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Relativistic potential model of proton-proton scattering spin observables

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A model has been constructed of the interaction between two protons and applied to the calculation of spin-dependent elastic-scattering parameters over a range of beam momenta from 3 to 24 GeV/c. The interaction is one-boson exchange, augmented by a zero-range core and an imaginary potential to model absorption into inelastic channels. The model provides excellent agreement with experimental measurements of the unpolarized differential cross section, and reproduces the qualitative features of the transverse asymmetry and spin correlation. The absorptive potential is found to dominate the real potential, greatly attenuating the low partial waves. This suggests that perturbative quantum chromodynamics does not apply to elastic scattering at the energy scale examined here, since absorptive effects prevent the quarks from approaching to appropriately small separations.

I. INTRODUCTION

The elastic scattering of protons has shown unexpected complexity in its spin structure at relativistic energies. The transverse asymmetry and spin correlation at several energies have been measured by the group led by Krisch,[1-3] who have obtained particularly intriguing results at 11.75 GeV/c for the spin correlation and at 24 GeV/c for the asymmetry. In each case, the observables take on unexpectedly large values at large momentum transfer. At their extremes, the asymmetry shows five protons scattering left for each three scattering right and the spin correlation shows four spin-parallel pairs scattering for each antiparallel pair.

Since the relative momentum of the two protons in the center-of-momentum frame is greater than the rest mass of the proton, and the unpolarized differential cross section obeys a scaling law derived from perturbative quantum chromodynamics,[4] it might be expected that the scattering is perturbative in nature. This turns out not to be the case.

Perturbation theory assumes that massless, effectively free quarks interact with single gluons, conserving the helicities of the interacting quarks. Therefore, the calculated asymmetry vanishes identically at all angles, and the spin correlation takes a roughly constant value of one-third.[5] Since the data do not do so, at least one part of the hypothesis must not hold at this length scale.

This work will approach the problem from the opposite direction, using the techniques of potential scattering in a relativistic framework, with the quark structure of the proton manifested by a form factor which modifies the proton-proton interaction. This ansatz explicitly incorporates the fact that the quarks are baryonic constituents, which constantly exchange information with their partners. In addition, the potential formalism allows the inclusion of the effects of the multitude of inelastic channels available to the system by means of an absorbing potential. The absorption, being a Lorentz scalar, necessarily discriminates between states of channel spin 0 and those of channel spin 1. Since the absorbing potential is found to dominate the scattering, large spin correlations are an immediate consequence of the potential model. Only the fine structure of the correlations depends upon the real potential and the form factors of the proton.

Relativistic dynamics are mandatory for this problem; they are incorporated here by means of a continuum form of the instantaneous Bethe-Salpeter equation.[6,7] This is a three-dimensional reduction of the full equation, but still retains part of the contribution due to the possibility of intermediate states of negative energy. Section II describes this equation and the Foldy-Wouthuysen transformation which renders it convenient for computation.

The potential is a simple form of the one-boson-exchange model with imaginary terms describing the inelastic processes resulting from meson production and quark rearrangement in the protons. It is presented in Sec. III.

The quark structure of the proton is derived from the chirally symmetric flux-tube model,[8] assuming that quarks couple directly to mesons which are approximated by elementary fields. The resulting form factors are derived in Sec. IV.

II. THE RELATIVISTIC SCATTERING EQUATION

In the center-of-mass frame, the initial protons move along the z axis and have relative momentum $k=\frac{1}{2}(p_a-p_b)$. The final state has the protons in the x-z plane, each of which has scattered through an angle Θ (or, indistinguishably, $\pi-\Theta$). The initial protons are polarized perpendicularly to the scattering plane and the final-state spins are not detected.

For the purposes of this work, the transition matrix (T matrix) will be defined as the solution of the operator equation

$$T=V+VGT \;, \tag{1}$$

where V is the interaction potential in the ladder approximation and G describes the propagation of two free particles. Using the basis of the Appendix, the T matrix is related to the scattering matrix and scattering amplitude by

$$S_{fi}=1+2\pi i\delta^4(P_a-P_b)T_{fi} , \quad (2)$$

$$f_{ij}=-2\pi^2 m T_{ij} . \quad (3)$$

The T matrix, and hence the scattering amplitude, for a system of two protons is a 4×4 matrix with rows and columns labeled by the helicities of the two protons. Due to the symmetry of the system, there are five independent elements in the scattering amplitude. Using the notation of Bystricky,[9] they are

$$\begin{aligned} M_1&=\langle ++|f|++\rangle ,\\ M_2&=\langle ++|f|--\rangle ,\\ M_3&=\langle +-|f|+-\rangle ,\\ M_4&=\langle +-|f|-+\rangle ,\\ M_5&=\langle ++|f|+-\rangle . \end{aligned} \quad (4)$$

In terms of these observables, the unpolarized cross section, asymmetry, and transverse spin correlation are

$$(d\sigma/dt)_0=\tfrac{1}{2}(|M_1|^2+|M_2|^2+|M_3|^2+|M_4|^2+4|M_5|^2) , \quad (5)$$

$$(d\sigma/dt)_0 A=-\mathrm{Im}[M_5^*(M_1+M_2+M_3-M_4)] , \quad (6)$$

$$(d\sigma/dt)_0 A_{nn}=2|M_5|^2+\mathrm{Re}(M_1M_2^*-M_3M_4^*) , \quad (7)$$

respectively.

The explicit meanings of the operators in Eq. (1) can be stated once the scattering equation is specified. When Salpeter's equation is used in momentum space, the free propagator has two terms—one for states of two positive-energy particles and one for negative-energy particles. Expressed with the aid of operators Λ which project states of definite energy sign, the propagator is

$$\begin{aligned}\langle q|G|q'\rangle&=\frac{E(q)^2}{m^2}\delta(\mathbf{q}-\mathbf{q}')G(q)\\ &=\frac{E(q)^2}{m^2}\delta(\mathbf{q}-\mathbf{q}')[G_+(q)\Lambda_+(q)\\ &\qquad +G_-(q)\Lambda_-(q)] ,\end{aligned} \quad (8)$$

where

$$G_+(q)=[E-2E(q)+i\epsilon]^{-1} , \quad (9a)$$

$$G_-(q)=[-E-2E(q)]^{-1} , \quad (9b)$$

$$\Lambda_\pm(q)=\frac{1}{2}\left[1\pm\frac{\alpha_a\cdot\mathbf{q}+\beta_a m}{E(q)}\right]\frac{1}{2}\left[1\pm\frac{-\alpha_b\cdot\mathbf{q}+\beta_b m}{E(q)}\right] . \quad (10)$$

E is the total energy of the system in the center-of-mass frame,

$$E(q)=(q^2+m^2)^{1/2} ,$$

m is the proton mass, α and β are Dirac matrices, and a and b are particle labels. In a helicity basis, the projection operators are simplified by the fact that

$$\begin{aligned}\alpha_a\cdot\mathbf{q}&=\lambda_a q\gamma_a^5 ,\\ -\alpha_b\cdot\mathbf{q}&=\lambda_b q\gamma_b^5 ,\end{aligned} \quad (11)$$

using Bjorken and Drell's representation of the Dirac matrices.[10] A further simplification arises from a change of basis effected by the Foldy-Wouthuysen transformation.[7] This unitary transformation, which will also be useful in evaluating matrix elements of the potential, is given by

$$\begin{aligned}U(q)&=(A_q+B_q\lambda_a\gamma_a^5\beta_a)(A_q+B_q\lambda_b\gamma_b^5\beta_b) ,\\ A_q^2&=[E(q)+m]/2E(q) ,\\ B_q^2&=[E(q)-m]/2E(q) .\end{aligned} \quad (12)$$

With respect to this new basis, the projection operators become

$$U\Lambda_\pm U^\dagger=\tfrac{1}{2}(1\pm\beta_a)\tfrac{1}{2}(1\pm\beta_b) . \quad (13)$$

With the operator product in the canonical T-matrix equation (1) expanded in plane waves, a matrix integral equation for the 16-equation system results

$$\langle p|T|k\rangle=\langle p|V|k\rangle+\int d^3q\langle p|V|q\rangle G(q)\langle q|T|k\rangle , \quad (14)$$

where, for elastic scattering, $|p|=|k|$.

In the transformed basis, the matrix $G(q)$ is mostly zeros. The eight rows and eight columns describing states in which the two particles have different energy signs are superfluous. They may be dropped from the matrices, so the symbols in Eq. (14) above will henceforth refer to 8×8 matrices.

The three-dimensional integral can be reduced to a radial integral by resolving the potential and the T matrix into partial-wave form. This is done according to the prescription of Jacob and Wick[11] so the states forming the basis of the matrices are characterized by the magnitude of the relative momentum, the total angular momentum J, the energy sign of the particles ϵ, and the two helicities. An element of the potential is expanded by

$$\langle p\lambda_c\lambda_d\epsilon|V|q\lambda_a\lambda_b\epsilon'\rangle=\sum_J[(2J+1)/4\pi]D^{J\dagger}_{\lambda\mu}(R)\times\langle pJ\lambda_c\lambda_d\epsilon|V|qJ\lambda_a\lambda_b\epsilon'\rangle . \quad (15)$$

Here, R is the rotation which turns the direction of q into the direction of p, D is the matrix associated with that rotation in an angular-momentum state labeled by J, $\lambda=\lambda_a-\lambda_b$, and $\mu=\lambda_c-\lambda_d$. Using the orthogonality properties of the rotation matrices,[11,27] the T matrix can be block diagonalized in J, and the integral equation for any given J becomes

$$\langle p\lambda_c\lambda_d\epsilon|T_J|k\lambda_a\lambda_b+\rangle=\langle p\lambda_c\lambda_d\epsilon|T_J|k\lambda_a\lambda_b+\rangle+\int_0^\infty dq\,q^2\sum_{\substack{\lambda_1\lambda_2\\ \eta}}\langle p\lambda_c\lambda_d\epsilon|V|q\lambda_1\lambda_2\eta\rangle G(q,\eta)\langle q\lambda_1\lambda_2\eta|T|k\lambda_a\lambda_b+\rangle , \quad (16)$$

or, in a matrix notation,

$$T_J(p,k)=V_J(p,k)+\int dq\, q^2 V_J(p,q)G(q)T_J(q,k)\ . \tag{17}$$

Summing the series in J is necessary at the last step to get the T matrix. In this case, the momentum in the ket is parallel to the z axis so R can be written in terms of the scattering angle Θ and the polar angle ϕ.[11]

The integral equations (17) are singular at the point where the intermediate momentum q equals the initial momentum k. To remove the pole and allow numerical quadrature of the integral, the half-shell T matrix is factored by a method due to Kowalski and Feldman[12–14] into its on-shell value and an off-shell factor $F(p)$:

$$T_J(p,k)=F(p)T_J(k,k)\ . \tag{18}$$

The off-shell factor is an 8×8 matrix which is equal to the identity matrix when $p=k$. The integral equation for the off-shell factor is (the subscript J is suppressed)

$$\begin{aligned}F(p)=&V(p,k)V^{-1}(k,k)\\&-\int dq\, q^2[V(p,k)V^{-1}(k,k)V(k,q)\\&\qquad -V(p,q)]G(q)F(q)\ .\end{aligned} \tag{19}$$

Once F is known, the on-shell T matrix is

$$\begin{aligned}T(k,k)=&\left[1-\int_0^\infty dq\, q^2 V(k,q)G(q)F(q)\right]^{-1}V(k,k)\\=&\left\{1-\int_0^\infty dq\, q^2[V(k,q)G(q)F(q)\right.\\&\qquad -\alpha(q)V(k,k)G(q)]\\&\left.-V(k,k)\begin{bmatrix}I_1 & 0\\ 0 & I_2\end{bmatrix}\right\}^{-1}V(k,k)\ ,\end{aligned} \tag{20}$$

where

$$\alpha(q)=k^2E(k)/q^2E(q)\ , \tag{21a}$$

$$I_1=\tfrac{1}{2}kE_k\left[\ln\left|\frac{k+E_k-1}{k-E_k+1}\right|-i\pi\right]\ , \tag{21b}$$

and

$$I_2=\tfrac{1}{2}kE_k\ln\left|\frac{k+E_k-1}{k-E_k+1}\right|\ . \tag{21c}$$

The function $\alpha(q)$ has been introduced so that the pole in the propagator can be subtracted from the integral in Eq. (21) and integrated analytically, giving the supermatrix with elements I_1 and I_2.

III. THE ONE-BOSON-EXCHANGE POTENTIAL

The T-matrix equation requires knowledge of the interaction potential when both the initial and final states may be off shell. A way to specify the potential completely is to relate it to an underlying physical process—in this case, the exchange of virtual mesons. To simulate the potential between protons below the inelastic threshold requires three mesons: a pseudoscalar pion, a scalar meson called "σ" to provide the attraction at medium ranges, and a vector meson which is a combination of the rho and omega mesons to provide short-range repulsion. The σ meson used here is the same as that of the Bonn potential,[15] with a mass of 500 MeV.

Additional mesons may be added to this minimum set to cause the energy dependence of the various parts of the potential to resemble the true interaction more closely. In particular, some of the scalar parts of the interaction may be due to the $a_0(980)$.

One more term must be added to this set to account for the very short-range behavior of the proton-proton system. At angles near 90° and center-of-mass momenta larger than 2 GeV/c, the differential cross section is larger than a pure meson-exchange model can account for. The extra term will be assumed to have zero range and transform as a Lorentz vector since its purpose is to augment the repulsive core.

A matrix element of the potential derived from the one-boson-exchange model factors neatly into separate parts, one describing the interaction of the boson with each proton, and the other describing the propagation of the boson through space. A typical matrix element is

$$\begin{aligned}\langle p\lambda_c\lambda_c\epsilon'=+|V|q\lambda_a\lambda_b\epsilon=+\rangle&\\=(g^2M^2/E_pE_q)y(p,q,\lambda)&\\\times[\bar{u}(p\lambda_c)\Gamma u(q\lambda_a)]&\\\times[\bar{u}(p\lambda_d)\Gamma u(q\lambda_b)]\ ,&\end{aligned} \tag{22}$$

where $y(p,q,\lambda)$ is a Yukawa potential between two eigenstates of helicity, and the Γ's signify the Lorentz-Dirac properties of the meson field. For the zero-range core, the function y is set equal to 1. When the Γ are factored into spin and energy-sign operators, the spin factors commute with the spinors. Hence, the bracketed terms are functions only of the magnitudes of the momenta and the helicities. All spin and angle dependence is in the spin factors and the Yukawa function.

An easy and compact way to calculate the bracketed terms is to use the Foldy-Wouthuysen transformation. As an example, consider the particle "a" term. Inserting $U_a^\dagger U_a$ on each side of the Dirac matrices yields

$$\begin{aligned}\bar{u}(p\lambda_c)\Gamma u(q\lambda_a)&=u^\dagger(p\lambda_c)U_a^\dagger U_a\beta\Gamma U_a^\dagger U_a u(q\lambda_a)\\&=(E_pE_q/m^2)^{1/2}[1,0]U_a\beta\Gamma U_a^\dagger\begin{bmatrix}1\\0\end{bmatrix}\ .\end{aligned} \tag{23}$$

The bracketed term is therefore the upper-left element of the transformed $\beta\Gamma$ operator. If negative-energy spinors had been used in the example, of course, different elements would have been selected.

For a pseudoscalar meson, the transformation goes like this for particle "a:"

$$\begin{aligned}U(p)\beta\Gamma_5U^\dagger(q)&=(A_p+B_p\lambda_c\beta\gamma_5)\beta\gamma_5(A_q+B_q\lambda_a\gamma_5\beta)\\&=(A_pA_q+B_pB_q\lambda_a\lambda_c)\beta\gamma_5\\&\quad+(A_pB_q\lambda_a-A_qB_p\lambda_c)\ .\end{aligned} \tag{24}$$

The part of the operator proportional to γ_5 connects kets of a given energy sign to bras of the opposite sign. We

TABLE I. Transition current elements for meson exchange.

Meson type	Dirac operators	Even	Odd
PS	$\beta\gamma_5$	$(A_pB_q\lambda_a-A_qB_p\lambda_c)(A_pB_q\lambda_b-A_qB_p\lambda_d)$	$(A_pA_q+B_pB_q\lambda_a\lambda_c)(A_pA_q+B_pB_q\lambda_b\lambda_d)$
SC	β	$(A_pA_q-B_pB_q\lambda_a\lambda_c)(A_pA_q-B_pB_q\lambda_b\lambda_d)$	$(A_pB_q\lambda_a+A_qB_p\lambda_c)(A_pB_q\lambda_b+A_qB_p\lambda_d)$
VE	1	$(A_pA_q+B_pB_q\lambda_a\lambda_c)(A_pA_q+B_pB_q\lambda_b\lambda_d)$	$(A_pB_q\lambda_a-A_qB_p\lambda_c)(A_pB_q\lambda_b-A_qB_p\lambda_d)$
fourth component			
VE space	γ_5	$(A_pB_q\lambda_a+A_qB_p\lambda_c)(A_pB_q\lambda_b+A_qB_p\lambda_d)$	$(A_pA_q-B_pB_q\lambda_a\lambda_c)(A_pA_q-B_pB_q\lambda_b\lambda_d)$

call it "odd in energy sign." The part which is not, connects only bras and kets of the same energy sign. We call it "even." The only terms in the potential which contribute to Salpeter's equation are those which are even for both particles or odd for both particles. Results for the transition current due to the various types of meson are collected in Table I.

Now, the matrix element can be written

$$\langle p\lambda_c\lambda_d\epsilon|V|q\lambda_a\lambda_b\eta\rangle=g^2Q(\epsilon,\eta)\Sigma y(p,q,\lambda)\ , \quad (25)$$

where Q comes from the table and Σ is the product of spin operators from the Γ matrices. For pseudoscalar and scalar mesons $\Sigma=1$. For vector mesons, the product of the fourth (time) components has $\Sigma=1$, and the dot product of the three-vector parts has $\Sigma=\sigma_a\cdot\sigma_b$. This separation facilitates the resolution into partial waves. The recoil term Q, with its helicity-dependent terms due to the recoil of the protons, is the same in all partial waves, so the expansion affects only Σ and y.

The resolution of the potential into states of good total angular momentum and helicity is given in Eq. (15), but its analytic form is not obvious in this basis. The Yukawa function has a simple form when resolved into eigenstates of orbital angular momentum, and the spin operators are simple in a total-spin basis, so the potential matrix element is most perspicuous when considered in the traditional LSJ basis. The transformation between the two representations is given by Jacob and Wick:[11]

$$|JM\lambda_1\lambda_2\rangle=\sum_{LS}\sqrt{(2L+1)/(2J+1)}C(\tfrac{1}{2}\tfrac{1}{2}S,\lambda_1\lambda_2\lambda)\times C(LSJ,0\lambda\lambda)|LSJM\rangle\ . \quad (26)$$

This model interaction conserves the total spin S, and the Yukawa potential y conserves orbital angular momentum L, so the transformation can be used twice to give the reduced matrix element

$$\langle p\lambda_c\lambda_dJ\|y\Sigma\|q\lambda_a\lambda_bJ\rangle=\sum_{LS}[(2L+1)/(2J+1)]C(LSJ,0\lambda\lambda)C(LSJ,0\mu\mu)\times C(\tfrac{1}{2}\tfrac{1}{2}S,\lambda_a\lambda_b\lambda)C(\tfrac{1}{2}\tfrac{1}{2}S,\lambda_c\lambda_d\mu)\langle pL\|y\|qL\rangle\langle S\|\Sigma\|S\rangle\ . \quad (27)$$

The spin matrix element is well known; if $\Sigma=1$ it is trivial and if $\Sigma=\sigma_a\cdot\sigma_b$ it is (4S-3). The orbital matrix is obtained from

$$y(p,q)=\sum_L\frac{2L+1}{4\pi}P_L(\cos\Theta)y_L(q,p)\ , \quad (28)$$

where

$$y_L(p,q)=2\pi\int_{-1}^{1}d(\cos\Theta)P_L(\cos\Theta)[(p-q)^2+\mu^2]^{-1}\ . \quad (29)$$

Numerical quadrature of the integral is the most efficient way to evaluate the partial-wave projection. Though the function $y(p,q)$ is a simple Yukawa function in this case, when form factors are included, the integral will no longer be analytic.

The potentials described above are purely real valued, so the only scattering they can produce is elastic, but when the energy of the system is above the threshold energy for pion production, inelastic processes contribute to the scattering. According to the optical model of elastic scattering, suppose there exists an operator W which can induce transitions to an inelastic state. Such states might contain extra mesons, or one or both of the protons might be rearranged into other hadrons. If the conservation of four-momentum permits it, the system may produce particles which satisfy the mass-energy condition for free propagation, and so escape to infinity, carrying momentum, charge, baryon number, and so forth. This possibility gives rise to the imaginary part of the potential, since it implies a decrease in the total density of elastically scattered protons.

This potential is energy dependent, since the energy available for the production of new particles determines the probability that the system will produce them. For total center-of-momentum energy E, it is

$$\langle p'|\Delta V+iV'|p\rangle=PV\int dE'\sum\langle p'|W|\text{inel}\rangle\langle\text{inel}|W|p\rangle(E-E')^{-1}-i\pi\int dE'\,\delta(E'-E)\sum\langle p'|W|\text{inel}\rangle\langle\text{inel}|W|p\rangle\ . \quad (30)$$

The sum is over all inelastic states of energy E' accessible by means of the operator W. The real term (ΔV) is a correction to the real potential. Since it is of higher order than the one-boson-exchange potential, it will be ignored in the ladder approximation. V' is the leading imaginary term, so it will be retained. This imaginary potential is negative definite in sign, which preserve the unitarity of the scattering matrix.

The model of the imaginary potential will have two terms corresponding to the two distinct slopes of the experimental differential cross section at high energies.[16] Both terms describe contact interactions. The first term is just a delta function (folded between quark densities) describing the rearrangement of quarks within a proton to form another baryon. The second term describes the production of a real meson with a Yukawa form factor at the point of contact between the protons. The radius of the meson will be determined by fitting the slope of the differential cross section at small angles.

The Lorentz-Dirac properties of the imaginary potential are unknown *a priori*, but some features can be noted, assuming that the process by which the system went into the inelastic channel is the same process by which it comes back. Since each such part of the coupling operator W has definite transformation properties, the imaginary potential is composed of the possible results of acting with W twice. For scalar and pseudoscalar mesons, the only possible result is a scalar. For vector mesons, the only possibility which is negative definite in all partial waves is a scalar. Therefore, the imaginary potential will be a pure scalar.

The strength of the absorbing potential is determined by the total scattering cross section. For momenta well above the inelastic threshold, the total cross section is roughly constant, about 40 mb. Requiring that the total cross section, as determined from the optical theorem, match its experimental value determines the strength of the absorbing potential.

IV. THE FLUX-TUBE MODEL

When the relative momentum of two protons is comparable to the proton mass, the quark degrees of freedom are relevant to the interaction between them. In this work, the interaction between protons is assumed to be the result of meson exchanges between quarks, double folded with the quark densities.

Quarks themselves are massless point particles confined to a neighborhood of some origin by a potential which increases linearly with the displacement of the quark from the origin. The potential is a scalar under Lorentz transformations, and so breaks chiral symmetry. The quarks are coupled to a pion field, which restores chiral symmetry to the system. This is the so-called "flux-tube model,"[8] since the linear potential is an approximation to the result from lattice gauge theory that gluons form themselves into narrow tubes between quarks when the quarks are relatively far apart.[17]

The Lagrangian for the flux-tube model is highly nonlinear[8] and therefore unsolvable, so it is assumed that the number of virtual pions in the cloud around the proton is of order 1 (this is true in the cloudy bag model[18]), and the Lagrangian may be linearized to give

$$L=\tfrac{1}{2}i(\bar{\psi}\gamma^{\mu}\partial_{\mu}\psi-\partial_{\mu}\bar{\psi}\gamma^{\mu}\psi)-\bar{\psi}S(r)\psi-\tfrac{1}{2}(\partial_{\mu}\phi)^2 -\tfrac{1}{2}m_{\pi}^2\phi^2-(i/f)\bar{\psi}\underline{\tau}\cdot\underline{\phi}\gamma_5 S(r)\psi \ . \tag{31}$$

Here, ψ is the quark field, ϕ is the pion field, f is the pion-decay constant measured to be 93 MeV, and $S(r)=\eta r$ is the confining potential. Here, $\underline{\tau}$ and $\underline{\phi}$ are vectors in isospin space.

When the protons approach one another closely, the hypothesis of a small number of pions need not be true and nonlinear terms which do not appear in (31) may be significant. The effect of such terms will be dealt with by means of heavy-meson exchanges. All the mesons used in this work can decay into states containing only pions, so they may be considered to be quasistable nonlinear excitations of the pion field, and thereby consistent with the flux-tube model.

Turning to the problem of the unperturbed quarks, the linear confinement problem for massless quarks has an approximate solution which is very simple and compact. Abe and Fujita[19] have shown that the solutions of the quark Lagrangian are well approximated by combinations of the eigenfunctions of the simple harmonic oscillator. In particular, the ground-state wave function is

$$\psi(r)=\begin{bmatrix} A\langle r|00\rangle \\ -iB\langle r|01\rangle\sigma\cdot\mathbf{n}\end{bmatrix}\chi_{\nu}\ , \tag{32}$$

where χ_{ν} is a Pauli spinor, $\mathbf{n}$ is a unit vector in the direction of r, $A=0.932$, $B=0.363$, and the radial functions $\langle r|nL\rangle$ are spherical harmonic-oscillator eigenfunctions with scale parameter $\sqrt{\eta}$.

The negative-energy states needed for Salpeter's equation will be constructed this way: the negative-energy proton is a system of three negative-energy quarks, coupled identically to the coupling in the positive-energy proton. The interaction of a meson with a quark can kick the quark into a negative-energy state, at which point, one of two things happens. A system of two positive-energy quarks and one negative-energy quark is not a proton, so either the chromodynamic interactions among the quarks in a baryon drag the spectators into negative-energy states as well, forming a negative-energy proton,[20] or the whole system is lost to an inelastic channel and the transition contributes to the absorptive potential.

Negative-energy quark states are constructed in the Abe-Fujita approximation by finding a unitary operator which anticommutes with the Hamiltonian. Such an operator is $\gamma_0\gamma_5$. A scalar potential has a spectrum which is symmetrical about $E=0$, so $\gamma_0\gamma_5\psi$ is the negative-energy partner to any positive-energy state ψ.

This model contains one parameter—the potential strength η. This parameter is set by requiring that the energy spectrum match the observed masses of the various baryons.[8] The best value $\eta=(420\text{ MeV})^2$ leads to agreement with the proton electric form factor as well, so it will be considered fixed.

The form factor for a given quark-meson interaction is

TABLE II. Densities and form factors for the mesons.

Transition type	Space density	Approximation form factor
PS even	$-2iABR_0R_1\sigma\cdot\mathbf{n}$	$e^{-q^2/3.14}-0.0123e^{-0.197(q-2.7)^2}$
PS odd	$(A^2+2/3B^2\mu r^2)R_0^2$	$e^{-q^2/3.14}-0.0832e^{-0.25(q-2.1)^2}$
SC even	$(A^2-2/3B^2\mu r^2)R_0^2$	$e^{-q^2/3.14}-0.040e^{-0.40(q-3.6)^2}$
SC odd	0	0
VE even (fourth)	$(A^2+2/3B^2\mu r^2)R_0^2$	$-$PS odd
VE odd (fourth)	$2iABR_0R_1\sigma\cdot\mathbf{n}$	$-$PS even
VE even (space)	$-2ABR_0R_1\sigma\times\mathbf{n}$	i PS even
VE odd (space)	$-A^2R_0^2\sigma+B^2R_1^2(\sigma\cdot\mathbf{n})\sigma(\sigma\cdot\mathbf{n})$	$-$SC even
ZR even (fourth)	$(A^2+2/3B^2\mu r^2)R_0^2$	$e^{-q^2/4}(1-B^2q^2/6)$
ZR odd (fourth)	$2iABR_0R_1\sigma\cdot\mathbf{n}$	$e^{-q^2/4}$
ZR even (space)	$-2ABR_0R_1\sigma\times\mathbf{n}$	$e^{-q^2/4}$
ZR odd (space)	$-A^2R_0^2\sigma+B^2R_1^2(\sigma\cdot\mathbf{n})\sigma(\sigma\cdot\mathbf{n})$	$e^{-q^2/4}(A^2-B^2+B^2q^2/6)$

derived from the appropriate term of the Lagrangian by expanding the quark fields in terms of the eigenstates of the potential and the meson fields in plane waves. The matrix element of the Lagrangian between a state with a quark only and a state with a quark and a meson of a given momentum will then have factors which appear in the interaction between a point proton and the meson and factors which do not. These latter are the form factors.

Thomas has given an excellent pedagogical derivation of pion-nucleon form factors in the context of the cloudy bag mode,[18] so only a cursory review will be given here. As an example, the pion-interaction term of the Lagrangian is used to obtain an operator in momentum space which is proportional to $\sigma\cdot\mathbf{k}$, which is the vertex factor for a point nucleon emitting a pion with momentum $\mathbf{k}$. We call it "K:"

$$K(\mathbf{k})=\int d\mathbf{x}\,\bar{q}(\mathbf{x})\gamma_5 q(\mathbf{x})S(r)e^{i\mathbf{k}\cdot\mathbf{x}}\ . \tag{33}$$

We define the form factor at zero momentum to be unity, so the constants may be ignored. Using the Abe and Fujita wave functions, $S(r)=\eta r$, $R_0=\langle r|00\rangle$, and $R_1=\langle r|01\rangle$,

$$K(\mathbf{k})=\int d\mathbf{x}(AR_0,-iBR_1\sigma\cdot\mathbf{n})\gamma_5\begin{bmatrix}AR\\-iBR_1\sigma\cdot\mathbf{n}\end{bmatrix}\eta re^{i\mathbf{k}\cdot\mathbf{x}} \tag{34}$$

$$=\int d\mathbf{x}\,2iABR_0R_1\sigma\cdot\mathbf{n}\eta re^{i\mathbf{k}\cdot\mathbf{x}} \tag{35}$$

$$=-32\pi^{3/2}AB\int dz\,z^4e^{-z^2}j_1(qz)\sigma\cdot\hat{\mathbf{q}}\ . \tag{36}$$

Here, $z=\sqrt{\eta}r$ and $\mathbf{q}=\mathbf{k}/\sqrt{\eta}$. This integral can be expressed in terms of tabulated functions, but doing so does not provide any particular illumination. What is important about it is that the form of the point-proton vertex has been recovered, with a momentum-dependent multiplier, independent of length scale,

$$(3/q)\int dz\,z^4e^{-z^2}j_1(qz)\ . \tag{37}$$

For purposes of computation, this form factor can be approximated by the sum of two Gaussians.

The form factors for the other mesons are calculated in the same manner, with different densities in the integral. All have been given the same approximation treatment. For the three types of mesons, each with transitions to positive- and negative-energy states, three form factors are needed (plus one which is identically zero). The various densities and approximate form factors are collected in Table II as functions of the momentum transfer scaled by the potential parameter. One further approximation should be noted: The negative-energy three-vector form factor is not used in its entirety. The speed of the numerical computation is vastly increased by the substitute in the table, which never deviates more than 50% from the correct factor.

The form factors for the zero-range (ZR) terms are obtained similarly, but the term $S(r)$ in Eq. (33) does not appear, so the integrals corresponding to (37) are analytic. These form factors are appended to Table II, labeled "ZR."

The "static" form factors thus derived are nonrelativistic (NR) in concept, since they take no account of the distortion of moving objects due to the Lorentz-Fitzgerald contraction. To simulate the effects of special relativity, Licht and Pagnamenta[21] derived a formula for scaling the momentum dependence of the form factor of a general cluster of particles.

TABLE III. Real potential parameters from phase-shift fit.

Meson	Mass (GeV/c^2)	Coupling
Pion	0.138	13.0
Sigma	0.500	5.9
Rho	0.783	5.0
Core		12.1

Brodsky and Farrar[4] have made an analysis of the actual high-momentum scaling laws that govern the form factors of hadrons. By counting the minimum number of gluon exchanges which are consistent with preservation of the color-singlet nature of a proton when it interacts with another, they conclude that the actual scaling law which is obeyed by a three-quark system is

$$\lim_{q^2 \to \infty} K(q^2) \propto (q^2)^{-2} , \qquad (38)$$

This asymptotic form can be incorporated into form factors in the region of interest here. Stanley and Robson[22] obtained good fits to the experimentally determined electromagnetic form factors of the proton, neutron, and pion using

$$K(q^2) = (1+q^2/4M^2)^{-2} K_{\rm NR}[q^2/(1+q^2/4M^2)] \qquad (39)$$

to give the correct limiting behavior. Hence, form (39) will be used here for the relativistic transition form factors corresponding to the form factors $K_{\rm NR}$ in Table II.

V. RESULTS AND CONCLUSION

The approximation to the proton-proton potential was made in two steps. First, the real part of the potential was determined from data below the inelastic threshold, then the imaginary potential strengths were varied at each beam momentum to fit the higher-energy data. The real potential parameters were adjusted to fit the scattering phase shifts below 300 MeV as determined by Arndt *et al.*,[23] which produced parameter set I, or to fit the transverse asymmetries at 210 and 310 MeV (Ref. 24) simultaneously, giving set II. The optimum values for the coupling strengths from a chi-squared fit are shown in Tables III and IV, and the fits are shown in Figs. 1 and 2. The fits are good for parameters set II. For set I, the $^3P_1$ and $^3P_2$ channels suffer from the inclusion of the zero-range core; were it not required, the fit could be made

TABLE IV. Real potential parameters from asymmetry fit.

Meson	Mass (GeV)	Coupling
Pion	0.138	13.0
Sigma	0.500	7.9
Rho	0.783	15.5
a_0	0.960	6.9
Core		9.9

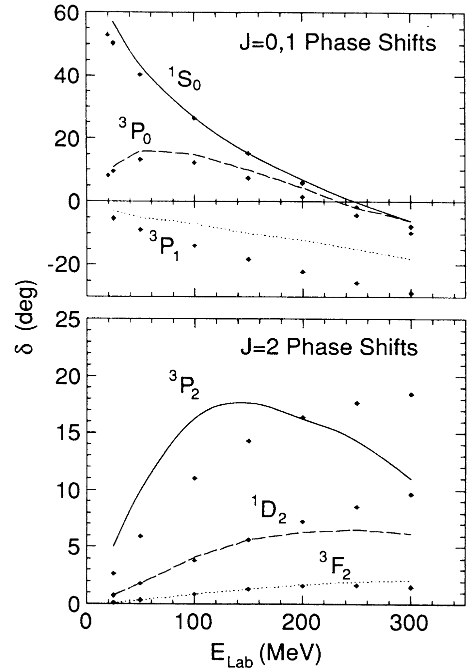

FIG. 1. The fit of parameter set I to the six lowest proton-proton phase shifts plotted versus the laboratory kinetic energy in MeV.

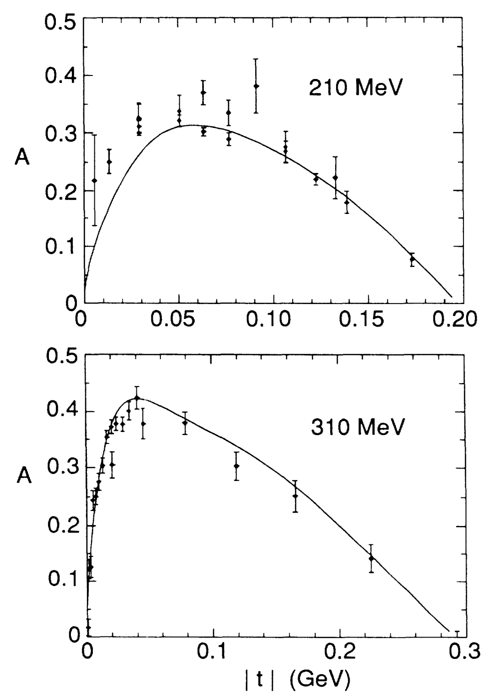

FIG. 2. The fit of parameter set II to the experimental asymmetries at 210 and 310 MeV plotted versus squared momentum transfer.

TABLE V. Imaginary potential parameters—phase-shift fit.

P_{lab}	Rho+omega	Core
3	9.86	7.11
6	34.5	88.9
9	49.4	118
11.75	123	296
18.5	123	834
24	1730	2960

comparable to that of the 1972 Bonn potential.[15]

The imaginary potentials work best when the Yukawa form factor describing production of a real meson has a mass parameter of 780 MeV. This is close to the pion cutoff mass used by the Bonn group,[15] implying that the mesons produced are primarily pions. Within the constraint of keeping the total cross section constant, the two coupling strengths were varied to fit the differential cross-section asymmetry and spin correlation by hand, due to the long running time of the computer program at higher-beam momenta. Since this model tends to overestimate the spin observables, the optimization was a process of balancing the couplings to obtain the maximum destructive interference between the two potentials. The values of the imaginary strengths are shown in Tables V and VI. They increase roughly as an exponential of the barycentric momentum.

Results of the calculation are shown in Figs. 3–9 for beam momenta of 3, 11.75, 18.5, and 24 GeV/c, together with data. The differential cross sections are well reproduced, though the breakdown of the assumption of a zero-range core can be seen in Fig. 8. The asymmetry and spin correlation are less well fitted. The 3-GeV data are sensitive to the exact form of the real potential or low-momentum transfer, but as the absorptive terms come to dominate at angles near 90°, the fit improves. At higher energies, the gross structures of the data are all that can be said to be reproduced. The model results show more oscillation than do the data at large angles, indicating that the smoothing influence of many interfering terms is absent from the model. Nevertheless, the spin observables show structure which is absent from previous treatments, due to the nature of the quarks as constituents of protons and the absorptive effects of inelastic channels in the proton-proton system.

Two predictions can be made from these calculations which may be relevant to experimental work in the near future. From Fig. 7, the value of A_{nn} at 90° for a beam of 18.5 GeV/c momentum can be estimated to be about 0.35, a decrease from the value at 11.75 GeV/c. From Fig. 9, at 24 GeV/c one can anticipate a rapid drop to zero in the asymmetry at a squared momentum transfer of 9 (GeV/c)^2, which corresponds to a squared transverse momentum of 7.5 (GeV/c)^2.

Since the imaginary potential dominates the real by as much as 2 orders of magnitude, the J dependence of the absorption dictates the form of the high-energy spin observables. The spin observables depend on angular momentum in two ways. First, the waves with lower J will suffer more attenuation, since they imply more overlap of the protons. The envelope of the J dependence in impact parameter space will mimic the scalar form factor of the proton. Second, and more important, there will be fine structure due to the Lorentz-scalar nature of the imaginary potential through the helicity operators in Table I. This is a general feature of a relativistic scalar imaginary potential, independent of the details of the

TABLE VI. Imaginary potential parameters—asymmetry fit.

P_{lab}	Rho+omega	Core
3	8.3	5.92
6	20.7	78.9
9	111	88.8
11.75	193	138
18.5	304	1090
24	2760	1970

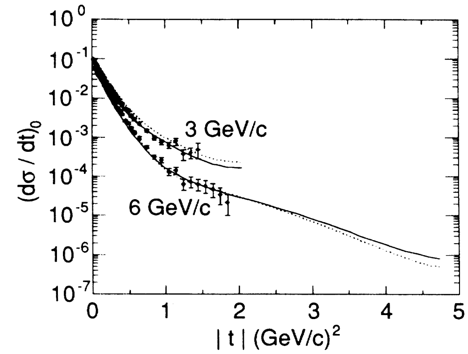

FIG. 3. The unpolarized differential cross section at beam momenta of 3 and 6 GeV/c plotted vs t. Dotted lines are parameter set I, solid lines are parameter set II. The data are from Ref. 24.

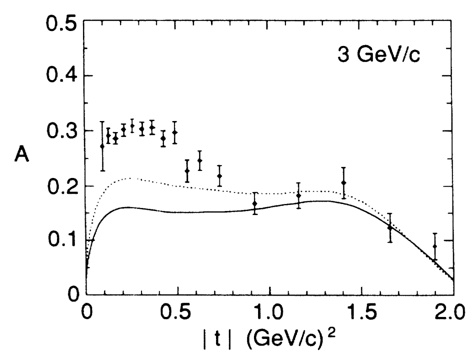

FIG. 4. The asymmetry at 3 GeV/c beam momentum plotted vs t. The data are from Ref. 25.

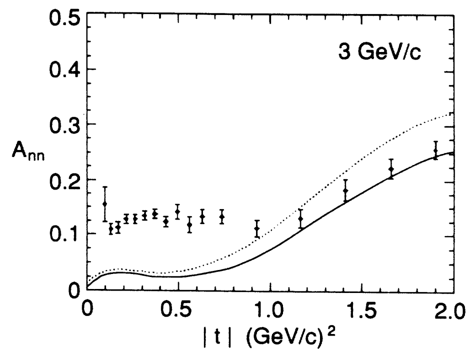

FIG. 5. The spin-correlation parameter at 3-GeV/c beam momentum plotted vs t. The data are from Ref. 25.

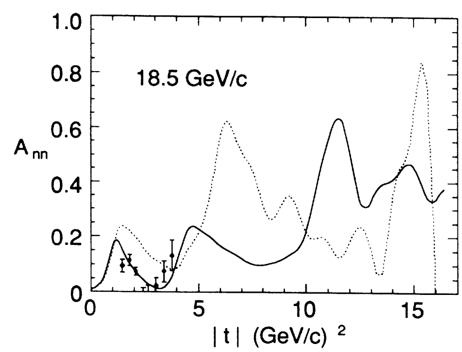

FIG. 7. The spin-correlation parameter at 18.5 GeV/c plotted vs t. The data are from Ref. 2.

quark-quark interaction. Figure 10 shows the effect of the absorptive potential as a function of angular momentum. When J is odd, the Pauli principle requires that the total spin of the system be 1. These states are less absorbed, so they are more likely to scatter elastically, giving rise to a large positive spin correlation. The low partial waves are almost completely absorbed, so elastic scattering is primarily due to grazing collisions between the protons.

This gives a clue to why the perturbative calculations fail for elastic events when they succeed so well with inclusive reactions. For perturbation theory to apply to elastic scattering, all six valence quarks must be at short distances from one another, which means the protons must be in a state of low angular momentum. Such states are highly suppressed by the absorption. Inclusive reactions, by contrast, make no such demand. A single pair of quarks can be very close together whenever the two protons overlap significantly, and can exchange a single gluon, giving rise to a state of unspecified hadrons. This is precisely the hypothesis upon which the imaginary potential of this work is based. Since the only momentum dependence of the imaginary potential is due to form factors which were forced to obey perturbative scaling laws at high-momentum transfer, it seems clear that the proper role of perturbative chromodynamics in modeling exclusive events is to determine the manner in which unobserved channels affect the reaction of interest.

The importance of negative-energy intermediate states does not increase monotonically with energy. They are most important in the range of beam momenta from 1 to 6 GeV/c, where they have the effect of softening the repulsive core. When the beam momentum is higher, the imaginary potentials dominate the real, and since the form factor for odd scalar transitions vanishes, the negative-energy states no longer have a visible effect.

In conclusion, the essential features of the elastic scattering of two protons, when their relative momentum is on the order of 1 GeV/c, are the chirally symmetric confinement of quarks and the absorption of flux into inelastic channels. Perturbation theory is essential to describing the interaction form factors at large momentum transfer, but is incapable of describing the processes which allow quarks to communicate their identities as hadronic constituents. This identity shows no sign of diminishing in importance at these momenta.

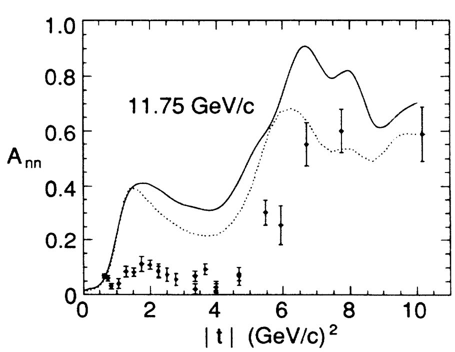

FIG. 6. The spin-correlation parameter at 11.75 GEV/c plotted vs t. The data are from Ref. 1.

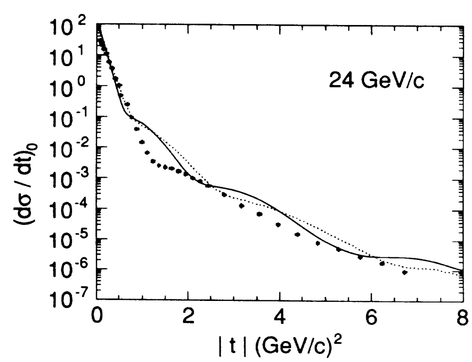

FIG. 8. The unpolarized differential cross section at 24 GeV/c. The data are from Ref. 24.

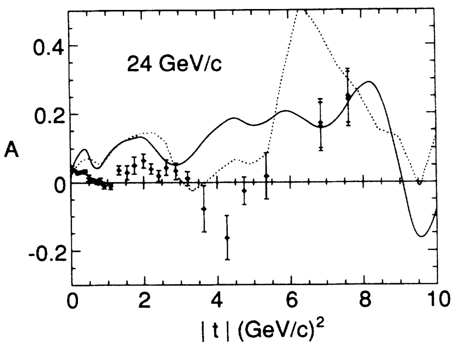

FIG. 9. The asymmetry at 24 GeV/c. The data are from Refs. 3 and 26. The doubled points are 28 GeV/c data.

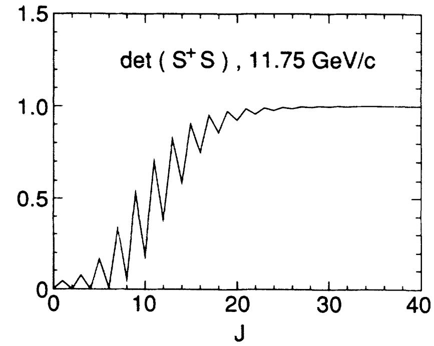

FIG. 10. The determinant of the absolute square of the scattering matrix for a given angular momentum as a function of angular momentum. The S matrix is that which produced the solid curve in Fig. 6. A value of 0 means total absorption, a value of 1 means elastic scattering.

APPENDIX

The eigenspinors of the Dirac equation are

$$u(k,s)=N\begin{bmatrix}1\\ \dfrac{\sigma\cdot\mathbf{k}}{E+m}\end{bmatrix}\chi_s,\quad V(k,s)=N\begin{bmatrix}\dfrac{-\sigma\cdot\mathbf{k}}{E+m}\\ 1\end{bmatrix}\chi_s \qquad \text{(A1)}$$

for a state with momentum **k** and spin given by the Pauli spinor χ_s.

The normalization constant N is determined by the requirement that the scalar density $u^{\dagger}(\mathbf{k},s)\gamma_0 u(\mathbf{k}',s)$ equal 1 when the two momenta **k** and **k**′ are equal, so

$$N^2=(E+M)/2m\ . \qquad \text{(A2)}$$

The only use of σ in the Dirac equation is in terms of the form $\sigma\cdot\mathbf{k}$, so the obvious spin quantization to use is the helicity basis. The helicity eigenvalue λ is defined by

$$\sigma\cdot\mathbf{k}|k,\lambda\rangle=k\lambda|k,\lambda\rangle \qquad \text{(A3)}$$

and takes on values ± 1.

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