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Extension of multiphonon theory to odd-mass nuclei

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The multiphonon method previously developed for a system containing an even number of fermions is extended to the case where this number is odd. Recursion formulas well suited for realistic applications to odd-mass nuclei are given for overlaps and matrix elements of one- and two-body operators.

I. INTRODUCTION

The multiphonon method (MPM) has been developed to study the observed anharmonicities of the vibrational motion in even-even deformed nuclei. The method presents two main advantages. First it is, to our knowledge, the only method that takes fully and properly the Pauli principle into account; second, it is an exact diagonalization of the model Hamiltonian and avoids therefore all of the problems inherent to any perturbative treatment.

Two general formulations of the multiphonon theory for a system containing an even number of fermions (even case) have been given in Ref. 1, hereafter quoted as paper I. The first one is a generalization of the Wick's theorem to phonons. It leads to compact and elegant expressions. The second one, based on recursion formulas, is more easily handled in realistic numerical calculations. The latter one has been successfully applied to nuclear structure problems such as quadrupole and octupole vibrations in even-even deformed nuclei.[2–4] A simple version of this method, where only one type of phonon is considered, has been extended to the case of an odd number of fermions (odd case)[5] and applied to the odd-mass light Actinium.[6]

The aim of this paper is to extend the general formulation of the MPM based on recursion formulas to the odd case. In Sec. II, it is shown that the matrix elements needed in the odd case can be expressed in terms of elements to be calculated in the even case. Among the latter some have been given in I. They are given once again in Sec. III in a slightly different form in addition to new terms needed to the odd case. Illustrative examples are given in Sec. IV, while conclusions are drawn in the last section.

II. EXPRESSIONS FOR THE ODD CASE

As in paper I the phonons $\mathcal{Q}_i^\dagger$ are defined as a superposition of two fermions

$$\mathcal{Q}_i^\dagger=\tfrac{1}{2}\sum_{\mu\nu}(X_i)_{\mu\nu}\alpha_\mu^\dagger\alpha_\nu^\dagger\ , \tag{1}$$

where the matrix X_i is antisymmetric and where the operator $\alpha_\mu^\dagger$ creates a fermion (e.g., a quasiparticle) characterized by quantum numbers summarized by label μ. In order to take the Pauli principle properly into account the full commutation rules of these phonons are considered

$$(\mathcal{Q}_1,\mathcal{Q}_2^\dagger)=-\tfrac{1}{2}\mathrm{Tr}(X_1X_2)+\sum_{\mu\nu}(X_1X_2)_{\mu\nu}\alpha_\nu^\dagger\alpha_\mu\ . \tag{2}$$

In the even case we introduce, as previously done, multiphonons states noted

$$\begin{aligned}|\mathbf{k}_r\rangle&=|k_1,k_2,\ldots,k_r\rangle\\&=(Q_1^\dagger)^{k_1}(Q_2^\dagger)^{k_2}\cdots(Q_r^\dagger)^{k_r}|0\rangle\ ,\end{aligned} \tag{3}$$

where, for convenience

$$(Q_1^\dagger)^{k_i}=\frac{(\mathcal{Q}_i^\dagger)^{k_i}}{k_i!}\ . \tag{4}$$

We shall further introduce the notation $|\mathbf{k}_r-k_i\rangle$ to label the multiphonon state (3), where k_i has been replaced by k_i-1.

It seems worthwhile to remind the reader that the multiphonon states (3) do not form a basis in the strict mathematical sense. As a consequence, we need to calculate the overlap matrix of states (3) in addition to the usual one- and two-body operator matrix elements.

As in I, we note by $A^{ij}_{\mu\nu\cdots\omega}(\mathbf{k}_r';\mathbf{k}_r)$ the matrix element of the product

$$P_{ij}=\underbrace{\alpha^\dagger\alpha^\dagger\alpha^\dagger\cdots\alpha^\dagger}_{i}\cdots\underbrace{\alpha\alpha\cdots\alpha}_{j}\ , \tag{5}$$

which contains i creation operators $\alpha^\dagger$ and j annihilation operators α. For instance

$$\begin{aligned}A^{02}_{\nu\mu}(\mathbf{k}_r';\mathbf{k}_r)&=-A^{02}_{\mu\nu}(\mathbf{k}_r';\mathbf{k}_r)\\&=\langle(Q_r)^{k_r'}\cdots(Q_2)^{k_2'}(Q_1)^{k_1'}\alpha_\mu\alpha_\nu\\&\quad\times(Q_1^\dagger)^{k_1}(Q_2^\dagger)^{k_2}\cdots(Q_r^\dagger)^{k_r}\rangle\ .\end{aligned} \tag{6}$$

It is clear that

$$A^{ij}_{\mu\nu\cdots\omega}(\mathbf{k}_r';\mathbf{k}_r)=A^{ji}_{\omega\cdots\nu\mu}(\mathbf{k}_r;\mathbf{k}_r')\ . \tag{7}$$

In the odd case, the multiphonon states write

$$|\mathbf{k}_r,\lambda\rangle=(Q_1^\dagger)^{k_1}(Q_2^\dagger)^{k_2}\cdots(Q_r^\dagger)^{k_r}\alpha_\lambda^\dagger|0\rangle\ . \tag{8}$$

To avoid confusion, we introduce in this case the notation

$$T^{ij}(\mathbf{k}'_r\lambda';\mathbf{k}_r\lambda)=\langle\mathbf{k}'_r\lambda'|P_{ij}|\mathbf{k}_r\lambda\rangle \ . \tag{9}$$

Here also one has a relation similar to (7)

$$T^{ij}_{\mu\nu\cdots\omega}(\mathbf{k}'_r\lambda';\mathbf{k}_r\lambda)=T^{ji}_{\omega\cdots\nu\mu}(\mathbf{k}_r\lambda;\mathbf{k}'_r\lambda') \ . \tag{10}$$

Note the appropriate order of the subindices in (7) and (10). As a consequence, we can restrict ourselves to elements A^{ij} and T^{ij} where $i\geq j$ or $j\leq i$.

Our task in this section will be to calculate the T^{ij} elements in terms of A^{kl} elements. We start with the overlap matrix elements

$$T^{00}(\mathbf{k}'_r\lambda';\mathbf{k}_r\lambda)=\langle\mathbf{k}'_r\alpha_{\lambda'}\alpha^\dagger_\lambda\mathbf{k}_r\rangle$$
$$=\delta_{\lambda\lambda'}A^{00}(\mathbf{k}'_r;\mathbf{k}_r)-A^{11}_{\lambda\lambda'}(\mathbf{k}'_r;\mathbf{k}_r) \ . \tag{11}$$

For the one-body matrix elements, one gets similarly the relations by writing $\alpha_{\lambda'}P_{ij}\alpha^\dagger_\lambda$ in normal order:

$$T^{20}_{12}(\mathbf{k}'_r\lambda';\mathbf{k}_r\lambda)=-T^{20}_{21}(\mathbf{k}'_r\lambda';\mathbf{k}_r\lambda)=\langle\mathbf{k}'_r\alpha_{\lambda'}\alpha^\dagger_1\alpha^\dagger_2\alpha^\dagger_\lambda\mathbf{k}_r\rangle$$
$$=\delta_{\lambda'1}A^{20}_{2\lambda}(\mathbf{k}'_r;\mathbf{k}_r)-\delta_{\lambda'2}A^{20}_{1\lambda}(\mathbf{k}'_r;\mathbf{k}_r)$$
$$+\delta_{\lambda\lambda'}A^{20}_{12}(\mathbf{k}'_r;\mathbf{k}_r)-A^{31}_{12\lambda\lambda'}(\mathbf{k}'_r;\mathbf{k}_r) \ , \tag{12}$$

$$T^{11}_{12}(\mathbf{k}'_r\lambda';\mathbf{k}_r\lambda)=\langle\mathbf{k}'_r\alpha_{\lambda'}\alpha^\dagger_1\alpha_2\alpha^\dagger_\lambda\mathbf{k}_r\rangle$$
$$=\delta_{\lambda'1}\delta_{\lambda2}A^{00}(\mathbf{k}'_r;\mathbf{k}_r)-\delta_{\lambda'1}A^{11}_{\lambda2}(\mathbf{k}'_r;\mathbf{k}_r)$$
$$-\delta_{\lambda2}A^{11}_{1\lambda'}(\mathbf{k}'_r;\mathbf{k}_r)+\delta_{\lambda\lambda'}A^{11}_{12}(\mathbf{k}'_r;\mathbf{k}_r)$$
$$-A^{22}_{1\lambda\lambda'2}(\mathbf{k}'_r;\mathbf{k}_r) \ . \tag{13}$$

It is seen that T^{20} can be deduced from A^{20} and A^{31}, while T^{11} is calculated from A^{00}, A^{11}, and A^{22}. We remind the reader that A^{31} and A^{22} can be obtained from A^{20} as shown in I. As a consequence the one-body matrix elements for the odd case can all be obtained with the help of formulas given in I.

For the two-body operators, the relations are a little bit more elaborated. But they are still easily obtained by writing $\alpha_{\lambda'}P_{ij}\alpha^\dagger_\lambda$ in normal order. One gets

$$T^{40}_{1234}(\mathbf{k}'_r\lambda';\mathbf{k}_r\lambda)=\langle\mathbf{k}'_r\alpha_{\lambda'}\alpha^\dagger_1\alpha^\dagger_2\alpha^\dagger_3\alpha^\dagger_4\alpha^\dagger_\lambda\mathbf{k}_r\rangle$$
$$=\delta_{\lambda'1}A^{40}_{234\lambda}-\delta_{\lambda'2}A^{40}_{134\lambda}+\delta_{\lambda'3}A^{40}_{124\lambda}-\delta_{\lambda'4}A^{40}_{123\lambda}+\delta_{\lambda\lambda'}A^{40}_{1234}-A^{51}_{1234\lambda\lambda'} \ , \tag{14}$$

$$T^{31}_{1234}(\mathbf{k}'_r\lambda';\mathbf{k}_r\lambda)=\langle\mathbf{k}'_r\alpha_{\lambda'}\alpha^\dagger_1\alpha^\dagger_2\alpha^\dagger_3\alpha_4\alpha^\dagger_\lambda\mathbf{k}_r\rangle$$
$$=\delta_{\lambda4}[\delta_{\lambda'1}A^{20}_{23}-\delta_{\lambda'2}A^{20}_{13}+\delta_{\lambda'3}A^{20}_{12}]$$
$$-\delta_{\lambda'1}A^{31}_{23\lambda4}+\delta_{\lambda'2}A^{31}_{13\lambda4}-\delta_{\lambda'3}A^{31}_{12\lambda4}+\delta_{\lambda\lambda'}A^{31}_{1234}-\delta_{\lambda4}A^{31}_{123\lambda'}-A^{42}_{123\lambda\lambda'4} \ , \tag{15}$$

$$T^{22}_{1234}(\mathbf{k}'_r\lambda';\mathbf{k}_r\lambda)=\langle\mathbf{k}'_r\alpha_{\lambda'}\alpha^\dagger_1\alpha^\dagger_2\alpha_3\alpha_4\alpha^\dagger_\lambda\mathbf{k}_r\rangle$$
$$=\delta_{\lambda'1}[\delta_{\lambda4}A^{11}_{23}-\delta_{\lambda3}A^{11}_{24}]+\delta_{\lambda'2}[\delta_{\lambda3}A^{11}_{14}-\delta_{\lambda4}A^{11}_{13}]$$
$$+\delta_{\lambda'1}A^{22}_{2\lambda34}-\delta_{\lambda'2}A^{22}_{1\lambda34}+\delta_{4\lambda}A^{22}_{12\lambda'3}-\delta_{3\lambda}A^{22}_{12\lambda'4}+\delta_{\lambda\lambda'}A^{22}_{1234}-A^{33}_{12\lambda\lambda'34} \ , \tag{16}$$

where, for simplification, the arguments $(\mathbf{k}'_r;\mathbf{k}_r)$ of all A have been omitted. It is found that T^{40} can be deduced from A^{40} and A^{51}, T^{31} from A^{20}, A^{31}, and A^{42} and finally T^{22} from A^{11}, A^{22}, and A^{33}.

In I it has been shown that A^{11}, A^{31}, and A^{22} can be expressed in terms of A^{20} and A^{40}. Using a similar method, we now show that the new quantities A^{51}, A^{42}, and A^{33} can be obtained from the knowledge of A^{40} and A^{60}. Explicitly one gets

$$A^{51}_{123456}(\mathbf{k}'_r;\mathbf{k}_r)=\langle\mathbf{k}'_r\alpha^\dagger_1\alpha^\dagger_2\alpha^\dagger_3\alpha^\dagger_4\alpha^\dagger_5\alpha_6\mathbf{k}_r\rangle=\sum_i\langle\mathbf{k}'_r\alpha^\dagger_1\alpha^\dagger_2\alpha^\dagger_3\alpha^\dagger_4(\mathbf{k}_r-k_i)[\alpha^\dagger_5\alpha_6,Q^\dagger_i]\rangle \ , \tag{17}$$

which according to the commutator given in relation (A1) of the Appendix leads to

$$A^{51}_{123456}(\mathbf{k}'_r;\mathbf{k}_r)=\sum_i\sum_\mu(X_i)(x_i)_{6\mu}A^{60}_{12345\mu}(\mathbf{k}'_r;\mathbf{k}_r-k_i) \ . \tag{18}$$

In a similar way

$$A^{42}_{123456}(\mathbf{k}'_r;\mathbf{k}_r)=\langle\mathbf{k}'_r\alpha^\dagger_1\alpha^\dagger_2\alpha^\dagger_3\alpha^\dagger_4\alpha_5\alpha_6\mathbf{k}_r\rangle=\sum_i\langle\mathbf{k}'_r\alpha^\dagger_1\alpha^\dagger_2\alpha^\dagger_3\alpha^\dagger_4(\mathbf{k}_r-k_i)[\alpha_5\alpha_6,Q^\dagger_i]\rangle+\sum_{i\leq j}\langle\mathbf{k}'_r\alpha^\dagger_1\alpha^\dagger_2\alpha^\dagger_3\alpha^\dagger_4(\mathbf{k}_r-k_i-k_j)C^{02}_{11}(i,j)\rangle \ , \tag{19}$$

where

$$C^{02}_{11}(i,j)=[[\alpha_5\alpha_6,Q^\dagger_i],Q^\dagger_j] \ .$$

If one uses relations (A2) and (A3) one gets

$$A^{42}_{123456}(\mathbf{k}'_r;\mathbf{k}_r)=-\sum_{i}(X_i)_{56}A^{40}_{1234}(\mathbf{k}'_r;\mathbf{k}_r-k_i)+\sum_{i\le j}\sum_{ij}^{\mathcal{P}}\sum_{\mu\nu}(X_i)_{5\mu}(X_j)_{6\nu}A^{60}_{1234\mu\nu}(\mathbf{k}'_r;\mathbf{k}_r-k_i-k_j)\ , \tag{20}$$

where $\sum_{ij}^{\mathcal{P}}$ is the sum over all permutations of indices i and j. Note that, for $i=j$, one gets a factor $2!(X_i)_{5\mu}(X_i)_{6\nu}$, and that this (2!) disappears with the factor $1/2!$ contained in $(Q_1^\dagger)^2$, as defined in (4). Finally

$$A^{33}_{123456}(\mathbf{k}'_r;\mathbf{k}_r)=\langle \mathbf{k}'_r\alpha_1^\dagger\alpha_2^\dagger\alpha_3^\dagger\alpha_4\alpha_5\alpha_6\mathbf{k}_r\rangle$$
$$=\sum_{i\le j}\langle \mathbf{k}'_r\alpha_1^\dagger\alpha_2^\dagger(\mathbf{k}_r-k_i-k_j)C^{13}_{11}(i,j)\rangle+\sum_{i\le j\le k}\langle \mathbf{k}'_r\alpha_1^\dagger\alpha_2^\dagger(\mathbf{k}_r-k_i-k_j-k_k)C^{13}_{111}(i,j,k)\rangle\ , \tag{21}$$

where

$$C^{13}_{11}(i,j)=[[\alpha_3^\dagger\alpha_4\alpha_5\alpha_6,Q_1^\dagger],Q_j^\dagger]\ ,$$

and

$$C^{13}_{111}(i,j,k)=[C^{13}_{11}(i,j),Q_k^\dagger]\ .$$

Using relations (A4) and (A5) one obtains

$$A^{33}_{123456}(\mathbf{k}'_r;\mathbf{k}_r)=-\sum_{i\le j}\sum_{ij}^{\mathcal{P}}\sum_{\mu}[(X_i)_{56}(X_j)_{4\mu}-(X_i)_{46}(X_j)_{5\mu}+(X_i)_{45}(X_j)_{6\mu}]A^{40}_{123\mu}(\mathbf{k}'_r;\mathbf{k}_r-k_i-k_j)$$
$$-\sum_{i\le j\le k}\sum_{ijk}^{\mathcal{P}}\sum_{\mu\nu\rho}(X_i)_{4\mu}(X_j)_{5\nu}(X_k)_{6\rho}A^{60}_{123\mu\nu\rho}(\mathbf{k}'_r;\mathbf{k}_r-k_i-k_j-k_k)\ , \tag{22}$$

where $\sum_{ijk}^{\mathcal{P}}$ is the sum over all permutations of indices i, j, and k.

Here also the $1/p!$ contained in $(Q^\dagger)^p$ compensates the $p!$ obtained from identical terms in $\sum^{\mathcal{P}}$ when $p=2$ (or $p=3$), i.e., when 2 (or 3) indices among i, j, and k are equal. To summarize this section, it has been shown that the overlaps and matrix elements of one- and two-body operators of a system with an odd number of fermions can all be expressed in terms of the quantities A^{00}, A^{20}, A^{40}, and A^{60} of the even case.

III. CALCULATION OF THE A^{20}, A^{40}, AND A^{60} MATRIX ELEMENTS

To calculate the matrix elements A^{r0} or, equivalently, A^{0r} we use the same techniques as in relations (17), (19), and (21). The game consists of commuting all annihilation operators α of P_{0r} with all the phonons $Q_i^\dagger$, so as to arrive to the action of the α on the vacuum. In this procedure one needs only to retain the successive commutators whose action on $|0\rangle$ is nonzero. Using the displayed formulas given in the Appendix one gets successively:

$$A^{02}_{21}(\mathbf{k}'_r;\mathbf{k}_r)=-A^{02}_{12}(\mathbf{k}'_r;\mathbf{k}_r)=\langle \mathbf{k}'_r\alpha_1\alpha_2\mathbf{k}_r\rangle=\sum_{i}\langle \mathbf{k}'_r(\mathbf{k}_r-k_i)C^{02}_{1}(i)\rangle+\sum_{i\le j}\langle \mathbf{k}'_r(\mathbf{k}_r-k_i-k_j)C^{02}_{11}(i,j)\rangle$$
$$=-\sum_{i}(X_i)_{12}A^{00}(\mathbf{k}'_r;\mathbf{k}_r-k_i)+\sum_{i\le j}\sum_{ij}^{\mathcal{P}}\sum_{\mu\nu}(X_i)_{1\mu}(X_j)_{2\nu}A^{20}_{\mu\nu}(\mathbf{k}'_r;\mathbf{k}_r-k_i-k_j)\ , \tag{23}$$

$$A^{04}_{4321}(\mathbf{k}'_r;\mathbf{k}_r)=A^{04}_{1234}(\mathbf{k}'_r;\mathbf{k}_r)=\langle \mathbf{k}'_r\alpha_1\alpha_2\alpha_3\alpha_4\mathbf{k}_r\rangle=\sum_{i\le j}\langle \mathbf{k}'_r(\mathbf{k}_r-k_i-k_j)C^{04}_{11}(i,j)\rangle$$
$$+\sum_{i\le j\le k}\langle \mathbf{k}'_r(\mathbf{k}_r-k_i-k_j-k_k)C^{04}_{111}(i,j,k)\rangle+\sum_{i\le j\le k\le l}\langle \mathbf{k}'_r(\mathbf{k}_r-k_i-k_j-k_k-k_l)C^{04}_{1111}(i,j,k,l)\rangle$$
$$=\sum_{i\le j}\sum_{ij}^{\mathcal{P}}\sum^{4}(-1)^{a+b+1}(X_i)_{ab}(X_j)_{cd}A^{00}(\mathbf{k}'_r;\mathbf{k}_r-k_i-k_j)$$
$$+\sum_{i\le j\le k}\sum_{ijk}^{\mathcal{P}}\sum_{\mu\nu}\sum^{4}(-1)^{a+b}(X_i)_{ab}(X_j)_{c\mu}(X_k)_{d\nu}A^{20}_{\mu\nu}(\mathbf{k}'_r;\mathbf{k}_r-k_i-k_j-k_k)$$
$$+\sum_{i\le j\le k\le l}\sum_{ijkl}^{\mathcal{P}}\sum_{\mu\nu\rho\sigma}(X_i)_{1\mu}(X_j)_{2\nu}(X_k)_{3\rho}(X_l)_{4\sigma}A^{40}_{\mu\nu\rho\sigma}(\mathbf{k}'_r;\mathbf{k}_r-k_i-k_j-k_k-k_l)$$
$$=\sum_{i\le j}\sum_{ij}^{\mathcal{P}}\sum^{4}(-1)^{a+b+1}(X_i)_{ab}(X_j)_{cd}A^{00}(\mathbf{k}'_r;\mathbf{k}_r-k_i-k_j)$$
$$+\sum_{i\le j\le k}\sum_{ijk}^{\mathcal{P}}\sum^{4}(-1)^{a+b+1}(X_i)_{ab}[X_jA^{20}(\mathbf{k}'_r;\mathbf{k}_r-k_i-k_j-k_k)X_k]_{cd}$$
$$+\sum_{i\le j\le k\le l}\sum_{ijkl}^{\mathcal{P}}[X_iX_jA^{40}(\mathbf{k}'_r;\mathbf{k}_r-k_i-k_j-k_k-k_l)X_kX_l]_{1234}\ , \tag{24}$$

where $\sum^4$ is the sum over all permutations P_4 of the four indices 1234 such as $a<b$ and $c<d$ and where obvious notations have been introduced for the summation over the greek indices.

$$
\begin{aligned}
A^{06}_{654321}(\mathbf{k}'_r;\mathbf{k}_r)&=-A^{06}_{123456}(\mathbf{k}'_r;\mathbf{k}_r)=\langle \mathbf{k}'_r\alpha_1\alpha_2\alpha_3\alpha_4\alpha_5\alpha_6\mathbf{k}_r\rangle\\
&=\sum_{i\le j\le k}\sum_{ijk}^{\mathcal{P}}\langle \mathbf{k}'_r(\mathbf{k}_r-k_i-k_j-k_k)C^{06}_{111}(i,j,k)\rangle+\sum_{i\le j\le k\le l}\sum_{ijkl}^{\mathcal{P}}\langle \mathbf{k}'_r(\mathbf{k}_r-k_i-k_j-k_k-k_l)C^{06}_{1111}(i,j,k,l)\rangle\\
&\quad+\sum_{i\le j\le k\le l\le m}\sum_{ijklm}^{\mathcal{P}}\langle \mathbf{k}'_r(\mathbf{k}_r-k_i-k_j-k_k-k_l-k_m)C^{06}_{11111}(i,j,k,l,m)\rangle\\
&\quad+\sum_{i\le j\le k\le l\le m\le n}\sum_{ijklmn}^{\mathcal{P}}\langle \mathbf{k}'_r(\mathbf{k}_r-k_i-k_j-k_k-k_l-k_m-k_n)C^{06}_{111111}(i,j,k,l,m,n)\rangle\\
&=-\sum_{i\le j\le k}\sum_{ijk}^{\mathcal{P}}\sum^{6}(-1)^{a+b+c+d}(X_i)_{ab}(X_j)_{cd}(X_k)_{ef}A^{00}(\mathbf{k}'_r;\mathbf{k}_r-k_i-k_j-k_k)\\
&\quad+\sum_{i\le j\le k\le l}\sum_{ijkl}^{\mathcal{P}}\sum^{6}\sum_{\mu\nu}(-1)^{a+b+c+d}(X_i)_{ab}(X_j)_{cd}(X_k)_{e\lambda}(X_l)_{f\mu}A^{20}_{\lambda\mu}(\mathbf{k}'_r;\mathbf{k}_r-k_i-k_j-k_k-k_l)\\
&\quad+\sum_{i\le j\le k\le l\le m}\sum_{ijklm}^{\mathcal{P}}\sum_{\lambda\mu\nu\rho}\sum^{6'}(-1)^{a+b}(X_i)_{ab}(X_j)_{c\lambda}(X_k)_{d\mu}(X_l)_{e\nu}(X_m)_{f\rho}\\
&\qquad\qquad\times A^{40}_{\lambda\mu\nu\rho}(\mathbf{k}'_r;\mathbf{k}_r-k_i-k_j-k_k-k_l-k_m)\\
&\quad+\sum_{i\le j\le k\le l\le m\le n}\sum_{ijklmn}^{\mathcal{P}}\sum_{\lambda\mu\nu\rho\sigma\tau}(X_i)_{1\lambda}(X_j)_{2\mu}(X_k)_{3\nu}(X_l)_{4\rho}(X_m)_{5\sigma}(X_n)_{6\tau}\\
&\qquad\qquad\times A^{60}_{\lambda\mu\nu\rho\sigma\tau}(\mathbf{k}'_r;\mathbf{k}_r-k_i-k_j-k_k-k_l-k_m-k_n)\\
&=-\sum_{i\le j\le k}\sum_{ijk}^{\mathcal{P}}\sum^{6}(-1)^{a+b+c+d}(X_i)_{ab}(X_j)_{cd}(X_k)_{ef}A^{00}(\mathbf{k}'_r;\mathbf{k}_r-k_i-k_j-k_k)\\
&\quad-\sum_{i\le j\le k\le l}\sum_{ijkl}^{\mathcal{P}}\sum^{6}(-1)^{a+b+c+d}(X_i)_{ab}(X_j)_{cd}[X_kA^{20}(\mathbf{k}'_r;\mathbf{k}_r-k_i-k_j-k_k-k_l)X_l]_{ef}\\
&\quad+\sum_{i\le j\le k\le l\le m}\sum_{ijklm}^{\mathcal{P}}\sum^{6'}(-1)^{a+b}(X_i)_{ab}[X_jX_kA^{40}(\mathbf{k}'_r;\mathbf{k}_r-k_i-k_j-k_k-k_l-k_m)X_lX_m]_{cdef}\\
&\quad+\sum_{i\le j\le k\le l\le m\le n}\sum_{ijklmn}^{\mathcal{P}}[X_iX_jX_kA^{60}(\mathbf{k}'_r;\mathbf{k}_r-k_i-k_j-k_k-k_l-k_m-k_n)X_lX_mX_n]_{123456}\,,
\end{aligned}
\tag{25}
$$

where $\sum^6$ is the sum over all permutations P_6 of the six indices 123456 such as $a<b, c<d$, and $e<f$, where $\sum^{6'}$ is the sum over all permutations P'_6 of the six indices 123456 such as $a<b$ and $c<d<e<f$, and where again obvious notations have been introduced for the summation over greek indices.

IV. ILLUSTRATIVE EXAMPLES

In the first example, where only one type of the collective phonon is considered, the formulas become much simpler, as shown in Ref. 5. For instance the overlap matrix formula decouples from the others and reads

$$T^{00}(k'\lambda';k\lambda)=\delta_{kk'}\sum_{l=0}^{k}(X^{2l})_{\lambda\lambda'}A^{00}(k-l;k-l) \tag{26}$$

with the definition

$$(X^0)_{\lambda\lambda'}=\delta_{\lambda\lambda'}\,. \tag{27}$$

As a second example, we choose the case where two types of collective phonons are introduced. This case may still be numerically tractable in realistic situations. The recursion formulas for A^{02} and A^{04} have been explicitly given in our previous works (see, e.g., Ref. 3). We therefore only indicate how to display the "new" relation (25) for $A^{06}(p'q';pq)$, where p and p' are relative to the first type of phonon, while q and q' deal with the second. The two phonons $Q_1^\dagger$ and $Q_2^\dagger$ are defined as in relation (1) by matrices X_1 and X_2, respectively. We first indicate the meaning of the sum $\sum_{i\le j\le k}$. It runs over four terms. The first one has $i=j=k=1$, contains a sum of 15 different products $X_1X_1X_1$ in factor of $A^{00}(p'q';p-3,q)$, the second one has $i=j=1$, $k=2$, contains all possible different products of the type $X_1X_1X_2$ in front of $A^{00}(p'q';p-2,q-1)$. The contribution of the two other terms where $i=1$, $j=k=2$, and $i=j=k=2$ can be deduced from the two preceding ones by interchanging the roles of p and q simultaneously with those of X_1 and X_2.

In a similar way, the sum $\sum_{i\le j\le k\le l}$ runs over five terms leading to $A^{20}(p'q';p-4,q)$, $A^{20}(p'q';p-3,q-1)$, $A^{20}(p'q';p-2,q-2)$, $A^{20}(p'q';p-1,q-3)$, and $A^{20}(p'q';p,q-4)$, with the appropriate coefficients. The sum $\sum_{i\le j\le k\le l\le m}$ contains products of five X and brings therefore six contributions in $A^{40}(p',q';p-a,q+a-5)$, where a varies from 0 to 5. Finally the sum $\sum_{i\le j\le k\le l\le m\le n}$ introduces products of six X in front of $A^{60}(p',q';p-b,q+b-6)$, with b varying from 0 to 6.

V. CONCLUSIONS

In this paper it has been shown that the general recursion formulation of the multiphonon method for a system with an even number of fermions can easily be extended to the case of an odd number of fermions. The price to pay is to calculate the matrix elements of the product of six annihilation operators between the multiphonon states of the even case.

In practical situations, the quality of the results to be obtained within this method greatly depends on the space of the multiphonon states (8). One needs to make an appropriate choice of the collective phonons used as the building blocks of the multiphonon theory *and* a suitable selection of the odd fermions involved. The corresponding limitations will essentially be of numerical order.

Several applications to nuclear structure problems are possible within this formalism. One of the most interesting is the study of vibrational states in odd-mass nuclei, where the odd particle couples to the vibrations of the core. In particular, it is worthwhile to search how the anharmonicities of the vibrations obtained in even-even deformed nuclei[3,4] are modified by the presence of the odd quasiparticle in odd-mass nuclei. For the octupole vibrations with $K=0$, the results obtained[4] in even-even nuclei have shown that one can safely restrict the multiphonon basis to one basic phonon. As a consequence, it is probable that the extension to odd-mass nuclei given in Ref. 5 can also be used for heavier Actinides. For the γ vibrations, where one has to introduce, at least two basic phonons, the extension to odd-mass nuclei leads to numerical problems that may still be tractable. Two open questions may find their answer in this application: (i) can one explain, in a microscopic way, why the $K-2$ vibrational level has always an energy lower than its $K+2$ partner? (ii) Where should one search for the $K\pm4$ levels arising from the "two γ phonon" state? We would like to emphasize two problems encountered in the applications of the multiphonon method. The first one is of numerical order: with the actual computer facilities, realistic calculations have to be restricted to two basic phonons. As a consequence, we cannot yet evaluate the importance of the coupling to the noncollective degrees of freedom and are forced to restrict the applications to nuclei where only two vibrational degrees of freedom are strongly collective. The second one is related to the cases where the pairing correlations play an important role. The fermions introduced in the multiphonon theory become quasiparticles and one has to take care of the particle nonconservation problem. From the applications to the even-even deformed nuclei, we learned that for the lowest vibrational states (e.g., those having main components of their wave function on one and two phonons states) the deviation of the number of particles remains small (of the order of 1). We may therefore expect, with some confidence, that the effects of the particle nonconservation may also be small in the odd-mass nuclei, at least for the lowest-lying states.

Finally, we would like to remind the reader, that in the multiphonon approach, the Pauli principle is fully taken into account. Furthermore, by taking into account the total model Hamiltonian (i.e., the total residual interaction) one can push the calculations further than the usual RPA and further than the quasiparticle-phonon nuclear model developed by Soloviev and his co-workers.[7]

APPENDIX

In this Appendix, we summarize the different commutators necessary to the proofs given in the main text. First, we write the commutation rules of the phonon operator $Q_i^\dagger$ defined in (1) with pairs of fermions

$$[\alpha_1^\dagger\alpha_2,Q_i^\dagger]=\sum_\mu (X_i)_{2\mu}\alpha_1^\dagger\alpha_\mu^\dagger \tag{A1}$$

and

$$[\alpha_1\alpha_2,Q_i^\dagger]=-(X_i)_{12}+\sum_\mu [(X_i)_{1\mu}\alpha_\mu^\dagger\alpha_2-(X_i)_{2\mu}\alpha_\mu^\dagger\alpha_1] .$$

From these relations, one deduces

$$[\alpha_1\alpha_2,Q_i^\dagger]|0\rangle=-(X_i)_{12} \tag{A2}$$

and

$$\begin{aligned}C_{11}^{02}(i,j)&=[[\alpha_1\alpha_2,Q_i^\dagger],Q_j^\dagger]\\&=\sum_{\mu\nu}[(X_i)_{1\mu}(X_j)_{2\nu}+(X_j)_{1\mu}(X_i)_{2\nu}]\alpha_\mu^\dagger\alpha_\nu^\dagger\\&=\sum_{ij}^{\mathcal{P}}\sum_{\mu\nu}(X_i)_{1\mu}(X_j)_{2\nu}\alpha_\mu^\dagger\alpha_\nu^\dagger ,\end{aligned} \tag{A3}$$

where $\sum_{ij}^{\mathcal{P}}$ is the sum over the 2! permutations of the indices i and j. Note that this permutation arises since $C(i,j)=C(j,i)$, which derives from

$$[\alpha_1\alpha_2,Q_i^\dagger Q_j^\dagger]=[\alpha_1\alpha_2,Q_j^\dagger Q_i^\dagger] .$$

Second, we give explicitly the commutators of operator $P_{13}=\alpha^\dagger\alpha\alpha\alpha$,

$$C_1^{13}(i)=[\alpha_1^\dagger\alpha_2\alpha_3\alpha_4,Q_i^\dagger]=-(X_i)_{34}\alpha_1^\dagger\alpha_2+(X_i)_{24}\alpha_1^\dagger\alpha_3-(X_i)_{23}\alpha_1^\dagger\alpha_4$$
$$+\sum_\mu[(X_i)_{2\mu}\alpha_1^\dagger\alpha_\mu^\dagger\alpha_3\alpha_4-(X_i)_{3\mu}\alpha_1^\dagger\alpha_\mu^\dagger\alpha_2\alpha_4+(X_i)_{4\mu}\alpha_1^\dagger\alpha_\mu^\dagger\alpha_2\alpha_3]\ ,$$

$$C_{11}^{13}(i,j)=[C_1^{13}(i),Q_j^\dagger]=\sum_{ij}^{\mathcal{P}}\left\{\sum_\mu[-(X_i)_{23}(X_j)_{4\mu}+(X_i)_{24}(X_j)_{3\mu}-(X_i)_{34}(X_j)_{2\mu}]\alpha_1^\dagger\alpha_\mu^\dagger\right.$$
$$\left.+\sum_{\mu\nu}[(X_i)_{3\mu}(X_j)_{4\nu}\alpha_\mu^\dagger\alpha_\nu^\dagger\alpha_1^\dagger\alpha_2-(X_i)_{2\mu}(X_j)_{4\nu}\alpha_\mu^\dagger\alpha_\nu^\dagger\alpha_1^\dagger\alpha_3+(X_i)_{2\mu}(X_j)_{3\nu}\alpha_\mu^\dagger\alpha_\nu^\dagger\alpha_1^\dagger\alpha_4]\right\}\ , \tag{A4}$$

and

$$C_{111}^{13}(i,j,k)=[C_{11}^{13}(i,j),Q_k^\dagger]=-\sum_{ijk}^{\mathcal{P}}\sum_{\mu\nu\rho}(X_i)_{2\mu}(X_j)_{3\nu}(X_k)_{4\rho}\alpha_\mu^\dagger\alpha_\nu^\dagger\alpha_\rho^\dagger\alpha_1^\dagger\ . \tag{A5}$$

The commutators C needed in Sec. III are more involved. It would be cumbersome to give their full explicit expressions. We therefore restrict ourselves to the parts of these that contribute to $C|0\rangle$.

The commutators for $P_{04}=\alpha_1\alpha_2\alpha_3\alpha_4$ have been given in I, we repeat them here in a slightly different presentation. The first commutator

$$C_1^{04}(i)=[\alpha_1\alpha_2\alpha_3\alpha_4,Q_i^\dagger]$$

contains six terms of the type $(X_i)_{ab}\alpha_c\alpha_d$ and four terms of the type $\sum_\mu(X_i)_{a\mu}\alpha_\mu^\dagger\alpha_b\alpha_c\alpha_d$ that do not contribute to $C_1^{04}(i)|0\rangle$. Here the set $(abcd)$ results from a permutation of set (1234). The second commutator

$$C_{11}^{04}(i,j)=[C_1^{04}(i),Q_j^\dagger]\ .$$

contains a constant term that will be given explicitly, 12 different terms of the type $\sum_\mu(X_i)_{ab}(X_j)_{c\mu}\alpha_\mu^\dagger\alpha_d$ and six terms of the type $\sum_{\mu\nu}(X_i)_{a\mu}(X_j)_{b\nu}\alpha_\mu^\dagger\alpha_\nu^\dagger\alpha_c\alpha_d$, which will all appear under the summation $\sum_{ij}^{\mathcal{P}}$,

$$C_{11}^{04}(i,j)|0\rangle=\sum_{ij}^{\mathcal{P}}[(X_i)_{12}(X_j)_{34}-(X_i)_{13}(X_j)_{24}+(X_i)_{14}(X_j)_{23}]|0\rangle\ . \tag{A6}$$

It can be seen easily that relation (A6) is equivalent to Eqs. (3.26) and (3.27) of I. In a similar way, the third commutator

$$C_{111}^{04}(i,j,k)=[C_{11}^{04}(i,j),Q_k^\dagger]$$

contains under the summation $\sum_{ijk}^{\mathcal{P}}$ six different terms of the type $\sum_{\mu\nu}(X_i)_{ab}(X_j)_{c\mu}(X_k)_{d\nu}\alpha_\mu^\dagger\alpha_\nu^\dagger$ that contribute to $C_{111}^{04}(i,j,k)|0\rangle$ and will be given below, and four different terms of the type

$$\sum_{\mu\nu\rho}(X_i)_{a\mu}(X_j)_{b\nu}(X_k)_{c\rho}\alpha_\mu^\dagger\alpha_\nu^\dagger\alpha_\rho^\dagger\alpha_d\ ,$$

$$C_{111}^{04}(i,j,k)|0\rangle=-\sum_{ijk}^{\mathcal{P}}\sum_{\mu\nu}\{(X_i)_{12}(X_j)_{3\mu}(X_k)_{4\nu}-(X_i)_{13}(X_j)_{2\mu}(X_k)_{4\nu}+(X_i)_{14}(X_j)_{2\mu}(X_k)_{3\nu}+(X_i)_{23}(X_j)_{1\mu}(X_k)_{4\nu}$$
$$-(X_i)_{24}(X_j)_{1\mu}(X_k)_{3\nu}+(X_i)_{34}(X_j)_{1\mu}(X_k)_{2\nu}\}\alpha_\mu^\dagger\alpha_\nu^\dagger|0\rangle\ . \tag{A7}$$

It can easily be seen that relation (A7) is equivalent to Eqs. (3.29) and (3.30) of I. Finally the fourth commutator gives

$$C_{1111}^{04}(i,j,k,l)=[C_{111}^{04}(i,j,k),Q_l^\dagger]$$
$$=\sum_{ijkl}^{\mathcal{P}}\sum_{\mu\nu\rho\sigma}(X_i)_{1\mu}(X_j)_{2\nu}(X_k)_{3\rho}(X_l)_{4\sigma}\alpha_\mu^\dagger\alpha_\nu^\dagger\alpha_\rho^\dagger\alpha_\sigma^\dagger\ . \tag{A8}$$

If we go to $P_{06}=\alpha_1\alpha_2\alpha_3\alpha_4\alpha_5\alpha_6$, we have to calculate the six successive commutators. $C_1^{06}(i)$ contains 15 terms of the type $(X_i)_{ab}\alpha_c\alpha_d\alpha_e\alpha_f$ and six terms of the type $\sum_\mu(X_i)_{a\mu}\alpha_\mu^\dagger\alpha_b\alpha_c\alpha_d\alpha_e\alpha_f$, where set $(abcdef)$ is deduced from set (123456) by permutation. As a consequence $C_1^{06}|0\rangle=0$. The second commutator $C_{11}^{06}(i,j)$ contains, under the summation $\sum_{ij}^{\mathcal{P}}$, 45 different terms of the form $(X_i)_{ab}(X_j)_{cd}\alpha_e\alpha_f$, 60 different terms of type $\sum_\mu(X_i)_{ab}(X_j)_{c\mu}\alpha_\mu^\dagger\alpha_d\alpha_e\alpha_f$, and 15 different contributions in

$$\sum_{\mu\nu}(X_i)_{a\mu}(X_j)_{b\nu}\alpha_\mu^\dagger\alpha_\nu^\dagger\alpha_c\alpha_d\alpha_e\alpha_f\ .$$

None of these terms contributes to $C_{11}^{06}(i,j)|0\rangle$. The third commutator contains, under the summation $\sum_{ijk}^{\mathcal{P}}$, 15 constant terms, 90 terms of the form $\sum_\mu (X_i)_{ab}(X_j)_{cd}(X_k)_{e\mu}\alpha^\dagger_\mu \alpha_f$, 90 terms

$$\sum_{\mu\nu}(X_i)_{ab}(X_j)_{c\mu}(X_k)_{d\nu}\alpha^\dagger_\mu \alpha^\dagger_\nu \alpha_e \alpha_f \ ,$$

and 20 terms in

$$\sum_{\mu\nu\rho}(X_i)_{a\mu}(X_j)_{b\nu}(X_k)_{c\rho}\alpha^\dagger_\mu \alpha^\dagger_\nu \alpha^\dagger_\rho \alpha_d \alpha_e \alpha_f \ .$$

Only the constant terms contribute to $C^{06}_{111}(i,j,k)|0\rangle$ and one has

$$\begin{aligned}
C^{06}_{111}(i,j,k)|0\rangle = -\sum_{ijk}^{\mathcal{P}} \{ & (X_i)_{12}(X_j)_{34}(X_k)_{56} - (X_i)_{12}(X_j)_{35}(X_k)_{46} + (X_i)_{12}(X_j)_{36}(X_k)_{45} \\
& -(X_i)_{13}(X_j)_{24}(X_k)_{56} + (X_i)_{13}(X_j)_{25}(X_k)_{46} - (X_i)_{13}(X_j)_{26}(X_k)_{45} \\
& +(X_i)_{14}(X_j)_{23}(X_k)_{56} - (X_i)_{14}(X_j)_{25}(X_k)_{36} + (X_i)_{14}(X_j)_{26}(X_k)_{35} \\
& -(X_i)_{15}(X_j)_{23}(X_k)_{46} + (X_i)_{15}(X_j)_{24}(X_k)_{36} - (X_i)_{15}(X_j)_{26}(X_k)_{34} \\
& +(X_i)_{16}(X_j)_{23}(X_k)_{45} - (X_i)_{16}(X_j)_{24}(X_k)_{35} + (X_i)_{16}(X_j)_{25}(X_k)_{36} \}|0\rangle \ .
\end{aligned} \tag{A9}$$

The fourth commutator contains, under the summation $\sum^{\mathcal{P}}_{ijkl}$, 45 different terms of the type $\sum_{\mu\nu}(X_i)_{ab}(X_j)_{cd}(X_k)_{e\mu}(X_l)_{f\nu}\,\alpha^\dagger_\mu \alpha^\dagger_\nu$ that contributes to $C^{06}_{1111}(i,j,k,l)|0\rangle$ and will be given below, 60 different terms of the type $\sum_{\mu\nu\rho}(X_i)_{ab}(X_j)_{c\mu}(X_k)_{d\nu}(X_l)_{e\rho}\alpha^\dagger_\mu \alpha^\dagger_\nu \alpha^\dagger_\rho \alpha_f$ and 15 different terms of the type $\sum_{\mu\nu\rho\sigma}(X_i)_{a\mu}(X_j)_{b\nu}(X_k)_{c\rho}(X_l)_{d\sigma}\alpha^\dagger_\mu \alpha^\dagger_\nu \alpha^\dagger_\rho \alpha^\dagger_\sigma \alpha_e \alpha_f$,

$$\begin{aligned}
C^{06}_{1111}(i,j,k,l)|0\rangle = \sum_{ijkl}^{\mathcal{P}}\sum_{\lambda\mu} \{ & [(X_i)_{12}(X_j)_{34} - (X_i)_{13}(X_j)_{24} + (X_i)_{14}(X_j)_{23}](X_k)_{5\lambda}(X_l)_{6\mu} \\
& -[(X_i)_{12}(X_j)_{35} - (X_i)_{13}(X_j)_{25} + (X_i)_{15}(X_j)_{23}](X_k)_{4\lambda}(X_l)_{6\mu} \\
& +[(X_i)_{12}(X_j)_{36} - (X_i)_{13}(X_j)_{26} + (X_i)_{16}(X_j)_{23}](X_k)_{4\lambda}(X_l)_{5\mu} \\
& +[(X_i)_{12}(X_j)_{45} - (X_i)_{14}(X_j)_{25} + (X_i)_{15}(X_j)_{24}](X_k)_{3\lambda}(X_l)_{6\mu} \\
& -[(X_i)_{12}(X_j)_{46} - (X_i)_{14}(X_j)_{26} + (X_i)_{16}(X_j)_{24}](X_k)_{3\lambda}(X_l)_{5\mu} \\
& +[(X_i)_{12}(X_j)_{56} - (X_i)_{15}(X_j)_{26} + (X_i)_{16}(X_j)_{25}](X_k)_{3\lambda}(X_l)_{4\mu} \\
& -[(X_i)_{13}(X_j)_{45} - (X_i)_{14}(X_j)_{35} + (X_i)_{15}(X_j)_{34}](X_k)_{2\lambda}(X_l)_{6\mu} \\
& +[(X_i)_{13}(X_j)_{46} - (X_i)_{14}(X_j)_{36} + (X_i)_{16}(X_j)_{34}](X_k)_{2\lambda}(X_l)_{5\mu} \\
& -[(X_i)_{13}(X_j)_{56} - (X_i)_{15}(X_j)_{36} + (X_i)_{16}(X_j)_{35}](X_k)_{2\lambda}(X_l)_{4\mu} \\
& +[(X_i)_{14}(X_j)_{56} - (X_i)_{15}(X_j)_{46} + (X_i)_{16}(X_j)_{45}](X_k)_{2\lambda}(X_l)_{3\mu} \\
& +[(X_i)_{23}(X_j)_{45} - (X_i)_{24}(X_j)_{35} + (X_i)_{25}(X_j)_{34}](X_k)_{1\lambda}(X_l)_{6\mu} \\
& -[(X_i)_{23}(X_j)_{46} - (X_i)_{24}(X_j)_{36} + (X_i)_{26}(X_j)_{34}](X_k)_{1\lambda}(X_l)_{5\mu} \\
& +[(X_i)_{23}(X_j)_{56} - (X_i)_{25}(X_j)_{36} + (X_i)_{26}(X_j)_{35}](X_k)_{1\lambda}(X_l)_{4\mu} \\
& -[(X_i)_{24}(X_j)_{56} - (X_i)_{25}(X_j)_{46} + (X_i)_{26}(X_j)_{45}](X_k)_{1\lambda}(X_l)_{3\mu} \\
& +[(X_i)_{34}(X_j)_{56} - (X_i)_{35}(X_j)_{46} + (X_i)_{36}(X_j)_{45}](X_k)_{1\lambda}(X_l)_{2\mu}\}\alpha^\dagger_\lambda \alpha^\dagger_\mu|0\rangle \ .
\end{aligned} \tag{A10}$$

The fifth operator contains under $\sum^{\mathcal{P}}_{ijklm}$, 15 different terms of the form

$$\sum_{\lambda\mu\nu\rho}(X_i)_{ab}(X_j)_{c\lambda}(X_k)_{d\mu}(X_l)_{e\nu}(X_m)_{f\rho}\alpha^\dagger_\lambda \alpha^\dagger_\mu \alpha^\dagger_\nu \alpha^\dagger_\rho \ ,$$

which contributes to $C^{06}_{11111}(i,j,k,l,m)|0\rangle$ and are given below and six different terms of the form

$$\sum_{\lambda\mu\nu\rho\sigma}(X_i)_{a\lambda}(X_j)_{b\mu}(X_k)_{c\nu}(X_l)_{d\rho}(X_m)_{e\sigma}\alpha^\dagger_\lambda \alpha^\dagger_\mu \alpha^\dagger_\nu \alpha^\dagger_\rho \alpha^\dagger_\sigma \alpha_f \ ,$$

$$C^{06}_{11111}(i,j,k,l,m)|0\rangle = -\sum_{ijklm}^{\mathcal{P}}\sum_{\lambda\mu\nu\rho}\{(X_i)_{12}(X_j)_{3\lambda}(X_k)_{4\mu}(X_l)_{5\nu}(X_m)_{6\rho}$$
$$-(X_i)_{13}(X_j)_{2\lambda}(X_k)_{4\mu}(X_l)_{5\nu}(X_m)_{6\rho}+(X_i)_{14}(X_j)_{2\lambda}(X_k)_{3\mu}(X_l)_{5\nu}(X_m)_{6\rho}$$
$$-(X_i)_{15}(X_j)_{2\lambda}(X_k)_{3\mu}(X_l)_{4\nu}(X_m)_{6\rho}+(X_i)_{16}(X_j)_{2\lambda}(X_k)_{3\mu}(X_l)_{4\nu}(X_m)_{5\rho}$$
$$+(X_i)_{23}(X_j)_{1\lambda}(X_k)_{4\mu}(X_l)_{5\nu}(X_m)_{6\rho}-(X_i)_{24}(X_j)_{1\lambda}(X_k)_{3\mu}(X_l)_{5\nu}(X_m)_{6\rho}$$
$$+(X_i)_{25}(X_j)_{1\lambda}(X_k)_{3\mu}(X_l)_{4\nu}(X_m)_{6\rho}-(X_i)_{26}(X_j)_{1\lambda}(X_k)_{3\mu}(X_l)_{4\nu}(X_m)_{5\rho}$$
$$+(X_i)_{34}(X_j)_{1\lambda}(X_k)_{2\mu}(X_l)_{5\nu}(X_m)_{6\rho}-(X_i)_{35}(X_j)_{1\lambda}(X_k)_{2\mu}(X_l)_{4\nu}(X_m)_{6\rho}$$
$$+(X_i)_{36}(X_j)_{1\lambda}(X_k)_{2\mu}(X_l)_{4\nu}(X_m)_{5\rho}+(X_i)_{45}(X_j)_{1\lambda}(X_k)_{2\mu}(X_l)_{3\nu}(X_m)_{6\rho}$$
$$-(X_i)_{46}(X_j)_{1\lambda}(X_k)_{2\mu}(X_l)_{3\nu}(X_m)_{5\rho}+(X_i)_{56}(X_j)_{1\lambda}(X_k)_{2\mu}(X_l)_{3\nu}(X_m)_{4\rho}\}\alpha^\dagger_\lambda\alpha^\dagger_\mu\alpha^\dagger_\nu\alpha^\dagger_\rho|0\rangle \; . \tag{A11}$$

Finally, the sixth and last commutator leads easily to

$$C^{06}_{111111}(i,j,k,l,m,n)=\sum_{ijklmn}\sum_{\mu\nu\rho\sigma\tau\upsilon}(X_i)_{1\mu}(X_j)_{2\nu}(X_k)_{3\rho}(X_l)_{4\sigma}(X_m)_{5\tau}(X_n)_{6\upsilon}\alpha^\dagger_\mu\alpha^\dagger_\nu\alpha^\dagger_\rho\alpha^\dagger_\sigma\alpha^\dagger_\tau\alpha^\dagger_\upsilon \tag{A12}$$

which contributes to $C^{06}_{111111}(i,j,k,l,m,n)|0\rangle$.

To shorten as much as possible the relations (A6), (A7), (A9), (A10), and (A11), we introduce the following abbreviated notations:

$$C^{04}_{11}(i,j)|0\rangle = \sum_{ij}^{\mathcal{P}}\sum^{4} sg(P_4)(X_i)_{ab}(X_j)_{cd}|0\rangle \; . \tag{A6$'$}$$

where $\sum^4$ is the sum over all permutations P_4 of the four indices 1234 such as $a<b$ and $c<d$. Note that $sg(P_4)=(-1)^{a+b+1}$;

$$C^{04}_{111}(i,j,k)|0\rangle = -\sum_{ijk}^{\mathcal{P}}\sum_{\mu\nu}\sum^{4} sg(P_4)(X_i)_{ab}(X_j)_{c\mu}(X_k)_{d\nu}\alpha^\dagger_\mu\alpha^\dagger_\nu|0\rangle \; , \tag{A7$'$}$$

$$C^{06}_{111}(i,j,k)|0\rangle = -\sum_{ijk}^{\mathcal{P}}\sum^{6} sg(P_6)(X_i)_{ab}(X_j)_{cd}(X_k)_{ef}|0\rangle \; , \tag{A9$'$}$$

where $\sum^6$ is the sum over all permutations P_6 of the six indices 123456 such as $a<b$, $c<d$, and $e<f$. Note that

$$sg(P_6)=(-1)^{a+b+c+d}$$
$$C^{06}_{1111}(i,j,k,l)|0\rangle = \sum_{ijkl}^{\mathcal{P}}\sum_{\mu\nu}\sum^{6} sg(P_6)(X_i)_{ab}(X_j)_{cd}(X_k)_{e\lambda}(X_l)_{f\mu}\alpha^\dagger_\mu\alpha^\dagger_\nu|0\rangle \; . \tag{A10$'$}$$

Finally,

$$C^{06}_{11111}(i,j,k,l,m)|0\rangle = \sum_{ijkl}^{\mathcal{P}}\sum_{\lambda\mu\nu\rho}\sum^{6'} sg(P'_6)(X_i)_{ab}(X_j)_{c\lambda}(X_k)_{d\mu}(X_l)_{e\nu}(X_m)_{f\rho}\alpha^\dagger_\lambda\alpha^\dagger_\mu\alpha^\dagger_\nu\alpha^\dagger_\rho|0\rangle \; , \tag{A11$'$}$$

where $\sum^{6'}$ is the sum over all permutations P'_6 of the six indices 123456 such as $a<b$ and $c<d<e<f$. Note that $sg(P'_6)=(-1)^{a+b}$.

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