Nuclear equation of state with derivative scalar coupling

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(Received 2 January 1990)

A relativistic nuclear mean field model is developed involving nucleons coupled to effective scalar and vector fields. It differs slightly from the usual Walecka model in the form of the coupling of the nucleon to the scalar meson. We calculate the equation of state for symmetric nuclear matter at zero temperature. The model, which has no arbitrary parameters, once we fit the empirical density and energy of nuclear matter, yields a compression modulus of 225 MeV and an effective nucleon mass=0.85.

I. INTRODUCTION

In order to describe the hadronic matter at both normal nuclear density ($n=n_0=0.16\ \text{fm}^{-3}$), and at high density produced in heavy-ion collisions, a complete field theoretical treatment would be very elegant and desirable. However, at least at present, it does not seem possible to construct such a treatment.

The next best approach could be relativistic effective mean-field description.[1] Since the early works of Walecka and Chin,[2] a lot of work has been done to investigate the properties of self-consistent relativistic field models. Such models, involving coupling of baryons to scalar and vector mesons, seem to be successful in describing many properties of nuclear matter. There is, however, a problem: at moderately high density and temperature, the effective mass of the nucleons become very small[3] (or even negative, if Δ particles are also included). This strong change in the effective mass has serious effects in the calculation of the production of new particles in heavy-ion collisions.

Our aim in the present paper is to investigate whether it is possible (a) to keep the general framework of the effective two-parameter relativistic mean-field model, and (b) by use of a less conventional form, for the coupling between the fields to avoid the problem of the too-small effective mass at high densities.

II. RELATIVISTIC FIELD EQUATIONS

Let us first briefly review the conventional model,[4] here referred to as the Walecka model (and in some of the literature as the σ-ω model), and then discuss our suggested modification. We follow the notation of Maruyama and Suzuki.[5] In the Walecka model, the Lagrangian density is given by

$$\mathcal{L}_W=[\bar{\psi}i\gamma_\mu\partial^\mu\psi-\bar{\psi}M_N\psi+\tfrac{1}{2}(\partial_\mu\sigma\partial^\mu\sigma-m_\sigma^2\sigma^2)]+g_\sigma\sigma\bar{\psi}\psi$$
$$-\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}+\tfrac{1}{2}m_\omega^2\omega_\mu\omega^\mu-g_\omega\bar{\psi}\gamma_\mu\psi\omega^\mu\ , \qquad (1)$$

where ψ,σ, and ω denote the fields of the baryon, the Lorentz scalar (σ) meson and vector (ω) meson, and $F_{\mu\nu}=\partial_\mu\omega_\nu-\partial_\nu\omega_\mu$. (We use the following notation of Bjorken and Drell:[6] $g^{00}=1$, $g^{11}=g^{22}=g^{33}=-1$, $\hbar=c=1$.)

For uniform matter, the time component ω^0 of the vector-field component is

$$\omega^0=g_\omega\rho/m_\omega^2 \qquad (2)$$

where ρ is the baryon density.

Our proposed model involves a change in the form of the coupling of the nucleons to the scalar σ field. We shall investigate a coupling of the form

$$\mathcal{L}_{\text{int}}=(g_\sigma\sigma/M_N)\bar{\psi}\gamma_\mu i\partial^\mu\psi\ , \qquad (3)$$

in addition to coupling to the vector field. Accordingly, let us consider the following modified Lagrangian:

$$\mathcal{L}_M=-\bar{\psi}M_N\psi+\left(1+\frac{g_\sigma\sigma}{M_N}\right)[\bar{\psi}i\gamma_\mu\partial^\mu\psi-g_\omega\bar{\psi}\gamma_\mu\psi\omega^\mu]$$
$$-\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}+\tfrac{1}{2}m_\omega^2\omega_\mu\omega^\mu+\tfrac{1}{2}(\partial^\mu\sigma-m_\sigma^2\sigma^2)\ . \qquad (4)$$

This gradient or derivative coupled Lagrangian is Lorentz invariant, since $\gamma_\mu\partial^\mu$ is a Lorentz scalar, but not renormalizable. The Lagrangian also contains a coupling between scalar and vector mesons. We will, however, not use $\mathcal{L}_M$ in this paper, but the Lagrangian obtained from it by rescaling the fermion[7] wave function,

$$\psi\rightarrow(1+g_\sigma\sigma/M_N)^{-1/2}\psi\ . \qquad (5)$$

Thus

$$\mathcal{L}_R=-\left(1+\frac{g_\sigma\sigma}{M_N}\right)^{-1}[\bar{\psi}M_N\psi]+\bar{\psi}i\gamma_\mu\partial^\mu\psi-g_\omega\bar{\psi}\gamma_\mu\psi\omega^\mu$$
$$-\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}+\tfrac{1}{2}m_\omega^2\omega_\mu\omega^\mu+\tfrac{1}{2}(\partial_\mu\sigma\partial\mu_\sigma-m_\sigma^2\sigma^2)\ . \qquad (6)$$

We may, in fact, use the rescaled Lagrangian (6) as a starting point of our mean field model rather than the original Lagrangian (4).

The coupling between scalar meson and nucleon is

$$\mathcal{L}_{\text{int}}=g_\sigma\sigma\bar{\psi}\psi/(1+g_\sigma\sigma/M_N)=m^*g_\sigma\sigma\bar{\psi}\psi\ , \tag{7a}$$

where the effective mass is now defined by

$$m^*=[1+g_\sigma\sigma/M_N]^{-1}\ . \tag{7b}$$

Note that in our Lagrangian, the m^* approaches zero only when σ becomes infinitely large. Up to first order in σ, the rescaled Lagrangian agrees with the conventional one, where

$$m^*=1-g_\sigma\sigma/M_N\ , \tag{8}$$

which vanishes when $\sigma=M_N/g_\sigma$. However, there are now also couplings between nucleon and scalar meson involving higher powers of σ:

$$\mathcal{L}_{\text{int}}=g_\sigma\sigma\bar{\psi}\psi-g_\sigma^2\sigma^2\bar{\psi}\psi/M_N+g_\sigma^3\sigma^3\bar{\psi}\psi/M_N^2+\cdots\ . \tag{9}$$

The field equations for nucleon and vector field have the same form as the conventional ones, and that for the scalar field differs only by a factor m^{*2}:

$$(i\gamma_\mu\partial^\mu-M_Nm^*-g_\omega\gamma_\mu\partial^\mu)\psi=0 \tag{10a}$$

$$\partial_\mu F^{\mu\nu}+m_\omega^2\omega^\mu=g_\omega\bar{\psi}\gamma^\mu\psi \tag{10b}$$

$$(\partial_\mu\partial^\mu+m_\sigma^2)\sigma=g_\sigma m^{*2}\bar{\psi}\psi \tag{10c}$$

The nucleon field equation can be written as

$$(\boldsymbol{\alpha}\cdot\mathbf{p}+\beta M_Nm^*+g_\omega\omega^0)=\psi=E\psi \tag{11a}$$

(neglecting the three space components of the vector field), or in terms of the scalar and vector potentials:

$$(\boldsymbol{\alpha}\cdot\mathbf{p}+\beta M^*)\psi=(E-V)\psi\ , \tag{11b}$$

where

$$M=M_n,\quad M^*=M(1+S/M)^{-1}\ ,\qquad S=g_\sigma\sigma,\quad V=g_\omega\omega^0\ . \tag{12}$$

Neglecting time derivatives, the Hamiltonian density can be written as follows:

$$\mathcal{H}=\psi^\dagger(\boldsymbol{\alpha}\cdot\mathbf{p}+\beta m_Nm^*)\psi+\tfrac{1}{2}(\nabla\sigma)^2+\tfrac{1}{2}m_\sigma^2\sigma^2+\tfrac{1}{2}(\nabla\omega^0)^2-\tfrac{1}{2}m_\omega^2(\omega^0)^2+g_\omega\bar{\psi}\gamma_0\psi\omega^0\ . \tag{13}$$

We can express the Hamiltonian density for uniform matter in terms of the effective mass and density:

$$\mathcal{H}=\psi^\dagger(\boldsymbol{\alpha}\cdot\mathbf{p}+\beta M_Nm^*)\psi+\frac{M_N^2m_\sigma^2}{2g_\sigma^2}\frac{(1-m^*)^2}{m^{*2}}+\frac{g_\omega^2}{2m_\omega^2}\rho^2\ . \tag{14a}$$

By comparison, in the Walecka model, we get the very similar expression:

$$\mathcal{H}=\psi^\dagger(\boldsymbol{\alpha}\cdot\mathbf{p}+\beta M_Nm^*)\psi+\frac{M_N^2m_\sigma^2}{2g_\sigma^2}(1-m^*)^2+\frac{g_\omega^2}{2m_\omega^2}\rho^2\ . \tag{14b}$$

For this case the nucleon field equation is the same as Eq. (11) except that the effective mass is defined slightly differently, namely $M^*=M-S$, which is a more sensitive function of S, vanishing for $S=M$.

The main difference between the two models is that for the gradient coupling, the effective mass is less sensitive to the strength of the scalar potential. Two alternative forms for the gradient coupling are discussed in the Appendix.

III. EFFECTIVE MASS AND ENERGY PER PARTICLE

For plane waves, our equation for the energy per nucleon, $W(\rho)$ can be written as

$$W(\rho)=\langle(M_N^2m^{*2}+p^2)^{1/2}\rangle+\frac{M_N^2m_\sigma^2}{2g_\sigma^2\rho}\frac{(1-m^*)^2}{m^{*2}}+\frac{g_\omega^2}{2m_\omega^2}\rho\ . \tag{15}$$

Here $\langle\ \rangle$ denotes the average over the Fermi sea.

At high densities where the amplitude of σ is large, the terms in M_N and σ become small, and we approach the extreme relativistic expression, except for the vector meson contribution

$$W(\rho)=\langle p\rangle+\frac{g_\omega^2}{2m_\omega^2}\rho\ . \tag{16}$$

We will find it convenient to express the energy in terms of the effective mass rather than σ. It is also convenient to define dimensionless coupling constants

$$B_v=\frac{g_\omega^2\rho_0}{m_\omega^2M_N}\ , \tag{17a}$$

$$B_s=\frac{g_\sigma^2\rho_0}{m_\sigma^2M_N}\ , \tag{17b}$$

where ρ_0 is the normal nuclear matter density, and M_N is the nucleon mass, and the normalized density

$$\hat{\rho}=\rho/\rho_0\ . \tag{17c}$$

Then the energy per particle can be written as

$$W(\hat{\rho})=(m^{*2}+2T_0\hat{\rho}^{2/3}/M_N)^{1/2}+\frac{1}{2B_s\hat{\rho}}\frac{(1-m^*)^2}{m^{*2}}+\tfrac{1}{2}B_v\hat{\rho}-1\ , \tag{18}$$

where T_0 is the average kinetic energy per particle at normal density. All energies are expressed in units of M_Nc^2. Replacing p^2 by $2M_NT_0\hat{\rho}^{2/3}$ is a good approximation up to high densities. Even in the extreme relativistic limit, the ratio of the two terms is 15/16.

For comparison, for the usual scalar coupling, and no self interaction between scalar mesons, i.e., the original Walecka model, we obtain

$$W_{\text{Walecka}}(\hat{\rho})=(m^{*2}+2T_0\hat{\rho}^{2/3}/M_N)^{1/2}+\frac{1}{2B_s\hat{\rho}}(1-m^*)^2+\tfrac{1}{2}B_v\hat{\rho}-1\ . \tag{19}$$

As can be seen these two expressions differ only by powers of m^* in the scalar term.

The field equations for σ are equivalent to the condition that $\partial W/\partial m^*(\hat{\rho})=0$ (for any value of $\hat{\rho}$). This leads to

$$\frac{1}{B_s\hat{\rho}}\frac{1-m^*}{m^{*3}}=\frac{m^*}{e^*} , \quad (20)$$

where

$$e^*=(m^{*2}+2T_0\hat{\rho}^{2/3})^{1/2} . \quad (21)$$

The saturation condition $(\partial W/\partial\hat{\rho})(1)=0$ gives

$$\frac{1}{2B_s}\frac{(1-m^*)^2}{m^{*2}}=\frac{1}{2}B_v+\frac{2T_0/3}{e^*} . \quad (22)$$

If the saturation energy is W_0 (<0), then we have

$$W(1)+\frac{\partial W}{\partial\hat{\rho}}(1)=B_v+e^*+\frac{2T_0/3}{e^*}-1=W_0 , \quad (23)$$

$$W(1)-\frac{\partial W}{\partial\hat{\rho}}(1)=\frac{1}{B_s}\frac{(1-m^*)^2}{m^{*2}}+e^*-\frac{2T_0/3}{e^*}-1=W_0 . \quad (24)$$

We can solve for B_s and B_v in terms of the values of $m^*(1)$.

$$B_s=\frac{(1-m^*)^2}{m^{*2}}\Big/\left[1+W_0-e^*+\frac{2T_0/3}{e^*}\right] , \quad (25a)$$

$$B_v\left[1+W_0-e^*-\frac{2T_0/3}{e^*}\right] . \quad (25b)$$

Substituting these results into the equation $(\partial W/\partial m^*)(\hat{\rho})=0$, we obtain an equation for m^* as a function of $\hat{\rho}$:

$$1+W_0-e^*+\frac{2T_0/3}{e^*}=\hat{\rho}m^*(1-m^*)\frac{m^*}{e^*} . \quad (26)$$

The nuclear compression modulus is defined as

$$K=9\rho_0^2\left[\frac{\partial^2 W}{\partial\rho^2}\right]_{\rho_0}=9\left[\frac{\partial^2 W}{\partial\hat{\rho}^2}\right]_1 . \quad (27)$$

Writing W as function of m^* and $\hat{\rho}$, it is readily shown that:

$$W_0''=\left[\frac{\partial^2 W}{\partial\hat{\rho}^2}\right]_1 =\left[\frac{\rho^2 W}{\partial\hat{\rho}^2}\right]_{m^*=m^*(1)}-\left[\frac{\partial^2 W}{\partial\hat{\rho}\partial m^*}\right]^2\Big/\left[\frac{\partial^2 W}{\partial m^{*2}}\right] . \quad (28)$$

We can obtain analytic expressions if we neglect the kinetic energy. In this case:

$$B_s=\frac{(1-m^*)^2}{m^{*2}}/[1+W_0-m^*] , \quad (29a)$$

$$B_v=\frac{1}{m^*}[1+W_0-m^*] . \quad (29b)$$

The equation for m^* becomes

$$\frac{1-m^*}{m^{*3}}=B_s\hat{\rho} , \quad (30)$$

and the equation for W is

$$W=m^*+\frac{1}{2B_s\hat{\rho}}\frac{(1-m^*)^2}{m^{*2}}+\frac{1}{2}B_v\hat{\rho}-1 . \quad (31)$$

Now let $m^*(1)=1-B$. Then we find, also with the expansion for small B:

$$B_s=\frac{B}{(1-B)^3}\to B(1+3B+6B^2+10B^3+\cdots) , \quad (32a)$$

$$B_v=B(1-B) , \quad (32b)$$

$$W_0=-B^2 , \quad (33a)$$

$$W_0''=2B^2\frac{(1-B)}{(1+2B)}\to 2B^2(1-3B+6B^2+\cdots) . \quad (33b)$$

Note that

$$W_0''/|W_0|=2(1-3B+6B^2+\cdots) . \quad (34)$$

We can also obtain an expansion for m^*:

$$m^*=1-B\hat{\rho}-3B^2\hat{\rho}(1-\hat{\rho})-6B^3\hat{\rho}(1-\hat{\rho})(1-2\hat{\rho}) -B^4\hat{\rho}(1-\hat{\rho})(10-53\hat{\rho}+55\hat{\rho}^2)+\cdots . \quad (35)$$

For comparison, in the Walecka model, we obtain the same result for B_v, while the result for B_s does not have the factor m^{*-2}.

IV. LOW DENSITY EXPANSION

Including the kinetic energy has only a small effect on m^*, giving an extra contribution of $(T_0\hat{\rho}^{5/3}M)$. The effect on the energy is, of course larger, and is included in our calculations.

We give here the low density expansions for the two models discussed here. For the Walecka model, we obtain

$$m^*=1-B_s\hat{\rho} , \quad (36)$$

$$W=\frac{1}{2}(B_v-B_s)\hat{\rho}+T_0\hat{\rho}^{2/3}+T_0B_s\hat{\rho}^{5/3}+\cdots . \quad (37)$$

Note that if the kinetic energy is neglected, then the model does not saturate at all; i.e., there is no stable minimum energy. Thus for this model, relativity is crucial for saturation.

For the derivative model, we have

$$m^*=1-B_s\hat{\rho}+3B_s^2\hat{\rho}^2-12B_s^3\hat{\rho}^3+55B_s^4\hat{\rho}^4+\cdots , \quad (38)$$

$$W_{\text{deriv}}=\frac{1}{2}(B_v-B_s)\hat{\rho}+B_s^2\hat{\rho}^2-3B_s^3\hat{\rho}^3+12B_s^4\hat{\rho}^4 +T_0\hat{\rho}^{2/3}+T_0B_s\hat{\rho}^{5/3}+\cdots . \quad (39)$$

We note that for this case, we can get saturation even in the nonrelativistic limit, i.e., if the kinetic energy is neglected. Expressing the coupling constants in terms of the parameter B, we obtain

$$W(\hat{\rho})=B^2(-2\hat{\rho}+\hat{\rho}^2)-3B^3\hat{\rho}(1-\hat{\rho})^2 -B^4\hat{\rho}(1-\hat{\rho})(5-11\hat{\rho}) +T_0(\hat{\rho}^{2/3}-\tfrac{2}{3}\hat{\rho})+T_0B(\hat{\rho}^{5/3}-\tfrac{5}{3}\hat{\rho}) . \quad (40)$$

Note that

$$W'(1)=0 . \quad (41)$$

This can be obtained in the Hartree approximation with the following density and momentum dependent zero-range interaction:

$$v(p_{12},\hat{\rho})=[B^2(-2+\hat{\rho})-6B^3(1-\hat{\rho})^2-\tfrac{4}{3}T_0 -\tfrac{10}{3}T_0B+2M_n^{-1}p_{12}^2B]\delta(\mathbf{r}) , \quad (42)$$

where p_{12} is the relative momentum. The term of order B^2 contains two- and three-body interactions, just like the Skyrme interaction.[8] However, the B^3 term contains four-body interactions as well. The terms proportional to p_{12}^2 correspond to momentum dependent terms in the Skyrme interaction.

We find for the energy at equilibrium

$$W(1)=-B^2+\tfrac{1}{3}T_0(1-2B)+ \cdots \quad (43)$$

and the compression modulus is

$$K=18B^2(1-3B)-2T_0(1-10B)+ \cdots . \quad (44)$$

The ratio is approximately given by

$$K/|W|=18(1-3B)+4T_0B^{-2}(1+2B)+ \cdots . \quad (45)$$

Thus the nonlinear terms in B tend to lower the ratio $K/|W|$, while the kinetic energy terms in T tend to increase it.

V. EXACT RESULTS FOR THE EQUATION OF STATE

We now give the results including the kinetic energy. For comparison, we also give results for the Walecka model. Also, we list the energy per particle for what we can call a standard model with the simple equation of state:[9]

$$W(\hat{\rho})=W_0(-2\hat{\rho}+\hat{\rho}^2) . \quad (46)$$

This leads to a compression modulus $K=18|W_0|=288$ MeV for $|W_0|=16$ MeV. For this model, the energy vanishes for twice normal density.

We use here the following values of the parameters: $M_n=938$ MeV, $T_0=22$ MeV, $W=-16.0$ MeV.

For the Walecka model: $B_s=0.487$, $B_v=0.368$, $m^*(1)=0.547$, $K=560$ MeV , (47a)

For our derivative model: $B_s=0.252$, $B_v=0.888$, $m^*(1)=0.855$, $K=225$ MeV . (47b)

The value of $K=225$ MeV agrees with Blaizot's value[9] extracted from energy of breathing modes. However, recent work by Sharma *et al.*[10] points to a higher value of K. This point remains to be settled. The value of 0.85 for the effective mass is in good agreement with a recent analysis by Mahaux and Sartor.[10] Both K and m^* agree closely with the results for one of the Skyrme interactions, namely SkM*, which is widely used in the literature.[11]

In Table I, we list the effective mass and energy per nucleon versus density (see also Fig. 1).

We find that for $B=1-m^*(1)=0.15$, $T=22$ MeV, the approximations (43) and (44) give $W=-16.0$ MeV, $K=230$ MeV, in good agreement with the exact values. Expansion (45) gives a ratio $K/|W|=15.3$, to be compared with exact value $220/16=13.8$.

At high density, derivative coupling implies an effective mass proportional to $\rho^{-1/3}$ neglecting kinetic en-

TABLE I. Effective mass and energy per nucleon for nuclear matter.

	$m^*(\hat{\rho})$		$W(\hat{\rho})$ (MeV)		
$\hat{\rho}$	Walecka	Deriv	Walecka	Standard	Deriv
0.1	0.951	0.977	−0.61	−3.04	−2.30
0.2	0.904	0.956	−2.87	−5.76	−5.47
0.5	0.762	0.908	−9.88	−12.00	−12.32
0.8	0.629	0.870	−14.82	−15.36	−15.47
1.0	0.547	0.850	−15.94	−16.00	−15.99
1.2	0.474	0.832	−14.59	−15.36	−15.51
1.5	0.386	0.809	−6.75	−12.00	−13.26
2.0	0.290	0.777	23.43	0	−6.34
3.0	0.203	0.730	133.1	48	15.48
5.0	0.140	0.669	438.8	240	76.74
7.0	0.112	0.630	781.2	560	149.9
10	0.089	0.591	1313	1280	270.2

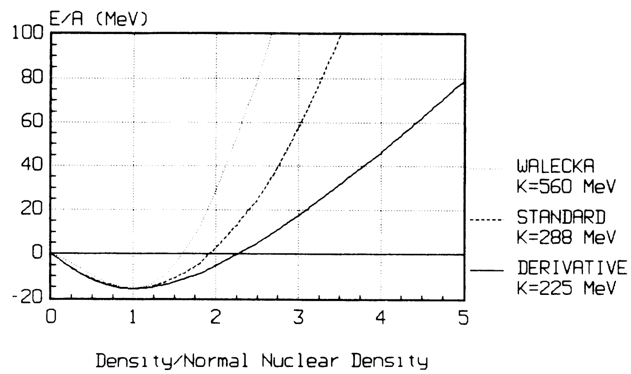

FIG. 1. Energy per particle for symmetric nuclear matter.

ergy, and $\rho^{-1/6}$ in the extreme relativistic limit, according to Eqs. (30) and (20). The dominant contribution to the energy comes from the vector mesons. At some density about 5 to 10 times normal, the energy exceeds that for a quark gluon plasma, so that the model considered here would be expected to lead to some kind of transition between the hadron and quark phase.

VI. CONCLUSIONS

The model developed in this paper, like the original Walecka model, has only two parameters, both of which are fixed by the nuclear matter density and binding energy. However, it yields what appear to be much more reasonable values for two key properties of nuclear matter, namely the compression modulus, and an effective mass. Although this equation is much softer than the Walecka model, it has in common with it the high density behavior dominated by vector meson exchange, implying some kind of phase transition to a quark gluon plasma at high density. An alternative model discussed briefly in the Appendix leads to essentially the same EOS near normal nuclear density, but a much softer EOS at very high densities.

ACKNOWLEDGMENTS

We wish to thank M. Banerjee, M. Barranco, G. Bertsch, M. Birse, A. R. Bodmer, G. E. Brown, D. T. Jaroszewicz, R. Lombard, N. Marcos, P. Ring, and N. Snyderman for helpful discussions and useful suggestions.

APPENDIX: ALTERNATIVE FORMS FOR DERIVATIVE COUPLING

The form of the Lagrangian (4) is somewhat arbitrary. For example, we have added a scalar coupling not only to the nucleon derivative term, but also to the interaction between nucleon and vector meson. Perhaps this might be understandable on basis of Gauge invariance. let us consider an alternative Lagrangian where only the nucleon term is modified by scalar coupling.

$$\mathcal{L}_M = -\bar{\psi}M_N\psi + \left(1+\frac{g_\sigma\sigma}{M_N}\right)\bar{\psi}i\gamma_\mu\partial^\mu\psi - g_\omega\bar{\psi}\gamma_\mu\psi\omega^\mu - \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + \tfrac{1}{2}m_\omega^2\omega_\mu\omega^\mu + \tfrac{1}{2}(\partial_\mu\sigma\partial^\mu\sigma - m_\sigma^2\sigma^2)\ . \tag{A1}$$

If we proceed just as before, we obtain the following for the energy per particle of nuclear matter:

$$W = (m^{*2} + 2T_0\hat{\rho}^{2/3}/M_n)^{1/2} + \frac{1}{2B_s\hat{\rho}}\frac{(1-m^*)^2}{m^{*2}} + \tfrac{1}{2}B_v m^{*2}\hat{\rho} - 1\ . \tag{A2}$$

As we will see, for this model, there would be no binding at all, were it not for the effect of kinetic energy. Let us consider this expression neglecting kinetic energy. We can write

$$W = m^* + \frac{1}{2B_s\hat{\rho}}\frac{(1-m^*)^2}{m^{*\alpha_s}} + \tfrac{1}{2}B_v\hat{\rho}\times m^{*\alpha_v} - 1\ . \tag{A3}$$

For the model considered in the main part of the text, we have $\alpha_s=2$ and $\alpha_v=0$, while for the alternative model $\alpha_s=\alpha_v=2$. For the Walecka model, $\alpha_s=\alpha_v=0$.

Minimizing W with respect to m^* and applying the saturation conditions at $\hat{\rho}=1$, we obtain the following expression for the ground state energy.

$$W_0 = \tfrac{1}{2}(\alpha_s-\alpha_v)B^2/\{1-[1-\tfrac{1}{2}(\alpha_s-\alpha_v)]B\}\ . \tag{A4}$$

Thus, if $\alpha_s=\alpha_v$, and if we neglect kinetic energy, there is no binding at all. For the case discussed in the text, i.e., $\alpha_s=2$, $\alpha_v=0$, we recover our earlier result $W_0=-B^2$.

A better alternative model of gradient coupling is one with $\alpha_s=2$, $\alpha_v=1$. Here the Lagrangian is given by

$$\mathcal{L}_m = -\bar{\psi}M_N\psi + \left(1+\frac{g_\sigma\sigma}{M_N}\right)[\bar{\psi}i\gamma_\mu\partial^\mu\psi - g_\omega\bar{\psi}\gamma_\mu\psi\omega^\mu - \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + \tfrac{1}{2}m_\omega^2\omega_\mu\omega^\mu] + \tfrac{1}{2}(\partial_\mu\sigma\partial^\mu\sigma - m_\sigma^2\sigma^2)\ . \tag{A5}$$

If we rescale the nucleon wave function as before, but also the vector meson

$$\psi\to\sqrt{m^*}\psi,\quad \omega_\mu\to m^*\omega_\mu\ , \tag{A6}$$

then the rescaled Lagrangian reads

$$\mathcal{L}_R = \bar{\psi}i\gamma_\mu\partial^\mu\psi - m^*[\psi M_N\psi + g_\omega\bar{\psi}\gamma_\mu\psi\omega^\mu - \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + \tfrac{1}{2}m_\omega^2\omega_\mu\omega^\mu] + \tfrac{1}{2}(\partial_\mu\sigma\partial^\mu\sigma - m_\sigma^2\sigma^2) \tag{A7}$$

The energy per particle of nuclear matter is given by

$$W = (m^{*2} + 2T_0\hat{\rho}^{2/3}/M_N)^{1/2} + \frac{1}{2B_s\hat{\rho}}\frac{(1-m^*)^2}{m^{*2}} + \tfrac{1}{2}B_v m^*\hat{\rho} - 1\ , \tag{A8}$$

which differs from the one used in the main part of our paper only in that the vector meson contribution is multiplied by m^*.

For this case, the nuclear matter equation of state near normal density is very similar, though slightly softer than for the main case discussed in this paper, $\alpha_s=2, \alpha_v=0$. Thus $K=205$ (225) MeV, and $m^*=0.82$ (0.85) for $\alpha_v=1(2)$. However, at very high density, the EOS is much softer for $\alpha_v=1$. This follows since the contribution of the vector meson to the energy is multiplied by $m^* \approx \hat{\rho}^{-2/3}$ at high $\hat{\rho}$, so it is proportional to $\hat{\rho}^{1/3}$, which is the same density dependence as for a quark-gluon plasma. In this model there might well not be any quark-hadron phase transition. There are no convincing *a priori* arguments for choosing one or the other of the Lagrangians. Our form (4) may be the simplest. On the other hand, the form with $\alpha_s=2$, $\alpha_v=1$, might simulate chiral symmetry restoration at high density.[14] However, *any* gradient coupling implies that $\alpha_s=2$, which, in turn, leads to an EOS with K slightly larger than 200 MeV, regardless of uncertainty in its behavior at very high density.

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