

Effects of isoscalar admixtures on the combined analysis of electron-nucleus and proton-nucleus “scissors mode” excitations in the f - p shell

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In the single j shell approximation, only the isovector amplitudes of electron-nucleus or proton-nucleus scattering contribute to the excitation of 1^+ states from the $J=0^+$ targets of even-even nuclei, provided the excitation is a one-step process. However, when configuration mixing is put in, one also obtains contributions from isoscalar amplitudes, which are here studied for various nuclei in the f - p shell— ^{44}Ti , ^{46}Ti , ^{48}Ti and ^{50}Cr . A comparison of the *cross-conjugate* pairs ^{46}Ti and ^{50}Cr is made. The decomposition of the electron-nucleus scattering into an orbital and spin part is made. The isoscalar effects are small and are unable to explain the discrepancies between the experimental analysis and the shell model calculations with regards to the variation of the orbit to spin ratio for various nuclei. However, there is probably hidden orbital strength in a state in ^{50}Cr for which the spin and orbit excitation amplitudes interfere destructively.

INTRODUCTION

Bohle *et al.*,¹ by a combined analysis of inelastic electron scattering and inelastic spin-flip proton scattering, discovered what are now commonly called “scissors mode” excitations in deformed nuclei such as ^{156}Gd . These are dominantly orbital magnetic dipole excitations which appear rather dramatically as isolated bumps in low-energy inelastic electron spectra (e.g., at 3.1 MeV in ^{156}Gd). Such states had been predicted by Lo Iudice and Palumbo² and pictured as isovector excitations in which the deformed neutron symmetry axis vibrates against the deformed proton symmetry axis. Such a picture, though, when taken literally yields too much orbital strength at too high an energy. Modifications in the interacting boson approximation (IBA) by Iachello³ and Dieperink⁴ lead to predictions that were in remarkable agreement with the subsequent experimental findings.

Zamick⁵ suggested that such excitations have a shell model analog and performed calculations in the even-even titanium isotopes, first in the single j shell approximation, and then with Liu⁶ putting in some configuration mixing. It was noted that in the single j shell approximation the excitation had to be purely isovector. Such states were found by a combined Darmstadt-Orsay (electron-proton) collaboration of Djalali and co-workers.⁷

It was, however, pointed out at the outset⁵ that in the single j shell model, the states could not be purely orbital in nature. If we write $B(M1)=|A_l+A_\sigma|^2$ where A_l is the orbital (or convection) contribution and A_σ the spin contribution, the absolute value of the ratio

$$R_{l\sigma} \equiv A_l / A_\sigma \quad (1)$$

is given by $[2l(g_{l\pi}-g_{lv})/(g_{s\pi}-g_{sv})]$. For the $f_{7/2}$ shell, $R_{l\sigma}=0.637$, i.e., the orbital contribution is smaller than the spin contribution. [It should be made clear though that this result includes the bias introduced by the probe, i.e., the fact that $(g_{s\pi}-g_{sv})/(g_{l\pi}-g_{lv})=9.412$. If we had an unbiased probe for which the above ratio were unity, we would have $R_{l\sigma}=2l=6$. Thus it is still fair to call these modes dominantly orbital.]

The Darmstadt-Orsay Collaboration,⁷ however, reported a ratio $R_{l\sigma}=1.7$. This stimulated some work on configuration mixing and other effects. It was shown by Oda, Hino, and Muto⁸ that in a shell model calculation in which one nucleon is allowed to be excited from the $f_{7/2}$ shell to the rest of the f - p shell, the ratio $R_{l\sigma}$ increases to a value close to unity. Using a rotational approach within the f - p shell, Zamick⁹ found a similar result, i.e., turning on a deformation (in the $\Delta N=0$ f - p shell) increases the ratio $R_{l\sigma}$ somewhat, but not enough. He suggested that perhaps one should use renormalized operators $(g_{l\pi}-g_{lv})=1.2$, $(g_{s\pi}-g_{sv})=0.7(g_{s\pi}-g_{sv})_{\text{free}}$. This will increase $R_{l\sigma}$ by a factor 1.71.

It was suggested by Moya de Guerra *et al.*¹⁰ that a rotational model (with pairing) in which $\Delta N=2$ admixtures were present could further enhance $R_{l\sigma}$. It is interesting that this mechanism will enhance the orbital part of magnetic dipole transitions but not of ground-state magnetic moments for $K \neq \frac{1}{2}$ bands. Calculations by Nojarov *et al.*,¹¹ however, have larger spin than orbit contributions in both ^{46}Ti and ^{48}Ti . Their calculations involve the

use of the quasiparticle random phase approximation (QRPA).

It was pointed out by McCullen, Bayman, and Zamick¹² that in the single j shell there is a close relationship—called the cross-conjugate symmetry—between nuclei that are related by the interchange of the protons with the neutron holes and proton holes with neutrons. Thus ^{46}Ti (two protons, four neutrons) and ^{50}Cr (two neutron holes, four proton holes) are a cross-conjugate pair. They should have identical spectra if the single j shell approximation is good. In the context of what we are dealing with here, the most relevant further property is that the value of $B(M1)$ between two corresponding states should be the same in both nuclei.

The nucleus ^{48}Ti is its own cross-conjugate and so, as noted by McCullen, Bayman, and Zamick,¹² the states can be classified by signature quantum numbers (even and odd). This corresponds to whether under the interchange of protons with neutron holes and proton holes with neutrons the wave function remains unchanged or changes sign.

The experimental situation with regard to cross-conjugate nuclei by Willis *et al.*¹³ is overall puzzling. That the 1^+ states in the two nuclei are not at the same excitation energy (3.6 MeV in ^{50}Cr and 4.3 MeV in ^{46}Ti) is not totally unexpected. Configuration mixing calculations go in that direction although it is hard to obtain the full spread in calculations in which only one or two nucleons are excited from the $f_{7/2}$ to the rest of the f - p shell. What is surprising is the large change in the ratio $R_{I\sigma}$. The “experimental” values for ^{46}Ti , ^{48}Ti , and ^{50}Cr are, respectively,¹³ 1.94, 0.71, and 0.58. The shell model calculations that give an enhancement of $R_{I\sigma}$ in ^{46}Ti tend also to give enhancements of this ratio in ^{48}Ti and ^{50}Cr .

In examining the combined proton-nucleus, electron-nucleus analysis performed by the Darmstadt-Orsay Collaboration,⁷ the only place where there is a question is in their assumption that the excitation is purely isovector. As the experimentalists note, this is true only in the single j shell limit. When one includes configuration mixing there can be isoscalar contributions as well. In this work we wish to investigate whether the isoscalar contributions are significant or not and whether they affect the analysis of $R_{I\sigma}$ at all.

ISOSCALAR AND ISOVECTOR AMPLITUDES FOR ELECTRON-NUCLEUS AND PROTON-NUCLEUS SCATTERING

For electron-nucleus scattering at zero momentum transfer we define the following quantities:

$$B(M1) = |A(M1)|^2 \mu_N^2 \quad (2)$$

and

$$A(M1) = \frac{1}{\sqrt{2J_i + 1}} \left[\frac{3}{4\pi} \right]^{1/2} \times \left\langle J_f \left\| \sum_i [g_l(i)l_i + g_s(i)s_i] \right\| J_i \right\rangle, \quad (3)$$

where we use the convention of Edmonds¹⁴ for reduced

matrix elements. The bare values of the g factors are

$$g_l(p) = 1, \quad g_s(p) = 5.586, \\ g_l(n) = 0, \quad g_s(n) = -3.826.$$

It is convenient to define the following elementary amplitudes corresponding to special values of the g factors in $A(M1)$:

	$g_s(p)$	$g_s(n)$	$g_l(p)$	$g_l(n)$
$A_\sigma(p)$	1	0	0	0
$A_\sigma(n)$	0	1	0	0
$A_I(p)$	0	0	1	0
$A_I(n)$	0	0	0	1

We also define the isoscalar (S) and isovector (V) elementary amplitudes:

$$A_\sigma^S = A_\sigma(p) + A_\sigma(n), \quad A_I^S = A_I(p) + A_I(n); \\ A_\sigma^V = A_\sigma(p) - A_\sigma(n), \quad A_I^V = A_I(p) - A_I(n).$$

We further define

$$g_\sigma^S = \frac{g_s(p) + g_s(n)}{2} = 0.880, \\ g_I^S = \frac{g_l(p) + g_l(n)}{2} = 0.500, \\ g_\sigma^V = \frac{g_s(p) - g_s(n)}{2} = 4.706, \\ g_I^V = \frac{g_l(p) - g_l(n)}{2} = 0.500.$$

Thus we have

$$A(M1) = [g_\sigma(p)A_\sigma(p) + g_\sigma(n)A_\sigma(n)] \\ + [g_I(p)A_I(p) + g_I(n)A_I(n)] \\ = A_\sigma + A_I \quad (4)$$

or

$$A(M1) = (g_\sigma^S A_\sigma^S + g_\sigma^V A_\sigma^V) + (g_I^S A_I^S + g_I^V A_I^V) \\ = A_\sigma + A_I. \quad (5)$$

We next consider proton-nucleus scattering. Several theorists have worked on the interrelationship of electron-nucleus and proton-nucleus scattering, e.g., Petrovitch *et al.*,¹⁵ Love and Franey,¹⁶ and Moss,¹⁷ and Shepard, Rost, and McNeil.¹⁸ We use the Wallace representation of the nucleon-nucleon scattering amplitude:¹⁹

$$M_{oi} = 2ik_{of} [A + B\sigma_o \cdot \sigma_i + iq(C_1\sigma_{oz} + C_2\sigma_{iz}) \\ + q^2 D\sigma_{ox}\sigma_{ix} + E\sigma_{oy}\sigma_{iy}], \quad (6)$$

where $q = |\mathbf{q}|$ with $\mathbf{q} = \mathbf{k}_{oi} - \mathbf{k}_{of}$, and

$$\hat{x} = \frac{\mathbf{q}}{q}, \quad \hat{y} = \frac{\mathbf{k}_{oi} + \mathbf{k}_{of}}{|\mathbf{k}_{oi} + \mathbf{k}_{of}|}, \quad \hat{z} = \hat{x} \times \hat{y}.$$

Note that the parameters A , B , C_1 , C_2 , D , and E are different for the target proton and the target neutron. In

the simplest approximation [plane wave impulse approximation (PWIA)], the nucleon-nucleus scattering amplitude is given by

$$F_{M_f, M_i, m_f, m_i}(\mathbf{q}) \propto \left\langle \psi_{f, M_f} \chi_{m_f} \left| \sum_i e^{i\mathbf{q} \cdot \mathbf{r}_i} M_{oi} \right| \chi_{m_i} \psi_{i, M_{oi}} \right\rangle, \quad (7)$$

where χ_m is the spin wave function of the incident (and scattered) proton.

Following the work of Zheng and Zamick,²⁰ we find for inelastic spin-flip proton-nucleus scattering (where the axis of polarization for the spin is in the direction $\hat{\mathbf{z}}$, perpendicular to the scattering plane), the scattering amplitude is given by

$$F_{M_f, M_i}(\mathbf{q}, \downarrow \uparrow) \propto \left\langle \psi_{f, M_f} \left| \sum_i e^{i\mathbf{q} \cdot \mathbf{r}_i} (\alpha \sigma_{i,+1} + \beta \sigma_{i,0} + \gamma \sigma_{i,-1}) \right| \psi_{i, M_i} \right\rangle \quad (8)$$

with

$$\alpha = -(2B + q^2 D + E), \quad \beta = 0, \quad \gamma = q^2 D - E.$$

The parameters α , β , and γ depend on the target nucleon index i ($i=p, n$). We are interested in the inelastic $M1$ scattering in the forward direction ($\mathbf{q}=0$) for which the expression simplifies to

$$F_{M_f}(\mathbf{q}=0, \downarrow \uparrow) \propto \left\langle 1_f^+ \left| \sum_i [(2B + E)\delta_{M_f, +1} + E\delta_{M_f, -1}] \sigma_i \right| 0_i^+ \right\rangle. \quad (9)$$

We get the cross section by summing over final states M_f of the target nucleus (it is not necessary to average over initial states since $J_i=0$):

$$\frac{d\sigma}{d\Omega} \propto \left| \left\langle 1_f^+ \left| \sum_i (2B + E) \sigma_i \right| 0_i^+ \right\rangle \right|^2 + \left| \left\langle 1_f^+ \left| \sum_i E \sigma_i \right| 0_i^+ \right\rangle \right|^2. \quad (10)$$

We then relate this to the elementary amplitudes by first defining the isoscalar and isovector amplitudes

$$B^S = \frac{B(pp) + B(pn)}{2}, \quad B^V = \frac{B(pp) - B(pn)}{2},$$

$$E^S = \frac{E(pp) + E(pn)}{2}, \quad E^V = \frac{E(pp) - E(pn)}{2},$$

where $B(pp)$ represents the parameter B in the proton-proton scattering amplitude, etc. We finally obtain

$$\frac{d\sigma}{d\Omega} \propto |(2B^S + E^S) A_\sigma^S + (2B^V + E^V) A_\sigma^V|^2 + |E^S A_\sigma^S + E^V A_\sigma^V|^2 \equiv \sigma^T. \quad (11)$$

Thus proton-nucleus scattering in the PWIA is related to two of the four elementary amplitudes that are present in electron-nucleus scattering.

To see the importance of the isoscalar admixture, we will examine the ratio

$$R_{VT} = \frac{\sigma^V}{\sigma^T}, \quad (12)$$

where σ^V is defined as

$$\sigma^V = |(2B^V + E^V) A_\sigma^V|^2 + |E^V A_\sigma^V|^2. \quad (13)$$

Note that σ^T and σ^V defined in Eqs. (11) and (13) are not the absolute differential cross section and they do not even have the dimension of the cross section ($length^2$).

The parameters B and E in the Breit frame¹⁹ can be read off from Table IV in Ref. 21 for the 200 MeV proton beam ($\mathbf{q}=0$):

$$B(pp) = 0.0 + i2.02 \text{ (GeV/c)}^{-2},$$

$$E(pp) = 1.55 + i1.13 \text{ (GeV/c)}^{-2},$$

$$B(pn) = -0.09 - i2.33 \text{ (GeV/c)}^{-2},$$

$$E(pn) = 1.18 - i1.37 \text{ (GeV/c)}^{-2}.$$

Thus

$$B^S = -0.045 - i0.155 \text{ (GeV/c)}^{-2},$$

$$E^S = 1.365 - i0.12 \text{ (GeV/c)}^{-2},$$

$$B^V = 0.045 + i2.175 \text{ (GeV/c)}^{-2},$$

$$E^V = 0.185 + i1.25 \text{ (GeV/c)}^{-2}.$$

Note that at 200 MeV the magnitude of B^V is much larger than that of B^S while the magnitudes of E^S and E^V are comparable. Overall the isovector combination to the spin-flip cross section is larger than the isoscalar one. Note further that whereas B^V has a larger magnitude than E^V , the reverse is true for the isoscalar case: $|E^S| > |B^S|$.

It is of interest to associate the nucleon-nucleon amplitudes with the nucleon-nucleon potential in the Born approximation. Obviously the term A can be associated with a spin-independent interaction $V_0(r)$, and the term B with a spin-dependent interaction $V_1(r)\sigma_1 \cdot \sigma_2$. The C terms can be associated with the two-body spin-orbit interaction $V_{s.o.}(r)(\sigma_1 + \sigma_2) \cdot \mathbf{l}$ where $\mathbf{l} = (\mathbf{r}_1 - \mathbf{r}_2) \times (\mathbf{p}_1 - \mathbf{p}_2)/2$. The $q^2 D$ term can be associated with the tensor interaction

$$V_i(r)[3(\sigma_1 \cdot r)(\sigma_2 \cdot r)/r^2 - \sigma_1 \cdot \sigma_2].$$

However, in the first-order Born approximation the E term cannot be associated with any interaction. Thus the E term must come purely from higher-order Born terms.

RESULTS

The results are presented in Tables I–IV corresponding, respectively, to ^{44}Ti , ^{46}Ti , ^{48}Ti , and ^{50}Cr . We use the OXBASH shell model code²² with the modified renormalized Kuo-Brown interaction.²³ The word “Mix n ” which appears in the tables tells us that up to n nucleons have been excited from the $f_{7/2}$ shell into the rest of the f - p shell (the term “Mix4*” is defined in Table II).

Ideally one should do the calculations up to “Mix N ” where N is the number of nucleons in the f - p shell. However, for the computer we are using this is impossible. Hence, whereas for ^{44}Ti we have indeed allowed all the valence nucleons to be anywhere in the f - p shell (Mix4), the best calculations for ^{46}Ti , ^{48}Ti , and ^{50}Cr are Mix4*, Mix2, and Mix1. Recently, a report has appeared by Caurier, Poves, and Zuker²⁴ where a Mix8 calculation has been performed for ^{48}Ti .

In Table I we consider ^{44}Ti . Since we consider final states with $J=1^+$, $T=1$, discussions about isoscalar admixtures are not relevant here. The amplitudes A_i^S and A_σ^S are of course identically zero for all cases. What is of interest is the fact that we can study the change in the ratio of the orbit to spin amplitudes $R_{l\sigma} = A_l/A_\sigma$ as the configuration space gets larger. For $n=0$ –4 the values of $R_{l\sigma}$ to the $1_1^+(T=1)$ state are, respectively, 0.637, 0.459, 0.632, 0.672, and 0.702. Note that this ratio does not increase monotonically; rather for $n=1$ it decreases relative to $n=0$, but then it increases slowly to a final value 0.702 for $n=4$. This is not so different from the

$n=0$ value. The value of $B(M1)$ for electron-nucleus scattering also changes in a nonmonotonic way. This teaches us that premature truncation can yield qualitatively misleading result.

In Table II we consider ^{46}Ti . We see that whereas for Mix0 the amplitudes A_i^S and A_σ^S vanish, they become nonzero when configuration mixing is turned on. However, for the transition $0_1^+ \rightarrow 1_1^+$ the isoscalar amplitudes are very small compared with the isovector ones. Note that in going from $n=0$ to $n=3$ (or 4*) the orbit to spin amplitude ratio $R_{l\sigma}$ changes from a number less than one [for $n=0$ it is $2l(g_l^V/g_s^V)=0.637$] to a value slightly larger than one (1.196 for $n=4^*$). The value of $B(M1)$ steadily decreases from 2.519 ($n=0$) to 0.968 ($n=3$) and to 0.854 ($n=4^*$).

The proton-nucleus spin-flip cross sections are given up to a common factor. σ^V corresponds to the case where the isoscalar amplitudes are neglected, while σ^T corresponds to the case where both the isoscalar and isovector amplitudes are included. We see that the ratio $R_{VT} = \sigma^V/\sigma^T$ is very close to unity (~ 0.985). Thus it would appear that the approximation of Willis *et al.*¹³ of ignoring the isoscalar contributions is a very good one for this transition.

Not only are the isoscalar amplitudes small but also the nucleon-nucleon isoscalar amplitudes B^S and E^S have absolute magnitudes that are smaller than the corresponding isovector ones B^V and E^V . This further adds to making the isoscalar contribution relatively unimportant for $0_1^+ \rightarrow 1_1^+$ transition.

The situation is more complicated for the $0_1^+ \rightarrow 1_2^+$ transition in ^{46}Ti . The ratio R_{VT} , which of course must be unity for Mix0, drops to 0.570 for Mix1 but then comes back up to 0.979 for Mix2. An examination of the other amplitudes clearly indicates that the 1_2^+ state is of a

TABLE I. The shell model calculations of the $M1$ transitions in ^{44}Ti . The various amplitudes A in unit μ_N , strengths B in unit μ_N^2 , and the ratio R are defined in the text. “Mix0” represents the single j shell calculation; “Mix n ” indicates that up to n nucleons can be excited from the $f_{7/2}$ shell to the rest of f - p shells. E_x in unit MeV is the excitation energy of the final 1^+ states. We use the modified renormalized Kuo-Brown interaction.

Final state Configuration	$0_1^+ \rightarrow 1_1^+$					$0_1^+ \rightarrow 1_2^+$				
	Mix0	Mix1	Mix2	Mix3	Mix4	Mix0	Mix1	Mix2	Mix3	Mix4
E_x	4.301	4.357	5.809	6.017	6.262	7.085	6.358	7.660	7.809	8.045
A_σ^p	-0.120	-0.163	-0.134	-0.130	-0.123	0.044	0.055	0.028	0.021	0.021
A_σ^n	0.120	0.163	0.134	0.130	0.123	-0.044	-0.055	-0.028	-0.021	-0.021
A_l^p	-0.717	-0.703	-0.797	-0.824	-0.815	0.263	0.260	0.362	0.381	0.408
A_l^n	0.717	0.703	0.797	0.824	0.815	-0.263	-0.260	-0.362	-0.381	-0.408
A_σ^S	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
A_l^V	-0.239	-0.325	-0.268	-0.260	-0.247	0.088	0.110	0.057	0.042	0.042
A_l^S	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
A_l^V	-1.434	-1.405	-1.595	-1.647	-1.630	0.527	0.521	0.724	0.761	0.815
A_l	-0.717	-0.703	-0.797	-0.824	-0.815	0.263	0.260	0.362	0.381	0.408
A_σ	-1.125	-1.530	-1.261	-1.226	-1.161	0.413	0.515	0.267	0.195	0.196
$R_{l\sigma}$	0.637	0.459	0.632	0.672	0.702	0.637	0.505	1.355	1.949	2.082
$B(M1)$	3.393	4.983	4.235	4.200	3.902	0.458	0.602	0.396	0.332	0.364

TABLE II. The shell model calculations of the $M1$ transitions in ^{46}Ti . The various amplitudes A in unit μ_N , strengths B in unit μ_N^2 , and the ratio R are defined in the text. "Mix0" represents the single j shell calculation; "Mix n " indicates that up to n nucleons can be excited from the $f_{7/2}$ shell to the rest of f - p shells. E_x in unit MeV is the excitation energy of the final 1^+ states. We use the modified renormalized Kuo-Brown interaction. The quantities σ^V, σ^T (arbitrary dimension) and $R_{VT} = \sigma^V/\sigma^T$ are for proton-nucleus scattering.

Final state Configuration	$0_1^+ \rightarrow 1_1^+$					$0_1^+ \rightarrow 1_2^+$		
	Mix0	Mix1	Mix2	Mix3	Mix4* ^a	Mix0	Mix1	Mix2
E_x	3.044	3.147	4.015	3.826	4.205	5.005	4.899	6.004
A_σ^p	0.103	0.088	0.059	0.045	0.041	0.013	0.011	-0.065
A_σ^n	-0.103	-0.104	-0.075	-0.057	-0.050	-0.013	0.030	0.090
A_l^p	0.618	0.630	0.557	0.517	0.503	0.075	-0.076	-0.319
A_l^n	-0.618	-0.614	-0.541	-0.505	-0.494	-0.075	0.035	0.294
A_σ^S	0.000	-0.016	-0.015	-0.012	-0.010	0.000	0.040	0.025
A_σ^V	0.206	0.193	0.134	0.102	0.091	0.025	-0.019	-0.155
A_l^S	0.000	0.016	0.016	0.013	0.010	0.000	-0.040	-0.025
A_l^V	1.236	1.243	1.098	1.022	0.997	0.150	-0.111	-0.613
A_l	0.618	0.630	0.557	0.517	0.503	0.075	-0.076	-0.319
A_σ	0.969	0.893	0.617	0.467	0.421	0.118	-0.054	-0.709
$R_{l\sigma}$	0.637	0.705	0.901	1.109	1.196	0.637	1.405	0.449
$B(M1)$	2.519	2.319	1.378	0.968	0.854	0.037	0.017	1.056
σ^V	1.402	1.228	0.594	0.340	0.275	0.021	0.012	0.797
σ^T	1.402	1.241	0.603	0.346	0.279	0.021	0.021	0.814
R_{VT}	1.000	0.990	0.985	0.984	0.986	1.000	0.570	0.979

^aFor $0^+ (1^+)$ state, up to 4 (3) valence nucleons are allowed to be excited.

TABLE III. The shell model calculations of the $M1$ transitions in ^{48}Ti . The various amplitudes A in unit μ_N , strengths B in unit μ_N^2 , and the ratio R are defined in the text. "Mix0" represents the single j shell calculation; "Mix n " indicates that up to n nucleons can be excited from the $f_{7/2}$ shell to the rest of f - p shells. E_x in unit MeV is the excitation energy of the final 1^+ states. We use the modified renormalized Kuo-Brown interaction. The quantities σ^V, σ^T (arbitrary dimension) and $R_{VT} = \sigma^V/\sigma^T$ are for proton-nucleus scattering.

Final state Configuration	$0_1^+ \rightarrow 1_1^+$			$0_1^+ \rightarrow 1_2^+$		
	Mix0	Mix1	Mix2	Mix0	Mix1	Mix2
E_x	2.821	2.857	3.692	6.353	5.628	6.492
A_σ^p	0.074	0.060	0.040	-0.017	-0.094	-0.073
A_σ^n	-0.074	-0.059	-0.038	0.017	-0.040	-0.018
A_l^p	0.442	0.429	0.355	-0.101	-0.224	-0.238
A_l^n	-0.442	-0.430	-0.357	0.101	0.358	0.329
A_σ^S	0.000	0.001	0.002	0.000	-0.134	-0.090
A_σ^V	0.147	0.119	0.078	-0.034	-0.054	-0.055
A_l^S	0.000	-0.001	-0.002	0.000	0.134	0.091
A_l^V	0.884	0.860	0.712	-0.202	-0.582	-0.567
A_l	0.442	0.429	0.355	-0.101	-0.224	-0.238
A_σ	0.694	0.562	0.369	-0.158	-0.371	-0.340
$R_{l\sigma}$	0.637	0.763	0.963	0.637	0.604	0.701
$B(M1)$	1.289	0.983	0.524	0.067	0.354	0.334
σ^V	0.718	0.470	0.200	0.037	0.095	0.101
σ^T	0.718	0.469	0.200	0.037	0.133	0.112
R_{VT}	1.000	1.001	1.003	1.000	0.715	0.905

TABLE IV. The shell model calculations of the $M1$ transitions in ^{50}Cr . The various amplitudes A in unit μ_N , strengths B in unit μ_N^2 , and the ratio R are defined in the text. "Mix0" represents the single j shell calculation; "Mix n " indicates that up to n nucleons can be excited from the $f_{7/2}$ shell to the rest of f - p shells. E_x in unit MeV is the excitation energy of the final 1^+ states. We use the modified renormalized Kuo-Brown interaction. The quantities σ^V , σ^T (arbitrary dimension) and $R_{VT} = \sigma^V/\sigma^T$ are for proton-nucleus scattering.

Final state Configuration	$0_1^+ \rightarrow 1_1^+$		$0_1^+ \rightarrow 1_2^+$		$0_1^+ \rightarrow 1_3^+$	
	Mix0	Mix1	Mix0	Mix1	Mix0	Mix1
E_x	3.043	2.989	5.004	5.213	7.434	5.410
A_σ^p	0.103	0.082	-0.012	-0.022	-0.007	-0.002
A_σ^n	-0.103	-0.072	0.012	0.011	0.007	-0.041
A_σ^p	0.618	0.655	-0.075	-0.079	-0.040	0.138
A_σ^n	-0.618	-0.665	0.075	0.089	0.040	-0.096
A_σ^S	0.000	0.010	0.000	-0.011	0.000	-0.043
A_σ^V	0.206	0.153	-0.025	-0.033	-0.013	0.039
A_σ^S	0.000	-0.010	0.000	0.011	0.000	0.043
A_σ^V	1.236	1.320	-0.150	-0.168	-0.079	0.234
A_σ	0.618	0.655	-0.075	-0.079	-0.040	0.138
A_σ	0.969	0.731	-0.117	-0.164	-0.062	0.145
$R_{I\sigma}$	0.637	0.896	0.637	0.479	0.637	0.953
$B(M1)$	2.519	1.921	0.037	0.059	0.010	0.081
σ^V	1.401	0.777	0.021	0.036	0.006	0.050
σ^T	1.401	0.772	0.021	0.035	0.006	0.063
R_{VT}	1.000	1.007	1.000	1.027	1.000	0.790

completely different nature in Mix1 and Mix2. For example, the amplitudes A_σ^p and A_σ^n are in phase for Mix1 and are out of phase for Mix2. The value of $B(M1)$ changes from 0.017 to 1.056. What is probably happening is that there are two closely lying levels 1_2^+ and 1_3^+ . Which one comes of the lowest depends very much on

the size of configuration space that is used.

The story for ^{48}Ti is much the same as for ^{46}Ti . The ratio $R_{I\sigma}$ changes from 0.637 for $n=0$ to 0.963 for $n=2$ (see Table III). The result for $n=8$ by *Caurier et al.* is 1.12 with the interaction "KB3".²⁴ Evidently the orbit to spin ratio increases slowly but persistently as the configuration

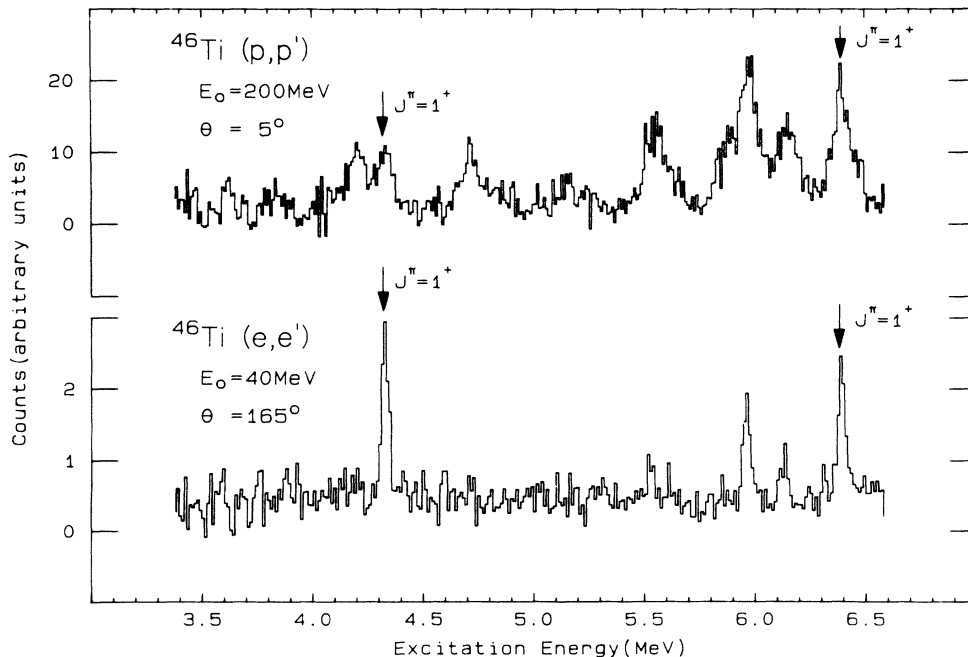


FIG. 1. A comparison of proton inelastic scattering and electron inelastic scattering from the $J=0^+$ ground state of ^{46}Ti to $J=1^+$ excited states.

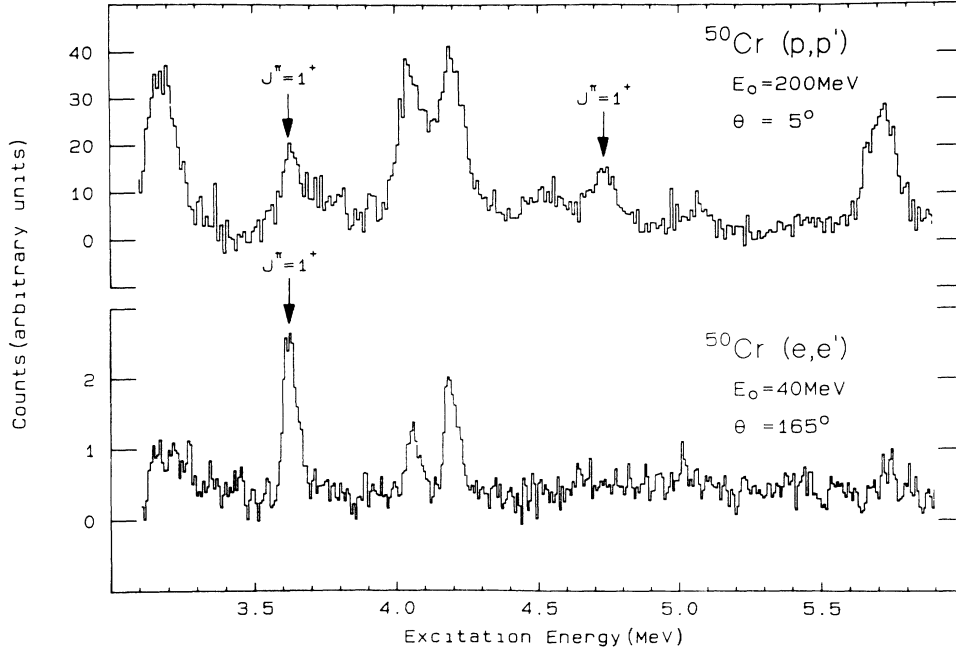


FIG. 2. The same as Fig. 1 but for the cross-conjugate nucleus ^{50}Cr .

space is enlarged.

Unfortunately the results disagree with the trend of the experimental analysis of Willis *et al.*¹³ As we mentioned before, they get a large decrease in $R_{I\sigma}$ from 1.94 in ^{46}Ti to 0.71 in ^{48}Ti and to 0.58 in ^{50}Cr .

In Table IV we give the results for ^{50}Cr . These should be compared with those of ^{46}Ti , the cross-conjugate nucleus. Unfortunately we can do this only for the Mix1 case. Nevertheless the relative trends are of interest.

As expected the Mix0 results are identical for ^{46}Ti and ^{50}Cr . When configuration mixing is included the energy of the ^{48}Ti 1^+ state goes up by 0.103 MeV and that of ^{50}Cr goes down by 0.055 MeV. This is certainly in the right direction but the effect is not large enough. We need a splitting of 0.8 MeV in the relative excitation energies, but at the Mix1 level we only get 0.158 MeV. But we note that in going from Mix1 to Mix3 in ^{46}Ti , the excitation energy changes by a large amount $3.826 - 3.147 = 0.679$ MeV.

The isoscalar contributions in ^{50}Cr , at the Mix1 level, are nonzero but too small to be of any significance. The ratio R_{VT} for proton-nucleus scattering is very close to unity.

It is nevertheless interesting to note that the relative phases of the isoscalar and isovector amplitudes are different in ^{46}Ti and in ^{50}Cr . The amplitudes A_σ^S and A_σ^V are of opposite sign in ^{46}Ti , but are of the same sign for ^{50}Cr . The amplitudes A_I^S and A_I^V are of the same sign in ^{46}Ti but of the opposite sign for ^{50}Cr . Thus if there somehow were a large isoscalar contribution there would be large differences in the behaviors of the $M1$ transitions in the cross-conjugate pair.

We now show results that were presented by Richter at the Argonne conference on the 40th anniversary of the

shell model. In Fig. 1 we have $^{46}\text{Ti}(p,p')$ and $^{46}\text{Ti}(e,e')$ to $J = 1^+$ excited states. In Fig. 2 we have a corresponding diagram for ^{50}Cr . Richter²⁵ noted that the second 1^+ state in ^{50}Cr at $E_x = 4.70$ MeV is excited in (p,p') but not in (e,e') . He therefore suggested that for electron scattering the spin and orbital amplitudes are nearly equal but opposite in sign. From the proton data one obtains $B_\sigma = 0.31 \mu_N^2$ for the 4.70 MeV state. By assuming that B_I is also equal to $0.31 \mu_N^2$, Richter noted that the summed orbital strength for the two states in ^{50}Cr at $E_x = 3.63$ and 4.70 MeV is $0.44 \mu_N^2$, which is essentially equal to $0.43 \mu_N^2$ for the transition to the 1^+ state in ^{46}Ti at 4.32 MeV.

We feel that the above analysis is correct. But the question is, do we obtain a state in the shell model calculation of ^{50}Cr for which the spin and orbit terms nearly cancel. If we examine Table IV we see that for the 1_1^+ , 1_2^+ , and 1_3^+ states of ^{50}Cr , A_I and A_σ are in phase. It should be remarked, however, that the calculation for ^{50}Cr is inadequate since it involves only one particle being excited from the $f_{7/2}$ shell.

It should be further noted that the shell model calculations often do yield such "hidden states" for which the spin and orbit terms cancel. Liu and Zamick²⁶ commented on such a case in ^{20}Ne : "For example, in ^{20}Ne there is a calculated state at 17.2 MeV which contains 22% of the total (orbital) strength (to all the states in ^{20}Ne) but for which $B(M1) = 0.003 \mu_N^2$ is truly negligible."

Large space shell model calculations for ^{50}Cr will have to be done to settle the issue of a "hidden state" in electron scattering. One should also keep open a possibility that highly deformed states in which nucleons are excited from the s - d shell into the f - p and possibly the g - d - s shell are playing an important role.

CLOSING REMARKS

In this work we have set up the formalism to handle combined isovector and isoscalar excitations of unnatural parity states especially 1^+ states and “scissors mode” excitations. For both electron-nucleus scattering and 200 MeV proton-nucleus spin-flip scattering, we find that the isoscalar contributions to the *first* excited 1^+ states in ^{46}Ti , ^{48}Ti , and ^{50}Cr are very small. There are two reasons for this. First, the basic nuclear structure amplitudes A_σ^S and A_l^S are much smaller than their isovector counterparts. Second, the probe parameters (g factors for electron-nucleus scattering and the nucleon-nucleon scattering amplitudes B and E for proton-nucleus scattering) are also larger overall in the isovector mode.

There is, however, some indication in our work that there are higher excited states for which the nuclear structure amplitudes are dominantly isoscalar. For such states the combined analysis presented here will be essential.

The trend of the shell model calculations^{6,8,24} is that as configuration space is increased, the orbit to spin ratio increases as well. Thus the shell model results are in qualitative disagreement with the QRPA results.¹¹

The suggestion that some of the missing orbital strength in ^{50}Cr is contained in a state at $E_x = 4.70$ MeV for which the orbit and spin amplitudes interfere destructively is probably correct and greatly clarifies matters.

We have, however, not yet obtained such a state in a microscopic calculation. In the future we could look at the effects of many-particle, many-hole deformed states admixtures (the particles are in the f - p shell and the holes in the s - d shell). Thus far it has been found that $K=0$ highly deformed states are very important at about the same energies that the $K=1$ scissors modes are found. For example, the first excited 0^+ states in ^{44}Ti and ^{46}Ti are dominantly $K=0$ highly deformed states and the corresponding state in ^{48}Ti is a strong admixture of a deformed and $(fp)^8$ state. To see if such deformed admixtures are important for 1^+ states, we must examine correspondingly highly deformed $K=1$ bands.

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