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COMMENTS

Comments are short papers which comment on papers of other authors previously published in the **Physical Review**. *Each Comment should state clearly to which paper it refers and must be accompanied by a brief abstract. The same publication schedule as for regular articles is followed, and page proofs are sent to authors.*

Comment on "testing" the Gamow-Teller sum rule

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Several recent papers purport to "test" the Gamow-Teller sum rule. They compare the differences between incomplete strength function integrals for the β^- and β^+ directions with the quantity $3(N-Z)$ and interpret discrepancies as degrees of failure of the sum rule. We point out that the Gamow-Teller sum rule is an exact operator relationship, not a model to be tested. It is useful for determining whether a measured strength function is complete.

Several recent papers purport to "test" the Gamow-Teller sum rule.[1–7] We address this Comment to the one that is published in Physical Review C,[5] but mean it to apply to all of the papers.

Some time ago it was pointed out by Gaarde *et al.*[8] that a certain operator identity involving spinors could be applied to the problem of determining whether empirically measured beta-decay strength functions are complete. The operator relationship was cast into a form that acquired the same "Gamow-Teller sum rule." In this form the relationship reads

$$S(\mathrm{GT}\beta^-)-S(\mathrm{GT}\beta^+)=3(N-Z)\ , \tag{1}$$

where $S(\mathrm{GT}\beta^-)$ is the total β^- Gamow-Teller transition probability originating from a given nuclear state and summed over all final states. It is the β^- strength function integral. $S(\mathrm{GT}\beta^+)$ is the same quantity evaluated in the β^+ direction. N is the number of neutrons in the parent state and Z is the number of protons in the parent state.

The sum rule derives from the properties of the nucleon isospin raising and lowering operators, t^+ and t^-. In the absence of a spin operator, as in Fermi β decay, the sum rule reads

$$S(\mathrm{F}\beta^-)-S(\mathrm{F}\beta^+)=N-Z\ . \tag{2}$$

It is assumed that the isospin component of the state vector for a nucleon is represented by a Pauli spinor. The strengths are sums over all i nucleons and over all j daughter states, $|D_j\rangle$:

$$S(\mathrm{F}\beta^-)=\sum \langle D_j|t_i^-|p\rangle^2\ , \tag{3}$$

$$S(\mathrm{F}\beta^+)=\sum \langle D_j|t_i^+|p\rangle^2\ . \tag{4}$$

For Gamow-Teller transitions the β^- and β^+ operators are taken to be $2st^-$ and $2st^+$ summed over all nucleons. When the spin is neither prepared in the initial state nor measured in the final state, summation over the three directions in the spin space introduces a factor of 3. Although the values of S^- and S^+ individually depend on the structure of the parent state, the sum rule tells us that the difference in strengths depends only on the neutron and proton numbers and not on how the neutrons and protons are built into the parent state. For this reason the sum rule is said to be model independent. For a pedagogical derivation and discussion of the Fermi and Gamow-Teller sum rules, see Ref. 9 and 10.

Gaarde *et al.*[8] pointed out that the GT sum rule could be used to make strong statements about missing GT strength from measurements of the β^- strength function alone. Rearrange the sum rule (1) to read

$$S(\mathrm{GT}\beta^-)=3(N-Z)+S(\mathrm{GT}\beta^+)\ . \tag{5}$$

Since $S(\mathrm{GT}\beta^-)$ and $S(\mathrm{GT}\beta^+)$ are positive quantities, Eq. (5) leads to

$$S(\mathrm{GT}\beta^-)\geq 3(N-Z)\ . \tag{6}$$

A definite correlation between medium-energy (p,n) cross sections, extrapolated to a momentum transfer

$q=0$, and allowed β^--decay rates has been demonstrated.[11,12] Thus, (p,n) measurements provide a means to measure beta-decay sum strengths. Because the strength function is measured only over a limited range of excitation energy and the identification of $L=0$ strength is imperfect, one must use discretion in interpreting the measured sum strength, which we shall call $S(\mathrm{GT}^-)$, as the equivalent of the quantity $S(\mathrm{GT}\beta^-)$ in the sum rule equation (5). However, since the sum rule specifies a minimum strength, if the measured value of $S(\mathrm{GT}^-)$ is less than $3(N-Z)$, one can conclude with certainty that the measured strength function is incomplete; some strength must lie outside the energy domain of the measurement or some strength inside the domain was not identified as such.

Typically, $S(\mathrm{GT}\beta^+)$ is much smaller than $S(\mathrm{GT}\beta^-)$ due to Pauli blocking, and the inequality (6) can be treated as an approximate equality. For some nuclei , notably in the iron region, $S(\mathrm{GT}\beta^+)$ may be large and the measured value of $S(\mathrm{GT}^-)$ can exceed $3(N-Z)$. In these rare cases a measurement of $S(\mathrm{GT}^-)$ alone does not establish the magnitude of the missing strength, and both $S(\mathrm{GT}^-)$ and $S(\mathrm{GT}^+)$ should be measured to discern whether strength is missing. In these regions the sum rule may be of little help in drawing any conclusions about missing strength. Nevertheless, several recent papers purport to "test" the sum rule by taking the difference between empirical β^- and β^+ strength functions.[1-7]

We wish to point out that the difference between two empirically determined and demonstrably incomplete strength functions not only does not test the sum rule, but also has no simply interpretable meaning. The missing strength problem was established with nuclei for which the β^+ strength was expected to be small. In these cases β^- strength functions were measured over the excitation energy range where the shell model places all the strength. In many cases less strength than $3(N-Z)$ was found. The sum rule can properly be used here to argue that additional strength must exist, undetected either because it lies outside the measured energy domain or because it is "hidden" in background.

Now consider a nucleus for which the β^+ strength may be large, as in the iron region. Assume that empirical values for $S(\mathrm{GT}^-)$ and $S(\mathrm{GT}^+)$ are available. It is likely that whatever mechanism quenched the measured β^- strength functions in the cases used to establish the missing strength problem operates here also. Thus, both the β^- and the β^+ strength functions are expected to be incomplete. In essence we already know that some β^- strength evades detection, so there is reason to believe that β^+ strength will evade detection also. Since the sum rule prescribes only the difference between complete strength functions, it provides no model for the difference between two incompletely measured strength functions.

A model has been considered by Delorme *et al.*[13] in which a relationship in the form of (1) can be applied over a limited region of excitation energy. In this model the axial vector coupling constant is locally renormalized and the sum rule becomes model dependent. Both the β^- and β^+ strengths are reduced by the same fraction. It follows that the difference would also be reduced by the same fraction. Accurate measurements of β^- and β^+ strength over the shell-model energy region would be useful in determining whether such a model can yield a satisfactory description of GT quenching. A relevant and outstanding question then is whether the quenching factor is the same for β^+ and β^-.

Some models based on random-phase-approximation calculations place the missing strength in a broadly dispersed continuum that, it is claimed, is mistaken for background in measurements.[14,15] In a particular calculation, Osterfeld *et al.*[15] claim that for ^{90}Zr, GT strength amounting to the difference between the measured value and $3(N-Z)$ is hidden in a broad continuum. They claim they have accounted for all the strength provided that $S(\mathrm{GT}\beta^+)$ is negligibly small. Yen *et al.*[2] measure $S(\mathrm{GT}^+)$ with the (n,p) reaction and find no observable strength. They claim this is a confirmation of the Osterfeld *et al.*[15] model. See also Brady *et al.*[16] for additional discussion on this point.

The measurement of Yen *et al.*[2] in fact, says very little about the Osterfeld model but is interesting in its own right. The ground-state correlations that are invoked to disperse the β^- strength imply the existence of β^+ strength. On the other hand, the Osterfeld model requires negligible β^+ strength to be self-consistent. The Osterfeld model may be consistent with the Yen measurement if that measurement somehow implies only a small amount of dispersed β^+ strength. Ground-state correlations necessary to account for nucleon transfer data imply the existence of β^+ strength. A calculation of Bloom *et al.*[17] place this strength in narrow peaks that should be detectable, although the total calculated β^+ strength is small compared to the total β^- strength. In any case, the nonobservance of this strength most likely signifies that it is dispersed rather than that it is nonexistent.

Various models designed to disperse GT strength can be roughly classified into two groups—those that introduce complex configurations of normal nucleons, and those that include excitations of the nucleons to the delta state. Of course, there are models that combine both features. However, neither the (p,n) nor the (n,p) measurements appear to distinguish between these different approaches.

In summary, we assert that the sum rule relates only the complete β^- strength function with the complete β^+ strength function, it cannot give information about partial strengths in a restricted energy region. Thus, insertion of empirically determined β^- and β^+ strength values for a limited excitation range into the sum rule equation cannot yield meaningful results.

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