Particle-hole symmetry and meson exchange corrections to the ^{6}He beta decay amplitude

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The meson exchange current contributions (MEC) to the $^6\mathrm{He}_{\mathrm{g.s.}} \rightarrow {}^6\mathrm{Li}_{\mathrm{g.s.}}$ beta decay are computed for various nuclear amplitudes. We use particle-hole symmetry properties of one- and two-body matrix elements and find relationships between the $A=6$ beta decay and the anomalously suppressed $^{14}\mathrm{C}_{\mathrm{g.s.}} \rightarrow {}^{14}\mathrm{N}_{\mathrm{g.s.}}$ beta transition, for which these have been studied previously. We find MEC contributions to be small for the mass-six case, bringing the calculated axial vector form factor closer to the experimental value. Implications for the positive pion photoproduction transition between the same states are discussed.

I. INTRODUCTION

In the Wigner supermultiplet scheme,[1] involving the group SU(4), the β-decay transitions in the ground-state multiplet of the $A=4n+2$ nuclei are superallowed. Thus, in the nuclear system with $A=6$ and 14, the Gamow-Teller (GT) β decays should be superallowed in the ground-state supermultiplet involving the permutation symmetry,[2] since the GT operator $\sigma\tau$ is a generator of the SU(4) group. In practice, while the ft values of the $\beta^{\pm}$ decays in the $A=6$ system are indeed characteristic of the superallowed β decay, the $\beta^{\pm}$ decays in the $A=14$ system are highly suppressed, due to nuclear configuration mixing effects[2] that violate the supermultiplet symmetry. However, the supermultiplet selection rules still persist,[3] albeit in a modified manner, and often have very interesting consequences in processes studied at the lepton and meson factories, such as nuclear muon capture,[4] radiative pion capture,[5] pion photoproduction,[6] and so on.

Common features of many electroweak processes in these systems have been turned around and exploited in another manner in recent years. Following the lead of the "elementary particle approach" by Kim and Primakoff,[7] Donnelly and Walecka[8] have exploited the β-decay and electron scattering processes to extract "effective" nuclear amplitudes. These then have been used to study other processes at intermediate energy, in interpretation of which the nuclear structure uncertainties have been thought to be reduced. Such phenomenological amplitudes have been particularly useful in the recent studies[9] involving the $A=14$ nuclei.

Meanwhile, Goulard, Lorazo, Primakoff, and Vergados[10] (GLPV) have examined β decays in the $A=14$ system, and found that the meson exchange corrections (MEC's) to the β^--decay amplitude are very large compared to the matrix elements of the GT operator, which is strongly suppressed due to nuclear structure effects. Thus, one must be careful not to incorporate β decay as a constraint in this case to determine the nuclear amplitudes, as the large MEC's need not be common to all electroweak processes.

The purpose of this paper is to exploit the particle-hole symmetry between the $A=6$ and 14 nuclear structures, and expand the work of GLPV to the mass-six system, to examine the MEC's in this case. This would give us a rigorous reason as to whether the β-decay processes in the $A=6$ system could provide a useful constraint in determining nuclear amplitudes phenomenologically. The MEC investigation of GLPV is based on the standard techniques of low-energy theorems.[11] We first rework the calculation of GLPV for the $(1p)^{-2}$ configuration, simulating the structural characteristic of the $A=6$ system. In this, we exploit the particle-hole symmetry[12] between the $(1p)^{-2}$ and $(1p)^2$ configurations, to draw conclusions on the MEC's for the $A=6$ system. We end with some remarks on implications of this for the processes studied with the medium-energy facilities.

II. ONE-BODY BETA-DECAY AMPLITUDE

The ground-state quantum numbers of ^{6}Li and ^{14}N are $J^{\pi}T=1^+0$, where J, π, and T are the angular momentum, parity, and isospin, respectively; the ground-state quantum numbers of ^{6}He and ^{14}C are $J^{\pi}T=0^+1$. The corresponding normalized wave functions in the LS representation are

TABLE I. $A=14$ phenomenological amplitudes.

	α	β	γ	x	y	$b_{1/2}$	$b_{3/2}$
RPI-NI (Ref. 9)	0.336	0.926	0.171	0.790	0.613	1.62	1.51
H1 (Ref. 15)	−0.362	0.348	0.864	−0.361	0.933	1.70	1.70
CK (Ref. 14)	−0.077	0.301	0.951	0.858	0.514		
ENS (Ref. 16)	0.403	−0.068	−0.913	−0.093	0.995	1.68	1.68

$$|1^+0\rangle=\alpha|^3S_1\rangle+\beta|^1P_1\rangle+\gamma|^3D_1\rangle\ ,\qquad |0^+1\rangle=x|^1S_0\rangle+y|^3P_0\rangle\ . \tag{1}$$

The normalized wave functions in the jj representation are

$$|1^+0\rangle=\sum_i c_i|\phi_i^\sigma\rangle=a|p_{1/2}^\sigma p_{1/2}^\sigma\rangle+b|p_{3/2}^\sigma p_{1/2}^\sigma\rangle+c|p_{3/2}^\sigma p_{3/2}^\sigma\rangle\ ,\qquad |0^+1\rangle=\sum_f c_f|\phi_f^\sigma\rangle=m|p_{1/2}^\sigma p_{1/2}^\sigma\rangle+n|p_{3/2}^\sigma p_{3/2}^\sigma\rangle\ , \tag{2}$$

where $\sigma=+1$ for $A=6$ and -1 for $A=14$, representing particle and hole, respectively. Shortly we shall return to the particle↔hole symmetry, in the context of matrix elements of one- and two-body operators.

Let us start with the discussion of GLPV on the impulse approximation (IA) estimate of the amplitude for the β^- decay

$$^{14}\mathrm{C}_{\mathrm{g.s.}}\rightarrow{}^{14}\mathrm{N}_{\mathrm{g.s.}}+e^-+\bar{\nu}_e\ .$$

This is given by the Gamow-Teller matrix element in the limit of zero momentum transfer:[10]

$$F_A^-(0)^{\mathrm{IA}}=g_A\left|\alpha x-\frac{1}{\sqrt{3}}\beta y\right|\ , \tag{3}$$

where the axial-vector coupling constant[13] $g_A=1.254$. In the $A=6$ system, the second term in Eq. (3) is a small correction to the first term; however, the mass-14 structure could make this matrix element vanish. We give values of $F_A^-(0)$ for the $A=14$ system in Table II for the Cohen-Kurath[14] (CK) wave function, and several phenomenologically determined effective wave function amplitudes: Huffman *et al.*[15] (H1), Ensslin *et al.*[16] (ENS), and Doyle[9] (RP1-N1) (Table I). The magnitude of $F_A^-(0)$ deduced form the experimental ft value[17] is 0.0016. $b_{1/2}$ and $b_{3/2}$ are harmonic oscillator parameters for the $p_{1/2}$ and $p_{3/2}$ radial wave functions, respectively.[9] They play little role here, due to the small momentum transfer involved in the decay. (In Table I, we do not assign values to the oscillator parameters for CK, as this approach does not explicitly specify their values.)

III. MESON EXCHANGE CURRENTS

The meson exchange corrections (MEC's) to the nuclear β decay are obtained by a procedure elaborated by Chemtob and Rho,[18] based on the techniques rooted in the low-energy theorems, arising from the hypothesis of partial conservation of the axial vector current. The Feynman graphs relevant to the calculation were discussed in GLPV. The dominant tree-level amplitudes are given by the intermediate Δ (1232) excitation decaying to a nucleon and a pion, and the intermediate ρ-π propagation, shown in Figs. 1(a) and 1(b), respectively. The $N-\bar{N}$ intermediate state [Fig. 1(c)] and ρ exchange [Fig. 1(d)] do not give important effects, the latter being due to the short range of the ρ meson. We also ignore other ρ-exchange effects such as the intermediate isobar decaying into a nucleon and a ρ meson.[11] Such a contribution would decrease the effect of diagram 1(a) slightly.

Repeating the calculation of GLPV, we summarize the MEC's for the β^- decay of $^{14}\mathrm{C}_{\mathrm{g.s.}}$ in Table II, merely updating it for the most recently obtained phenomenological amplitudes. The primary conclusion on the MEC's, arrived at in GLPV, remains unchanged: the MEC for the $A=14$ β^- decay dominates over the impulse approximation estimate for the Gamow-Teller matrix element. The MEC from the Δ contribution largely sets the scale of the correction. While insensitive to the variation of

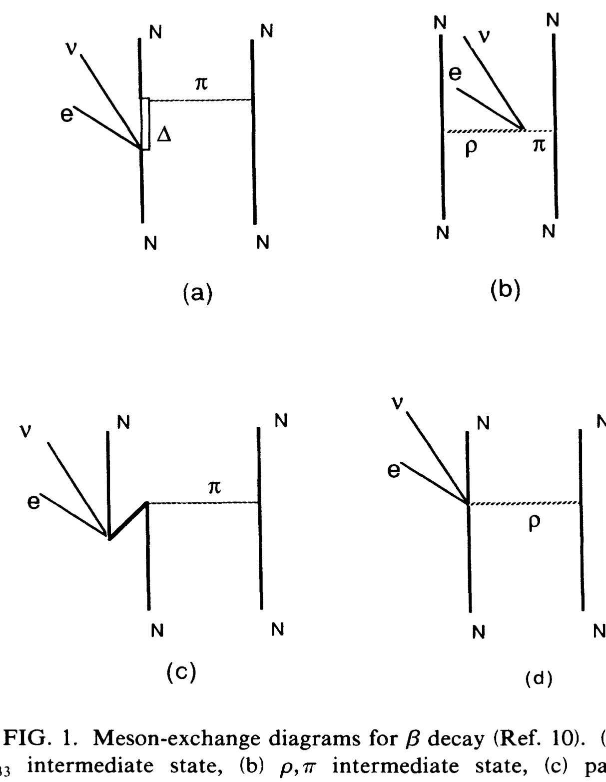

FIG. 1. Meson-exchange diagrams for β decay (Ref. 10). (a) Δ_{33} intermediate state, (b) ρ,π intermediate state, (c) pair current, and (d) ρ exchange.

TABLE II. Meson-exchange contributions to $F_A^-(0)$ for $A=14$.

	$F_A^-(0)^{\rm IA}$	$F_A^-(0)^{\rm MEC}$	$F_A^-(0)$
RPI-NI (Ref. 9)	−0.078	−0.036	−0.114
H1 (Ref. 15)	−0.071	−0.034	−0.105
CK (Ref. 14)	−0.195	−0.079	−0.274
ENS (Ref. 16)	0.002	−0.039	−0.037

the nuclear amplitudes at the order of magnitude level, the MEC can easily vary by a factor of 2, for the chosen wave function amplitudes.

Note that none of the phenomenological amplitudes reproduce the experimental value for $F_A^-(0)$. This requires a very sensitive cancellation between the nuclear one- and two-body contributions. The interference is constructive for all the amplitudes except for that of Ensslin *et al.*,[16] which is already constrained to reproduce the observed ft value at the one-body level. The agreement of theory[9] for positive pion photoproduction on $^{14}N_{\rm g.s.}$ to $^{14}C_{\rm g.s.}$ with experiment at low photon energies using the phenomenological amplitude of Huffman *et al.*,[15] suggests that the amplitude H1 is a reliable determination of the one-body transition density, since the pion photoproduction reaction is expected to be insensitive to MEC effects at these energies. This casts some doubt on the reliability of estimating $F_A^-(0)$ using the one-body and MEC amplitudes alone. Given the small magnitude of the β-decay matrix elements, one may have to consider other contributions such as relativistic effects and excitations outside of the 1p shell, before quantitative conclusions can be drawn.

IV. PARTICLE-HOLE CONJUGATION OF ONE- AND TWO-BODY AMPLITUDES

We now come to the crux of our paper: the MEC in the β^- decay of $^6He_{\rm g.s.}$. The crucial question is, how do we relate the $F_A^-(0)$ and the MEC, obtained for the $(1p)^{-2}$ configuration in the $A=14$ system, with those for the $(1p)^2$ configuration, relevant for the $A=6$ system? We consider the symmetry between particles and holes, a problem similar[19] to that in atomic physics. The fundamental notion is that the creation of a hole state $|\alpha^{-1}\rangle$ is equivalent to the annihilation of the time-reversed particle state $|\tilde{\alpha}\rangle$. Thus the single-hole creation operator $b_\alpha^\dagger$ and the single-particle annihilation operator $a_{-\alpha}$ satisfy the phase relation, defined by the time-reversal operation,

$$b_\alpha^\dagger = a_{\tilde{\alpha}} = (-1)^{j_\alpha + m_\alpha + (1/2) + \kappa_\alpha} a_{-\alpha} = S_\alpha a_{-\alpha} \; . \tag{4}$$

Here α labels the set $\{j_\alpha m_\alpha (1/2)\kappa_\alpha\}$, and $\frac{1}{2}$ is the nucleon isospin and κ is the isospin projection. The negative subscript denotes spin and isospin projections of opposite sign. Thus, the $b_\alpha^\dagger$'s form[12] the components of a spherical tensor operator of rank j.

A. Particle-hole symmetry of one-body amplitude

A one-body operator f can change only one particle or hole state per interaction. The number representation F of the one-body operator f for transitions between particle states is given by

$$F = \sum_{\beta\alpha} \langle \beta | f | \alpha \rangle a_\beta^\dagger a_\alpha \; . \tag{5}$$

In the case where the transition is between hole states, the number representation $\hat{F}$ of f is given by

$$\hat{F} = \sum_{\beta\alpha} \langle \beta^{-1} | f | \alpha^{-1} \rangle b_\beta^\dagger b_\alpha \; . \tag{6}$$

We now relate the single hole matrix element in Eq. (6) to the corresponding single particle matrix element:[12]

$$\begin{aligned} \langle \beta^{-1} | f | \alpha^{-1} \rangle &= \langle \hat{0} | b_\beta f b_\alpha^\dagger | \hat{0} \rangle \\ &= \langle \hat{0} | \tilde{a}_\beta^\dagger f \tilde{a}_\alpha | \hat{0} \rangle \\ &= -\langle 0 | \tilde{a}_\alpha f \tilde{a}_\beta^\dagger | 0 \rangle + \langle \hat{0} | f | \hat{0} \rangle \delta_{\beta\alpha} \\ &= -\langle \tilde{\alpha} | f | \tilde{\beta} \rangle + \langle \hat{0} | f | \hat{0} \rangle \delta_{\beta\alpha} \; , \end{aligned} \tag{7}$$

where $|\hat{0}\rangle$ is the hole vacuum and the anticommutation relation $\tilde{a}_\beta^\dagger \tilde{a}_\alpha = -\tilde{a}_\alpha \tilde{a}_\beta^\dagger + \delta_{\beta\alpha}$ has been used. The vacuum expectation value (or contraction) in the last term on the right-hand side of Eq. (7) vanishes if f is an operator of nonzero rank. Introducing the time-reversal operator $\mathcal{T} = e^{i\pi(\sigma_2+\tau_2)/2} K$, where K is the complex conjugation operation and $\mathcal{T}|\alpha\rangle = |\tilde{\alpha}\rangle$, we obtain

$$\begin{aligned} -\langle \tilde{\alpha} | f | \tilde{\beta} \rangle &= -\langle \alpha | \mathcal{T}^{-1} f \mathcal{T} | \beta \rangle^* \\ &= -\langle \beta | (\mathcal{T}^{-1} f \mathcal{T})^\dagger | \alpha \rangle = \langle \beta | f_c | \alpha \rangle \; , \end{aligned}$$

where (8)

$$f_c \equiv -(\mathcal{T}^{-1} f \mathcal{T})^\dagger \; .$$

If $f = \sigma\tau$, then $f_c = -f$, since σ and τ have the same time-reversal transformation properties. Thus we obtain

$$\langle \beta^{-1} | \sigma\tau | \alpha^{-1} \rangle = -\langle \beta | \sigma\tau | \alpha \rangle$$

and (9)

$$\hat{F} = -\sum_{\beta\alpha} \langle \beta | \sigma\tau | \alpha \rangle b_\beta^\dagger b_\alpha \; .$$

We now compare the nuclear matrix element of F for nuclear states having n valence particles to the nuclear matrix element of $\hat{F}$ for nuclear states having n valence holes. The hole vacuum is constructed from the particle vacuum by

$$|\hat{0}\rangle = \prod_\alpha a_\alpha^\dagger |0\rangle \; , \tag{10}$$

where α labels the states contained in a closed shell. In a closed shell all the magnetic substates for a given j are occupied for both protons and neutrons and thus $J_{\rm vac} = T_{\rm vac} = 0$. In the case under study, the $1p$-shell particle vacuum is $|^4He\rangle$, which is composed of a fully occupied $1s$ shell. The holes states are created in $|^{16}O\rangle$, which has all twelve $1p$-shell states occupied. Using the Wigner-Eckart theorem[20] for the one-body matrix element,

$$F_{JM}^{T\Lambda}=\sum_{\beta\alpha} S_{-\alpha}\langle j_\beta m_\beta j_\alpha -m_\alpha|JM\rangle\langle \tfrac{1}{2}-\kappa_\beta \tfrac{1}{2}k_\alpha|T\Lambda\rangle \times\langle\beta|||f^{TJ}|||\alpha\rangle a_\beta^\dagger a_\alpha/([J][T])$$
$$=\sum_{j_\beta j_\alpha}\langle\beta|||f^{TJ}|||\alpha\rangle[a_\beta^\dagger\otimes S_{-\alpha}a_\alpha]_{JM}^{T\Lambda}/([J][T]) , \quad (11)$$

where Λ is the total nuclear isospin projection, and $[x]=\sqrt{2x+1}$. Similarly, for the hole operator,

$$\hat{F}_{JM}^{T\Lambda}=\sum_{j_\beta j_\alpha}\langle\beta^{-1}|||f^{TJ}|||\alpha^{-1}\rangle[b_\beta^\dagger\otimes S_{-\alpha}b_\alpha]_{JM}^{T\Lambda}/([J][T])$$
$$=\sum_{j_\beta j_\alpha}\langle\beta|||f_c^{TJ}|||\alpha\rangle[b_\beta^\dagger\otimes S_{-\alpha}b_\alpha]_{JM}^{T\Lambda}/([J][T]) . \quad (12)$$

Evaluating the nuclear matrix element of the one-body operator for the nuclear states with n valence particles yields

$$\langle J_fT_f|||F^{TJ}|||J_iT_i\rangle[J][T]$$
$$=\sum_{j_\beta j_\alpha}\sum_{c_fc_i}\langle\beta|||f^{TJ}|||\alpha\rangle\langle 0|[a_{1f}\otimes a_{2f}\otimes\cdots a_{nf}]_{T_f}^{J_f}|||c_f[a_\beta^\dagger\otimes S_{-\alpha}a_\alpha]_J^Tc_i|||[a_{1i}^\dagger\otimes a_{2i}^\dagger\otimes\cdots a_{ni}^\dagger]_{T_i}^{J_i}|0\rangle , \quad (13)$$

where c_i and c_f label the basis states in the initial and final nuclear states, respectively, as introduced in Eq. (2). Evaluating the reduced matrix element with the same initial and final state quantum numbers and with the valence particles replaced by holes,

$$\langle J_fT_f|||\hat{F}^{TJ}|||J_iT_i\rangle[J][T]$$
$$=\sum_{j_\beta j_\alpha}\sum_{c_fc_i}\langle\beta|||f_c^{TJ}|||\alpha\rangle\langle\hat{0}|[b_{1f}\otimes b_{2f}\otimes\cdots b_{nf}]_{T_f}^{J_f}|||c_f[b_\beta^\dagger\otimes S_{-\alpha}b_\alpha]_J^Tc_i|||[b_{1i}^\dagger\otimes b_{2i}^\dagger\otimes\cdots b_{ni}^\dagger]_{T_i}^{J_i}|\hat{0}\rangle . \quad (14)$$

To compute the doubly reduced nuclear transition matrix elements in Eqs. (12) and Eq. (14), the destruction operators are brought to the right via anticommutation to act on the vacuum state. The hole operators have the same anticommutation relations among themselves as do the particle operators. Also, the basis states are constructed with holes merely replacing particles [see Eq. (2)]. Therefore, the remaining c-number must be the same in the hole and particle case, for the same basis component input. (For example, $\langle\hat{0}|b_\alpha b_\beta^\dagger b_\gamma b_\delta^\dagger|\hat{0}\rangle=\delta_{\alpha\beta}\delta_{\gamma\delta}=\langle 0|a_\alpha a_\beta^\dagger a_\gamma a_\delta^\dagger|0\rangle$.) In order to transform the nuclear matrix element of a one-body operator from the hole to the particle representation, we need only to particle-hole conjugate the single-particle matrix element. Given Eq. (9), it can be seen that the nuclear matrix element of the Gamow-Teller operator for $A=6$ and 14 have the same dependence on the wave function basis components, except for an overall sign difference. The value of $F_A^-(0)^{\mathrm{IA}}$ for the decay

$$^6\mathrm{He}_{\mathrm{g.s.}}\rightarrow{}^6\mathrm{Li}_{\mathrm{g.s.}}+e^-+\bar{\nu}_e$$

is then given by

$$F_A^-(0)^{\mathrm{IA}}=-g_A\left[\alpha x-\frac{1}{\sqrt{3}}\beta y\right] . \quad (15)$$

B. Particle-hole symmetry of two-body amplitude

A two-body operator g, such as the meson-exchange current operator, has the following number representation G for transitions between particle states:

$$G=\tfrac{1}{4}\sum_{\alpha\beta\gamma\delta}\langle\alpha\beta|g|\gamma\delta\rangle_a a_\alpha^\dagger a_\beta^\dagger a_\delta a_\gamma . \quad (16)$$

The matrix element of g in Eq. (16) is between antisymmetrized two-particle states.[12] For transitions between hole states,

$$\hat{G}=\tfrac{1}{4}\sum_{\alpha\beta\gamma\delta}\langle\alpha^{-1}\beta^{-1}|g|\gamma^{-1}\delta^{-1}\rangle_a b_\alpha^\dagger b_\beta^\dagger b_\delta b_\gamma . \quad (17)$$

The relation between the two-particle matrix element of g and the two-hole matrix element is given by

$$\langle\alpha^{-1}\beta^{-1}|g|\gamma^{-1}\delta^{-1}\rangle=\langle\hat{0}|g|\hat{0}\rangle\delta_{\beta\gamma}\delta\alpha\delta-\langle\hat{0}|g|\hat{0}\rangle\delta_{\alpha\gamma}\delta_{\beta\delta}-\langle\tilde{\delta}|g|\tilde{\alpha}\rangle\delta_{\beta\gamma}$$
$$+\langle\tilde{\delta}|g|\tilde{\beta}\rangle\delta_{\alpha\gamma}-\langle\tilde{\gamma}|g|\tilde{\alpha}\rangle\delta_{\beta\delta}+\langle\tilde{\gamma}|g|\tilde{\beta}\rangle\delta_{\alpha\delta}+\langle\tilde{\delta}\tilde{\gamma}|g|\tilde{\beta}\tilde{\alpha}\rangle . \quad (18)$$

Once again, we are considering only nonscalar operators so that all but the last term on the right-hand side of Eq. (18) vanish. We then have

$$\langle\tilde{\delta}\tilde{\gamma}|g|\tilde{\beta}\tilde{\alpha}\rangle=\langle\delta\gamma|\mathcal{T}^{-1}g\mathcal{T}|\beta\alpha\rangle^*$$
$$=\langle\alpha\beta|(\mathcal{T}^{-1}g\mathcal{T})^\dagger|\gamma\delta\rangle=\langle\alpha\beta|g_c|\gamma\delta\rangle . \quad (19)$$

In general, a nonscalar n-body operator will pick up a phase $(-1)^n$ from the n^2 anticommutations of the time-reversed particle operators so that $O_c=(-1)^n(\mathcal{T}^{-1}O\mathcal{T})^\dagger$. Since the phase resulting from the time-reversal transformation of an operator depends only on the rank of the operator in spin and isospin space,[12] and g has the same rank as f, $g_c=g$. The two-body nuclear reduced matrix

TABLE III. $A=6$ phenomenological amplitudes.

	α	β	γ	x	y	$b_{1/2}$	$b_{3/2}$
RPI-L1 (Ref. 9)	0.928	0.366	0.071	0.996	0.094	2.30	1.93
STAN-HO (Ref. 8)	0.924	0.369	0.102	1.00	0.028	2.03	2.03
CK (Ref. 14)	0.958	0.076	0.276	1.00	−0.024		
SASK-A (Ref. 21)	0.924	0.369	0.101	0.844	−0.537	1.80	1.80
SASK-B (Ref. 21)	0.924	0.369	0.100	1.00	−0.010	1.85	1.85

element must be the same for the particle and hole case, for the same reasons as in the one-body case. Thus, the MEC matrix elements for $A=6$ and 14 have the same dependence on the wave function parameters. We have assumed that the radial integrals in the $A=14$ MEC are identical to those for $A=6$. This is justified, since the momentum transfer in the β-decay process is of the order of a few MeV/c.

V. RESULTS AND CONCLUSIONS

In Table III, we display the Cohen and Kurath[14] wave function amplitude relevant to the mass-six case, and several phenomenologically determined wave function amplitudes: STAN-HO of Donnelly and Walecka,[8] SASK-A and SASK-B of Bergstrom *et al.*,[21] and RPI-L1 of Ref. 9 [the basis components in the LS representation were defined in Eq. (1)]. RPI-L1 is obtained from a systematic study of how well the phenomenological amplitude simultaneously describes low momentum transfer electromagnetic and weak observables such as electron scattering, ground-state electromagnetic moments, and the M1 radiative decay. RPI-L1 contains additional radial physics in allowing the $p_{1/2}$ and $p_{3/2}$ radial wave functions to be nondegenerate. In Table IV, the impulse approximation estimate of $F_A^-(0)$, given by Eq. (8), and the MEC's are compared. We treat the wave functions of Table III as a pure $(1p)^{-2}$ configuration and follow the exact procedure of Ref. 10 for the $A=14$ system to obtain the MEC contributions. The column denoted % Dev is the percent deviation of the theoretical estimate for $F_A^-(0)$ from the experimental value[22] (having a magnitude of 1.1775). We should note that the ft value for this decay is six orders of magnitude smaller than that of mass-14, the β-decay rate being correspondingly larger.

Let us summarize our conclusions. The IA contribution to $F_A^-(0)$ for $A=6$ arises almost entirely from the ${}^1S_0 \rightarrow {}^3S_1$ transition. We observe that the MEC's here are small, only at the few percent level, and bring the total matrix element *closer* to the experimental value, *in contrast* to the mass-14 case, except for the amplitude CK. We regard the CK amplitude as unreliable in this case, owing to its overestimation of the first maximum of the ${}^6\mathrm{Li}_{\mathrm{g.s.}}$ $(e,e')\mathrm{Li}^*(3.56$ MeV) form factor and its poor description of the magnetic and quadrupole moments of ${}^6\mathrm{Li}_{\mathrm{g.s.}}$ The above calculation indicates that fitting the mass-six phenomenological amplitude to the β-decay amplitude is justified as a means to determine the one-body transition amplitude.

Nuclear pion photoproduction is not likely to be influenced by meson-exchange current effects, except at sufficiently high energy for multiple pion production, since in the former reaction the photon couples to the same single particle currents as found in the lowest order MEC diagrams for the corresponding electromagnetic process where pions are not emitted. Thus if the phenomenological amplitude is determined from observables that have negligible meson exchange current effects, such an amplitude should be appropriate for use in a nuclear pion photoproduction calculation. Accordingly, phenomenological amplitudes that describe the mass-six β-decay well should yield agreement[23] with the low-energy ${}^6\mathrm{Li}_{\mathrm{g.s.}}(\gamma,\pi^+){}^6\mathrm{He}_{\mathrm{g.s.}}$ data. These amplitudes should be also useful to compute reliably nuclear muon capture observables in the process ${}^6\mathrm{Li}(\mu^-,\nu_{\mu^-}){}^6\mathrm{He}_{\mathrm{g.s.}}$ There is continuing interest in this process at the muon factories in the context of determining[4,24] the axial vector form factor in this superallowed Gamow-Teller transition. This is a test of muon-electron universality in a complex nuclear environment.

TABLE IV. Meson-exchange contributions to $F_A^-(0)$ for $A=6$.

	$F_A^-(0)^{\mathrm{IA}}$	$F_A^-(0)^{\mathrm{MEC}}$	$F_A^-(0)$	% Dev
RPI-L1 (Ref. 9)	−1.13	−0.034	−1.16	1
STAN-HO (Ref. 8)	−1.15	−0.035	−1.18	1
CK (Ref. 14)	−1.20	−0.047	−1.25	6
SASK-A (Ref. 21)	−1.12	−0.022	−1.14	3
SASK-B (Ref. 21)	−1.16	−0.035	−1.20	2

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