respect, it is interesting to note that our fitted NN amplitudes tend to be closer to the phase-shift analysis of Arndt[20] (open triangles) than to the AB data. We therefore conclude that the present model can give a fairly realistic description of all the above three processes in the $J = 2^+$ channel.

With the model now established we go on to investigate the Sopkovich approximation, Eq. (2.1), where the Born term is given by the first two terms of the series (2.8). Specifically for the $NN \rightarrow \pi d$ reaction, we therefore define

$$T^S_{\pi d,NN} \equiv e^{i\delta_{\pi d}} B^S_{\pi d,NN} e^{i\delta_{NN}} \ , \quad (3.1)$$

where

$$B^S_{\pi d,NN} \equiv v_{\pi d,NN} + v_{\pi d,N\Delta} G_{N\Delta} v_{N\Delta,NN} \quad (3.2)$$

and we are interested to see how well the Sopkovich modified Born term $T^S_{\pi d,NN}$ approximates the exact $NN \rightarrow \pi d$ t-matrix $T_{\pi d,NN}$.

In Fig. 4(a) we compare the real and imaginary parts of the exact t-matrix $T_{\pi d,NN}$, obtained on the basis of the separable potential model (filled circles), with the Sopkovich approximation $T^S_{\pi d,NN}$ (solid line). The dashed line gives the unmodified Born term $B^S_{\pi d,NN}$. It is also convenient to show the same comparison in terms of the modulus and phase of amplitudes; this we do in Fig. 4(b).

An immediate conclusion from Fig. 4 is that the Born term itself is very large, and bears little relation to the data—especially around the resonance region. This observation is in common with the relativistic calculation of Locher and Švarc.[9] It is thus essential to account for initial- and final-state interactions in a Born calculation of $NN \rightarrow \pi d$. In comparing the Sopkovich approximation $T^S_{\pi d,NN}$ with the exact $T_{\pi d,NN}$, a number of different features become apparent corresponding to whether we are below, on, or above the Δ resonance peak.

Below the $N\Delta$ threshold, our Born term is purely real. This is a consequence of the fact that our Δ has no width. Also in this region, both NN and πd amplitudes are essentially elastic (of course the NN amplitudes are exactly elastic below pion production threshold). Below Δ threshold, therefore, the Sopkovich prescription equation (3.1), reduces to a statement of two-body unitarity (i.e., Watson's theorem[18]) and this is reflected in the identity of the phases of $T^S_{\pi d,NN}$ and $T_{\pi d,NN}$ in this region. In reality, because of deuteron breakup and the nonzero width of the Δ, the above phases are not expected to be identical. We note that even though the phases of $T^S_{\pi d,NN}$ and $T_{\pi d,NN}$ are identical below threshold, their magnitudes are not. Thus within the scope of our model, the Sopkovich prescription does not appear to be applicable in the energy region below the delta peak.

At the delta peak, $E_k \approx 160$ MeV, the Born term is very large. Although the Sopkovich prescription definitely helps to bring down the peak, there is no agreement with the exact results. Indeed, if one looks at the energy dependence of $T^S_{\pi d,NN}$, one sees a rapid oscillation around the peak, while the exact results vary smoothly. This oscillating behavior is due to the strong inelasticity in πd elastic scattering. Indeed the inelasticity parameter $\eta_{\pi d}$, defined as usual by

$$e^{i\delta_{\pi d}} = \eta_{\pi d} e^{i \,\mathrm{Re}(\delta_{\pi d})} \ , \quad (3.3)$$

shows a very strong dip at the Δ resonance peak. By its construction, $T^S_{\pi d,NN}$ contains a factor $e^{i\delta_{\pi d}}$ and therefore will itself show a sharp dip at the resonance peak. This behavior is not displayed by the exact amplitude. As discussed later, this disagreement can be attributed partly to off-shell effects and partly to the large size of the Born term.

Above the resonance peak, the Sopkovich approximation first rapidly approaches to within 10% of the exact t-matrix value, and then continues converging, although rather slowly. This is explicitly shown in Fig. 5 where we plot the value $|T_{\pi d,NN} - T^{S}_{\pi d,NN}|$ as a function of energy. From Figs. 3–5 one can conclude that the Sopkovich approximation works reasonably well above about 200 MeV total c.m. kinetic energy.

We now examine the nature of the Sopkovich approxi-

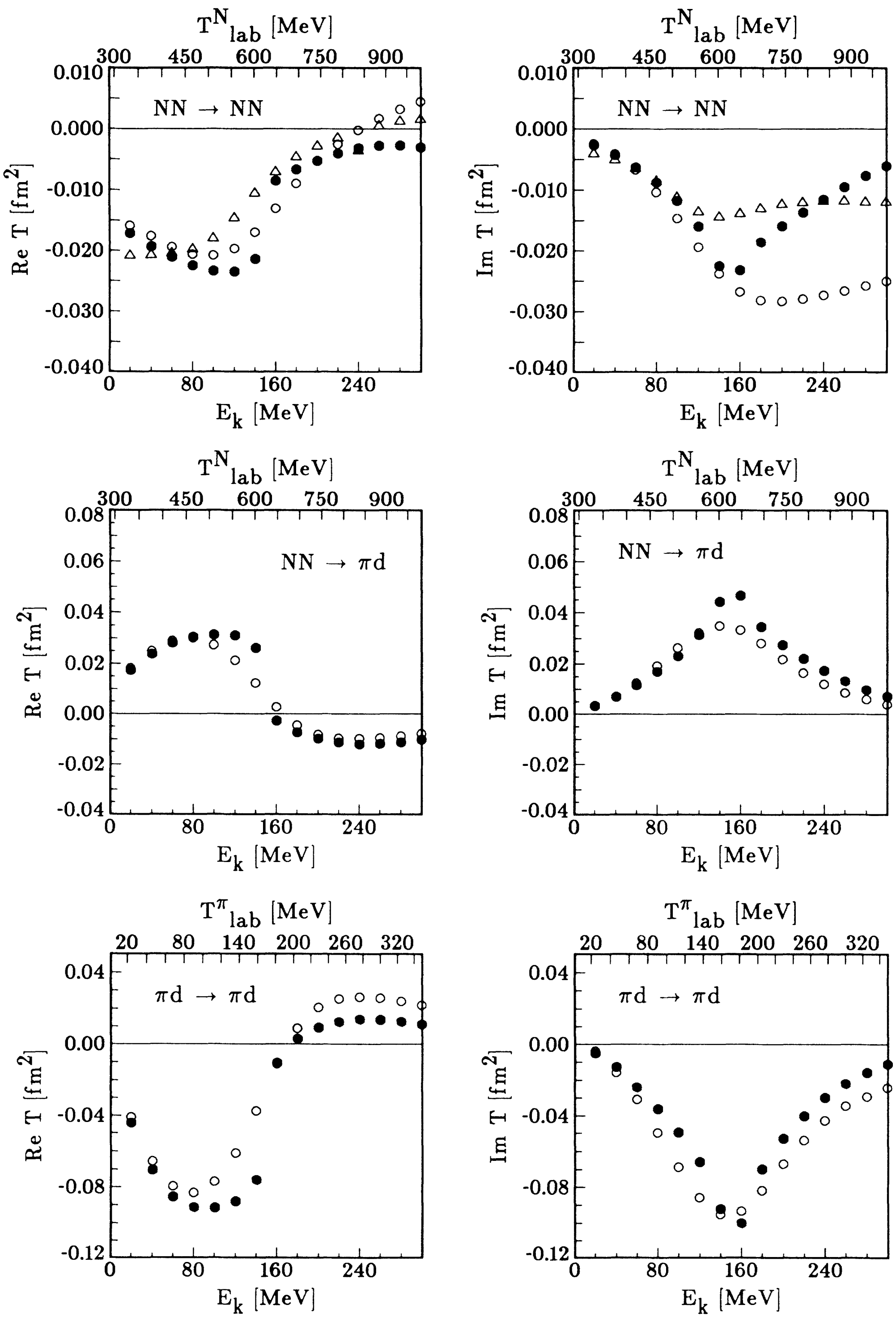

FIG. 3. Comparison of fitted t matrices for $NN \rightarrow NN$, $NN \rightarrow \pi d$, and $\pi d \rightarrow \pi d$ using our coupled-channels model (filled circles), with the theoretical values of Ref. 4 which are here taken as data (open circles). The phase-shift results from Ref. 20 are given as open triangles for comparison. The bottom horizontal scale gives the total c.m. kinetic energy E_k while the top scale gives the corresponding laboratory kinetic energy of the beam particle.

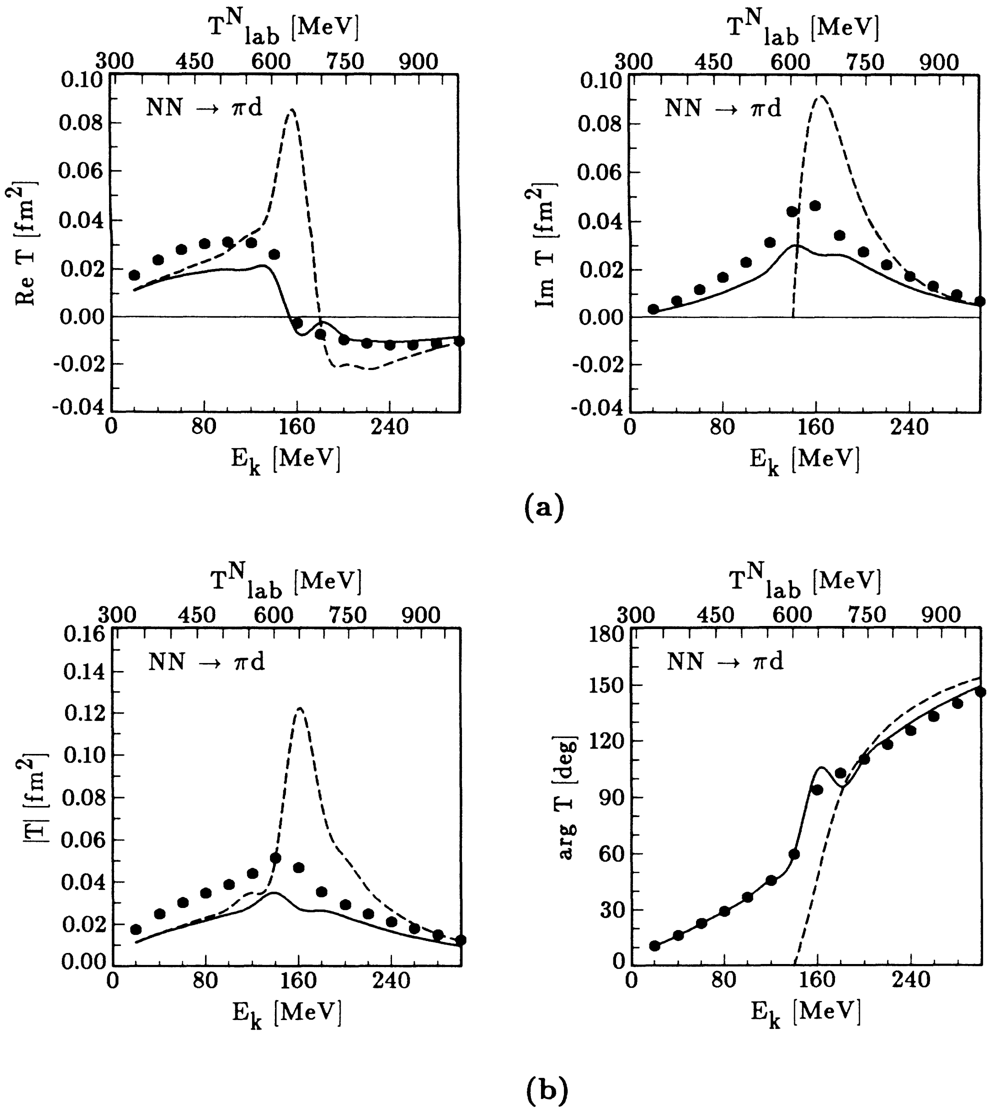

FIG. 4. Comparison of the exact t matrix for the process $NN \rightarrow \pi d$ (filled circles) with the Sopkovich modified Born term (solid line). Also shown is the unmodified Born term (dashed line).

mation a little more closely. As was stated in the previous section, the usefulness of the approximation is based on the rapid convergence of the Born series in Eq. (2.8). For our fitted model, this turns out indeed to be the case. If we use the full series $B_{\pi d,NN}$ instead of just the first two terms $B^S_{\pi d,NN}$, then our results are virtually indistinguishable from the ones shown in Fig. 4. As our best fits are essentially unique, we conclude that, within our model, the data demands that the contribution from the potential $v_{N\Delta,N\Delta} \equiv |g_3\rangle\lambda_{33}\langle g_3|$ be small. Thus the sign of λ_{33} is not well determined in our model. For the fits shown in Figs. 3 and 4, λ_{33} was chosen to be $+1$. If we set $\lambda_{33}=-1$ then we obtain a fit very similar to the one already shown. It must be said, however, that the best fit with $\lambda_{33}=-1$ gives a small but significant value for $v_{\pi d,N\Delta}G_{N\Delta}v_{N\Delta,N\Delta}G_{N\Delta}v_{N\Delta,NN}$—the third term in the Born series of Eq. (2.8). As a consequence, the Sopkovich approximation with this solution is slightly worse than the

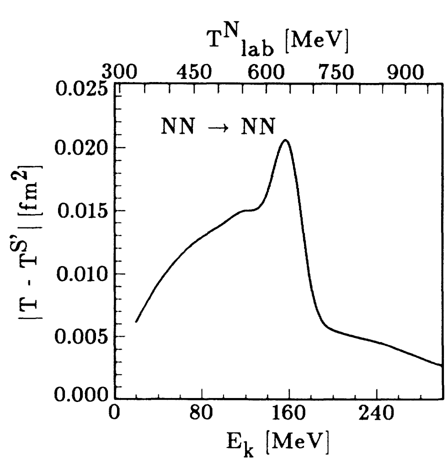

FIG. 5. The deviation of the Sopkovich modified Born term (T^S) from the exact solution (T) for the $NN \rightarrow \pi d$ process.

one with $\lambda_{33}=+1$, although within the accuracy of our model, it is difficult to attach much significance to this small difference. This point is better left to more accurate studies of the $N\Delta$ potential.

As previously discussed, a number of assumptions underlie the Sopkovich approximation. If all these assumptions are met, evidently, off-shell effects in initial- and final-state interactions do not contribute. One of the assumptions made is that the transition potential V_{ij} ($\equiv B_{\pi d,NN}$) is small. It is interesting to examine the consequences of making this assumption alone. In this case, a simple approximation may be derived. Starting with Eq. (2.6), the equation for the exact $NN\rightarrow\pi d$ t-matrix is

$$T_{21}=V_{21}+V_{21}G_1T_{11}+V_{22}G_2T_{21} \tag{3.4}$$

which can also be written in the form

$$(1-V_{22}G_2)T_{21}=V_{21}(1+G_1T_{11})\ . \tag{3.5}$$

Multiplying through on the left by $1+T_{22}G_2$, and assuming that all terms quadratic or higher in V_{21} may be neglected, we obtain the expression

$$T_{21}\approx T_{21}^{S'}\equiv(1+T_{22}G_2)V_{21}(1+G_1T_{11}) \tag{3.6}$$

which is exact in the limit $V_{21}\rightarrow 0$. Although Eq. (3.6) shares the smallness of V_{21} assumption with the Sopkovich prescription, it retains full off-shell information in its initial and final distortion factors. In Figs. 6 and 7 we compare the exact solution for $T_{\pi d,NN}$, with the one using the approximation of Eq. (3.6). Except around the resonance point, we find excellent agreement between the exact and approximate values. This comparison suggests that, except at the resonance position, the assumption that the Born term is "small" in $NN\rightarrow\pi d$ is justified in the Sopkovich approximation. Below the resonance peak, however, it is evident that the off-shell contributions are important, and need to be included in any calcu-

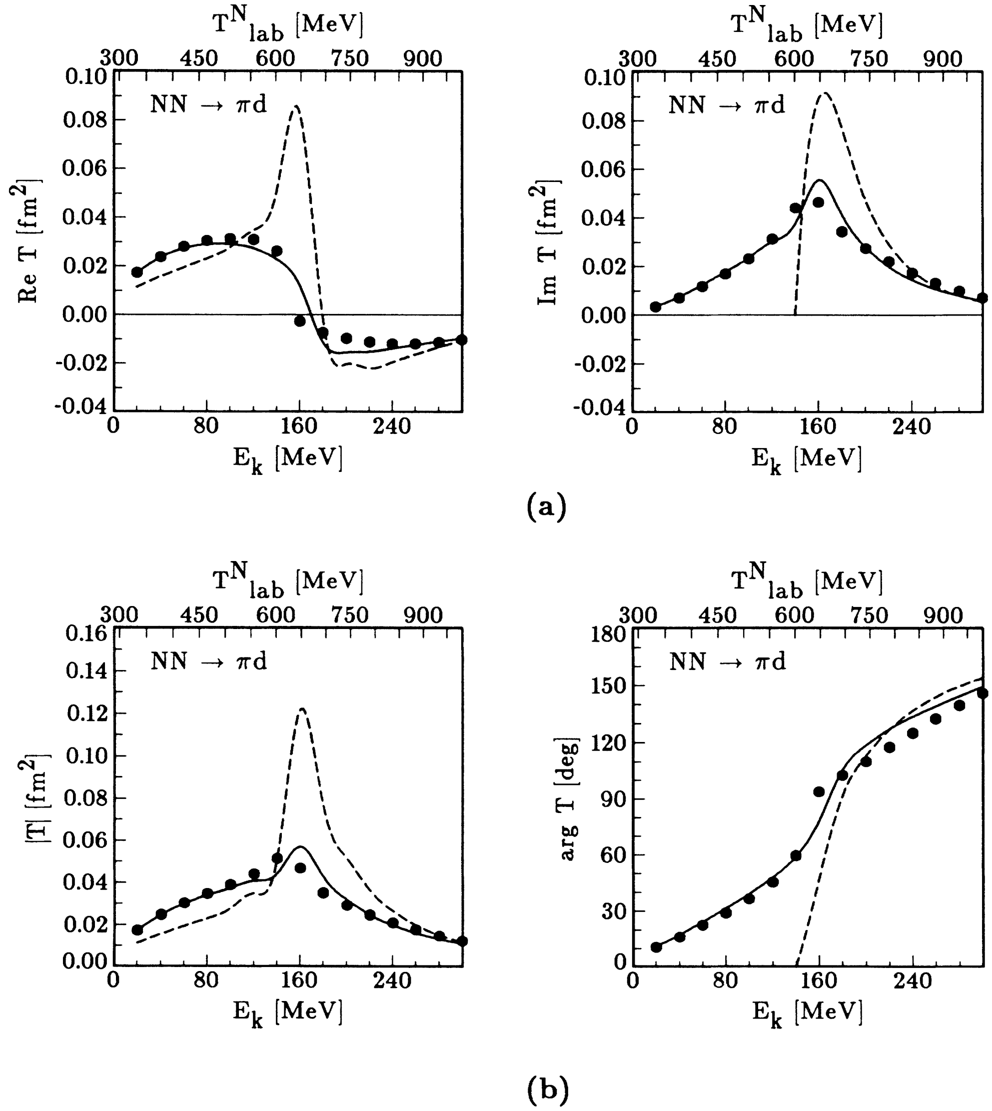

FIG. 6. Comparison of the exact t matrix for the process $NN\rightarrow\pi d$ (filled circles) with the approximation of Eq. (3.6) (solid line). Also shown is the unmodified Born term (dashed line).

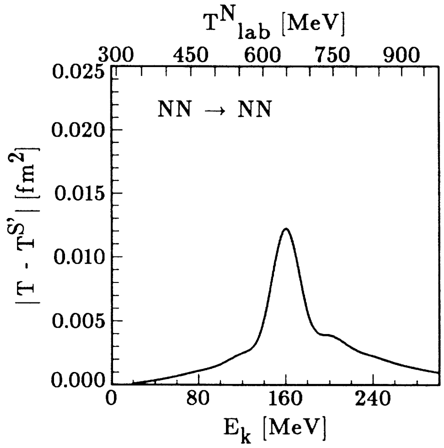

FIG. 7. The deviation of the approximation of Eq. (3.6) ($T^{S'}$) from the exact solution (T) for the $NN\to\pi d$ process.

lation of distortion effects.

This concludes our observations for the $J=2^+$ partial wave. We have also performed a similar analysis for the smaller $J=2^-$, 3^-, and 4^+ partial waves. For these the minimization procedure gives essentially two types of solutions, differing mainly in the size of the potential $v_{N\Delta,N\Delta}$. We have already pointed out that this potential would need to be smaller for practical applications of the Sopkovich prescription. Indeed the contribution of this potential was neglected from the Born term in the relativistic model of Ref. 9. To see if this neglect of $v_{N\Delta,N\Delta}$ is justified, we have examined the effect of setting this term to zero in the three-body model of AB. In this case $v_{N\Delta,N\Delta}$ is given by one pion exchange, and we find its effect to be relatively small for all partial waves with the biggest effect being in the $J=2^+$ channel. For the purposes of this paper we are, therefore, led to keep only those minimization solutions that have a small value for $v_{N\Delta,N\Delta}$.

For the $J=3^-$ partial wave, the $NN\to\pi d$ Born term tends to be already close to the full t matrix, except around the resonance region where it has a notable peak. This peak is similar but smaller than the one shown in Fig. 4 for the $J=2^+$ case. The Sopkovich corrected Born term then tends to agree quite well with the exact t matrix across the whole $0\le E_k\le 300$ MeV energy region. For the $J=2^-$ and 4^+ partial waves the Born terms do not display significant peaking but tend to be very close to the full t matrix. The additional Sopkovich correction then has a minimal effect. For the above partial waves we therefore find that the validity of the Sopkovich prescription is limited essentially by the $J=2^+$ partial wave.

As mentioned in the Introduction, our three-channel model is not appropriate for $J=0^+$ and $J=1^-$. Indeed, reasonable fits for these partial waves appear not to be possible using the model as described above. This is in fact expected, because these partial waves couple strongly to s-wave πN states, and these are not explicitly taken into account in the three-channel model. Moreover, as these s states are nonresonant, we would not expect a description in terms of effective two-body states to be reliable. We therefore leave the validity of the Sopkovich prescription in the 0^+ and 1^- partial waves as an open question.

IV. SUMMARY

In order to study the validity of on-shell distortion factors for $NN\to\pi d$, we have constructed an exactly solvable two-body coupled channels model for all the reactions $NN\to NN$, $NN\to\pi d$, and $\pi d\to\pi d$. The model has eight parameters, describing transition potentials between NN, πd, and $N\Delta$ states, and is used to fit given amplitude data in the dominant $J=2^+$ partial wave. Despite the phenomenological nature of the model, we are able to obtain good fits to all the above reactions; in addition, our results do not appear to differ greatly from those of more microscopic approaches. With this model we have examined the frequently used Sopkovich prescription for the on-shell distortion factors—Eq. (1.1). Although the theoretical conditions for the validity of this prescription are known—Born term must be small and short range, and energy must be large, we have been able to quantify the conditions for the $NN\to\pi d$ reaction. We find that the Sopkovich prescription works well only above 200 MeV total center-of-mass kinetic energy, although it does help reduce the large differences between the $NN\to\pi d$ Born term and the exact t matrix also at the lower energies. To better understand the inaccuracy of the Sopkovich prescription below 200 MeV, we have also examined an off-shell distortion approximation, Eq. (3.6), that shares with the Sopkovich prescription the assumption that the Born term is small. Despite the inclusion of off-shell contributions, we find that the assumption of a small Born term appears to be invalid close to the Δ resonance peak. Below the resonance, although the Born term here appears small enough, off-shell effects become important making the Sopkovich prescription inaccurate.

A similar investigation has been performed for the smaller $J=2^-$, 3^-, and 4^+ partial waves. For these we find that the Sopkovich prescription works at least as well as for the 2^+ channel. Excluding the $J=0^+$ and 1^- channels for which our model is inappropriate, we therefore find that the validity of the Sopkovich approximation is mainly determined by the 2^+ channel.

We would like to thank M. Locher for many useful discussions.

[1]A. M. Green and J. A Niskanen, Nucl. Phys. **A271**, 503 (1976); A. M. Green, J. A. Niskanen, and M. E. Sainio, J. Phys. G **4**, 1055 (1978); J. A. Niskanen, Nucl. Phys. **A298**, 417 (1978).

[2]T. Mizutani and A. Koltun, Ann. Phys. (N.Y.) **109**, 1 (1977); Y. Avishai and T. Mizutani, Nucl. Phys. **A326**, 352 (1979); **A338**, 372 (1980); **A352**, 399 (1981); T. Mizutani, C. Fayard, G. H. Lamot, and R. S. Nahabetian, Phys. Lett. **80B**, 177 (1981).

[3]A. W. Thomas and A. S. Rinat, Phys. Rev. C **20**, 216 (1979); A.

S. Rinat, Y. Starkand, and E. Hammel, Nucl. Phys. **A364**, 486 (1981).

[4]I. R. Afnan and B. Blankleider, Phys. Lett. **93B**, 367 (1980); Phys. Rev. C **22**, 367 (1980); **24**, 1572 (1981); **31**, 1380 (1985).

[5]T. Ueda, Prog. Theor. Phys. **76**, 729 (1986); **76**, 959 (1986).

[6]H. Garcilazo, Phys. Rev. Lett. **53**, 652 (1984).

[7]B. Goplen, W. R. Gibbs, and E. L. Lomon, Phys. Rev. Lett. **32**, 1012 (1974).

[8]O. V. Maxwell, W. Weise, and M. Brack, Nucl. Phys. **A348**, 388 (1980); **A348**, 429 (1980).

[9]W. Grein, A. König, P. Kroll, M. P. Locher, and A. Švarc, Ann. Phys. (N.Y.) **153**, 301 (1984); M. P. Locher and A. Šarc, J. Phys. G **11**, (1985).

[10]T. Yao, Phys. Rev. **134B**, 454 (1964).

[11]J. N. Chahoud, G. Russo, and F. Selleri, Nuovo Cimento **45A**, 38 (1966).

[12]D. Schiff and J. Tran Thanh Van, Nucl. Phys. **B5**, 529 (1968).

[13]G. W. Barry, Ann. Phys. (N.Y.) **73**, 482 (1972).

[14]N. J. Sopkovich, Nuovo Cimento **26**, 186 (1962).

[15]M. H. Ross and G. L. Shaw, Phys. Rev. Lett. **12**, 627 (1964).

[16]K. Gottfried and J. D. Jackson, Nuovo Cimento **34**, 735 (1964).

[17]L. Durrand III and Yam Tsi Chiu, Phys. Rev. **139**, B646 (1965).

[18]K. M. Watson, Phys. Rev. **95**, 228 (1954).

[19]M. L. Goldberger and K. M. Watson, *Collision Theory* (Wiley, New York, 1964).

[20]R. A. Arndt, L. D. Roper, R. A. Bryan, R. B. Clark, B. J. VerWest, and P. Signell, Phys. Rev. D **28**, 97 (1983); and values from their program SAID (1988 version).