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ARTICLES

n-p mass difference and charge-symmetry breaking in the trinucleons

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The effect of unequal neutron and proton masses on the binding-energy difference of ^{3}He and ^{3}H is calculated using a wide variety of nuclear models. We find that this charge-symmetry-breaking mechanism contributes 14(2) keV to the 764 keV difference. We also demonstrate the importance of the mixed-symmetry wave function components in making the contribution of this mechanism fairly small, but rather model dependent.

CHARGE-SYMMETRY BREAKING

Charge symmetry can be viewed as the process of interchanging neutrons and protons. Charge-symmetry breaking (CSB) reflects differences that arise in this process. Because ^{3}He and ^{3}H are charge-symmetry mirror images, differences in their observables often can be directly interpreted in terms of charge-symmetry breaking.[1]

The triton has two n-p pairs and one n-n pair, while ^{3}He has two n-p pairs and one p-p pair. Thus, naively, charge-symmetry breaking in the trinucleons involves the differences between the p-p and n-n pair potentials. Obviously, the Coulomb potential between the two protons is the primary component of the CSB force. The strong n-n and p-p forces are nevertheless different and reflect CSB at an elementary hadronic level. One of the most important recent developments in this field was the $\pi^- d \rightarrow n + n + \gamma$ experiment,[2] which led to a larger value of the $^1S_0$ scattering length for the n-n system (and a more attractive potential) than for the p-p system strong interaction (in the absence of the Coulomb force). Previous experiments based on three-nucleon reactions had suggested the opposite conclusion, although the systematic uncertainty in analyzing these complex reactions remains controversial. Scattering length values of $-18.5(4)$ fm, $-23.75(1)$ fm, and $-17.1(2)$ fm for the n-n,[2] n-$p(T=1)$,[3] and p-p (Ref. 1) systems have been obtained.

The binding-energy difference of ^{3}H and ^{3}He is 764 keV (^{3}H is more bound), of which roughly 640(10) keV is due to the Coulomb interaction and is rather well understood. The latter number was obtained two decades ago using an approximation which allows the Coulomb energy to be calculated from the experimental triton and ^{3}He charge form factors.[4,5] This approximation has been studied[6] and overestimates the Coulomb energy by 1% (6–7 keV); it also miscounts the second-order Coulomb energy by an amount (6 keV) which almost exactly compensates. Direct calculation of the Coulomb energy from first-order and second-order perturbation theory[6] leads to $E_C = 652 - 4 = 648(3)$ keV, a result confirmed recently by Ref. 7. The remaining 120 keV is less well understood, although it can be attributed to a variety of small mechanisms not traditionally included in the N-N forces used to calculate trinucleon properties: magnetic and other small forces between nucleons which are contained in the nucleon-nucleon Breit interaction,[5,7] the different masses of neutrons and protons,[5,7,8] and the aforementioned difference of the p-p and n-n forces. All of these mechanisms are believed to produce contributions to CSB in the trinucleons that have the same sign. Studies of the latter mechanism[7,9,10] suggest that a (n-n)-(p-p) scattering length difference of -1 fm produces CSB of roughly 45(5) keV in the energy difference of ^{3}He and ^{3}H. Thus, the experimental scattering length values listed above would generate 65(20) keV. Magnetic and other Breit-interaction effects have been estimated to be 35(3) keV in size.[7] Our purpose here is to study the effect of the n-p mass difference for a variety of models. Adding this contribution to those listed earlier, one finds that the theoretical prediction is in nearly perfect agreement with experiment, although with a large (20 keV) uncertainty.

KINETIC ENERGY CSB

We assume that the effect of the n-p mass difference is small and can be calculated in perturbation theory. We write the mass of the ith nucleon in the form

$$m_i = m - \Delta m \, \tau_z(i)/2 \, , \tag{1a}$$

where

$$m = (m_n + m_p)/2 \tag{1b}$$

and

$$\Delta m = (m_n - m_p) \tag{1c}$$

produce m_n or m_p from $i = n$ or p in Eq. (1a). The nonrelativistic kinetic energy operator can be written in terms of the individual momenta of the nucleons in the nuclear center-of-mass frame:

$$\hat{T} = \sum_i \frac{\pi_i^2}{2m_i} \cong \sum_i \frac{\pi_i^2}{2m} + \frac{\Delta m}{2m} \sum_i \frac{\pi_i^2}{2m} \tau_z(i) \equiv \hat{T}_0 + \Delta \hat{T} \ . \tag{2}$$

The first term ($\hat{T}_0$) is the usual kinetic energy, while the second (isospin-dependent) term is the CSB interaction, ΔT, which contributes to the mass difference of ^{3}He and ^{3}H in the first-order perturbation theory:

$$\Delta T = \langle {}^3\text{He} | \Delta \hat{T} | {}^3\text{He} \rangle - \langle {}^3\text{H} | \Delta \hat{T} | {}^3\text{H} \rangle \cong 2 \langle \Delta \hat{T} \rangle \ , \tag{3}$$

with equal and opposite contributions from the two trinucleons only if we neglect the Coulomb interaction effect in ^{3}He. Another interesting limit is obtained by assuming that the large space-symmetric S state ($P_S \sim 90\%$) dominates ΔT as well. In that case we can use symmetry to produce

$$\Delta T_S = \frac{\Delta m}{3m} \langle \hat{T}_0 \rangle \ . \tag{4}$$

Using $\Delta m = 1.2935$ MeV, $m = 938.926$ MeV, and $\langle \hat{T} \rangle \sim 55$ MeV, we obtain $\Delta T_S \sim 25$ keV. This is known to be a substantial overestimate, because the mixed-symmetry components of the wave function are important, as we demonstrate below. The reason is that the n-p and n-n (or p-p) forces are somewhat different; because the n-p force is stronger, the unlike nucleon feels a stronger force. Consequently, the wave function (and kinetic energy) associated with the like particles is different from (less than) that associated with the unlike particle. This leads to a substantial reduction in ΔT by the mixed-symmetry S'-state and D-state components in the wave function.

CALCULATIONS AND RESULTS

Naively, there should be a fairly strong dependence of the kinetic energy for a particular model on the binding energy of the triton (E_B) for that model. The uncertainty principle guarantees this because the size decreases as binding increases. Naively, we have for a nucleus with spatial extent, x, and bound-state wave number, κ

$$p^2 \sim 1/x^2 \sim \kappa^2 \sim E_B \ . \tag{5}$$

A plot of the triton kinetic energy for a variety of models versus binding energy is shown in Fig. 1. The uppermost points (circles) are Reid soft core (RSC) models.[11] The filled circles correspond to 34 channels (i.e., "exact"), and those points with binding energy greater than 7.4 MeV include a Tucson-Melbourne three-body force.[12] The solid curve is a simple linear fit to these points, motivated by Eq. (5). The second set of points (squares) correspond to the Argonne V_{14} (AV14) model,[13] with a Brazil three-body force[14] for cases with E_B greater than 7.7 MeV. A

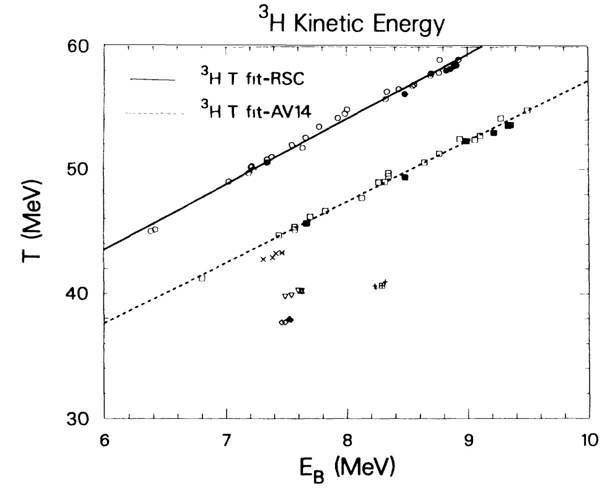

FIG. 1. ^{3}H kinetic energy for a wide variety of models plotted versus the model triton binding energy.

few additional points without a three-body force are shown in four clusters, and correspond to the super soft core C (SSCC),[15] Paris,[16] Bonn,[17] and Nijmegen[18] potential models, with the SSCC model having the smallest value. It is clear from this plot that details of the force, such as the stiffness for small separations, can play a significant role, and extrapolating various parts of these results to the physical $E_B = 8.48$ MeV would not lead to a unique result. A plot of ΔT vs T has a similar structure, showing that Eq. (4) is too naive.

Figure 2 displays ΔT plotted vs E_B for ^{3}H. We have doubled this value and changed its sign so that it can be directly compared to Eq. (4) and the ^{3}He case. Clearly, no model-independent prediction can be obtained. One obtains the extrapolated (to $E_B = 8.48$ MeV) values of $\Delta T = 15.2$ keV (RSC) and 14.2 keV (AV14), and possibly 13 keV for the other group of models. We therefore esti-

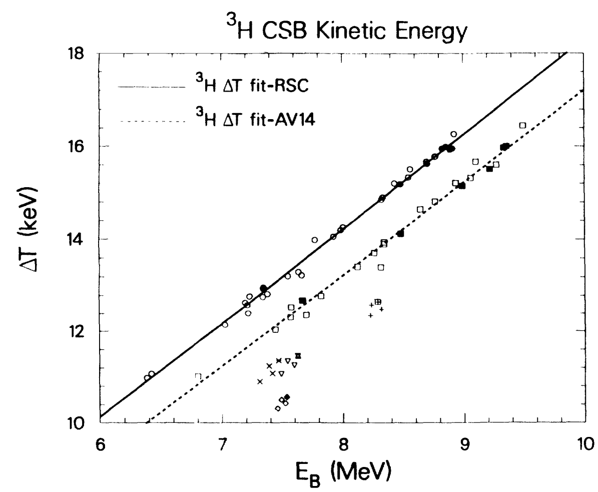

FIG. 2. ^{3}H CSB kinetic energy for a wide variety of models plotted versus the model triton binding energy.

TABLE I. Contributions to the kinetic energy and CSB kinetic energy decomposed according to the symmetry of the wave function component.

	S^2	SS'	S'^2	D^2	Other	Total
T (MeV):	35.3	0	0.7	19.4	0.7	56.1
ΔT (keV):	16.2	−3.2	0.3	1.6	0.5	15.2

mate $\Delta T = 14$ keV with a 10% uncertainty. The ^{3}He case is very similar, but the lines are somewhat offset. The extrapolated (to $E_B = 7.72$ MeV) values of ΔT are 14.6 and 13.6 keV for the RSC and AV14 models with the other models approximately one keV lower than AV14. The effect on ΔT of the much stronger CSB Coulomb interaction is approximately 0.6 keV. Our best estimate for ΔT is the average of the ^{3}He and ^{3}H values according to Eq. (3):

$$\Delta T \cong 14(2)\ \text{keV} . \tag{6}$$

This value is somewhat larger than the recent calculation of Ref. 7, and is rather model dependent. It is also much larger than the 9 keV estimate in Ref. 8 based on separable potentials, and larger than the prediction in Ref. 5 based on a single (underbound) RSC calculation.

Finally, we explicate why this value is much smaller than the naive estimate of 25 keV based (solely) upon the dominant wave function component. We decompose contributions to ΔT and T into S^2-, S'^2-, SS'-, and D^2-wave parts, corresponding to S, S', or D waves or the SS' overlap. Results for an RSC model for ^{3}H which has a Tucson-Melbourne three-nucleon force added are displayed in Table I. The small S' state generates a non-negligible (negative) overlap with the S state. The larger D-state contribution to T is not reflected in ΔT because two (large) separate components cancel, due to the mixed-symmetry nature of the D waves. The resulting values of ΔT are nearly a factor of 2 smaller than the naive S-state estimate. Unlike the S state which is generated by the average N-N force, the mixed-symmetry (including D states) components depend strongly on details of the force, and this causes the model dependence of ΔT.

SUMMARY

We have shown that the n-p mass difference CSB mechanism in the nucleons' kinetic energy is 14(2) keV, larger and more model dependent than previous estimates.

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