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Theoretical angular distributions from coherent subthreshold pion production

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For the first time, we have calculated theoretical angular distributions for exclusive π^0 production in C+C collisions at 84 MeV/nucleon using a quantum-coherent, microscopic, many-body formalism. We use a model spin, transition-spin, Δ-isobar-formation interaction that is expanded in angular momentum multipoles and we now have the results from a complete code that includes these multipoles to all orders which couple to shell-model information. Also the first time, we calculate pion contributions coming from both the target and projectile and find that these contributions are important in determining the shape of the theoretical angular distributions. We compare the theoretical calculations to π^0 data and find that our results are similar to the shape of the data at higher pion energies and bear some similarity in trend to the pion bremsstrahlung model. We also find that the shape of the theoretical angular distribution is sensitive to shell-model information.

There continues to be interest in finding a sensitivity to collective or coherent mechanisms in subthreshold heavy-ion pion production.[1,2] This interest is enhanced by the experimental ability to measure neutral pions[3–5] which leads to data that are not complicated by final-state Coulomb effects on the outgoing pions. The present model interaction has been described briefly in previous works[6,7] where pion spectra were calculated over a range of incident energies from subthreshold to relativistic values for ^{16}O on ^{12}C; however, those calculations used only the lowest-order ($k=0$) angular momentum multipole. We have now included all orders and discovered that the multipole expansion is highly convergent in the subthreshold region. For the first time with this model we calculate the angular distributions for coherent π^0 production in C+C collisions at 85 MeV/nucleon at three different energy cuts.

The Lorentz-invariant differential cross section calculated in the *projectile* rest frame for a given target isospin $M(T)$ is given by

$$\frac{d^3\sigma}{d^3p_\pi/E_\pi}=\frac{2\pi}{\hbar v}\frac{E_\pi V^3}{(2\pi\hbar)^6}\frac{d}{dE_F}\int d^3p_{T^*}\,|C_{FI}|^2\,, \qquad (1)$$

where v is the incident speed of the target as seen in the projectile rest frame, E_π is the total pion energy, and V is the quantization volume which cancels an identical term coming from the second-order amplitude C_{FI}. The total energy of the final state E_F is the sum of energies from the pion, recoil projectile, and excited target of momentum p_{T^*}. Making the high momentum, forward scattering approximation that $p_{T^*}\gg P_{\pi R'}$ where $p_{\pi R'}$ is the momentum of the recoil projectile plus pion as seen from the projectile rest frame, then $d/dE_F\simeq(E_F/p_{T^*}c^2)d/dp_{T^*}$. Furthermore, the scattering amplitude is assumed to be approximately independent of the target solid angle. These assumptions are consistent with peripheral collisions of low recoil and simplify the integrations over the target momentum p_{T^*}. The sums over all target isospins $M(T)$ of the differential cross section are then taken. The second-order amplitude

$$C_{FI}=\frac{\sum_N\langle F|V_2|N\rangle\langle N|V_1|I\rangle}{E_\pi+M_Nc^2-M_\Delta c^2+i\Gamma_\Delta/2}\,, \qquad (2)$$

where $|I\rangle$ refers to the initial state of target, projectile, and the kinetic energy of the target, $|N\rangle$ is the intermediate state where the target is excited to a $J^\pi=1^+$, $T=1$ spin-, isospin-coherent, isobar-analog, nuclear state, and the projectile is excited to a similar state except that a nucleon has been excited to a $\Delta(3,3)$ isobar of mass $M_\Delta c^2=1232$ MeV and width $\Gamma_\Delta=115$ MeV. The final state $|F\rangle$ contains the pion that decays from the Δ and it is assumed that the projectile returns to its ground state. Also, it is assumed that the decay of the Δ from the projectile does not influence the target after the Δ is created so that the center-of-mass plane waves and the internal target wave functions between the intermediate and final states are orthogonal. A nonrelativistic Breit-Wigner propagator for the Δ resonance is used where the energy dependence of the width[8] has been neglected since we are concerned here mainly with angular distributions.

The matrix elements for Δ formation and decay in Eq. (2) are evaluated using the techniques of second quantization and involves a great deal of Racah algebra. The formation matrix $\langle N|V_1|I\rangle$ which excites a particle-hole state in the target and creates a Δ-hole state in the projectile is reduced by Wick's theorem to matrix elements of an interaction between particle hole and scattering states. In order to do an estimate calculation, a separable model Δ-formation interaction was chosen to be

$$v_1=2v_0g(r)g(\xi_p)g(\xi_t)(\mathbf{S}_p\cdot\boldsymbol{\sigma}_t)(\mathbf{T}_p\cdot\boldsymbol{\tau}_t)\times\sum_k\mathbf{C}_k(\Omega_r)\cdot\mathbf{C}_k(\Omega_p)\,, \qquad (3)$$

where $g(r)$, $g(\xi_p)$, and $g(\xi_t)$ are Gaussian shapes which are functions of the coordinate magnitudes of the nucleus-nucleus center of mass, projectile nucleon, and target nucleon, respectively. This form was motivated by the work of Gaarde, Kammuri, and Osterfeld,[9] but simplified to take into account target Lorentz contraction. The main features of this model interaction are the spin and isospin flip where the target is excited to a giant resonance as well as the projectile with the added feature that a Δ is created in the projectile through the transition spin and isospin. The scalar product of spherical tensors of rank k allows for an exchange of orbital angular momentum $\hbar\mathbf{k}$ between the projectile nucleon and target center of mass. The target nucleons, although spin flipped, are left in their spatial ground state so that the matrix element in the target can be approximated by a Lorentz-contracted form factor. This model represents a compromise where orbital angular momentum changes are neglected in the target single-particle states, but allows for a simple handling of the target Lorentz contraction as described previously.[6] The strength of the interaction is given by v_0 which is initially determined from two-body scattering and the factor 2 has been included since the strength of the $\Delta N\pi$ vertex is approximately twice as large as the $NN\pi$ vertex where quark theory gives $g_{\Delta N\pi}=(6\sqrt{2}/5)g_{NN\pi}$.[10] This model represents a step forward compared to our previous model.[11] The calculation of the Δ-decay matrix element $\langle F|V_2|N\rangle$ has also been described previously.[11]

The nuclear, particle-hole states are spin-orbit coupled to produce the j values of the particle and hole and then coupled to produce the total angular momentum of the nucleus. The isospins of the particle-hole state are also coupled to produce the total isospin of the nucleus. Then, linear combinations of these states with particle-hole coefficients $X_{\rm ph}$ are taken to produce the total nuclear state and each matrix element contains $9j$ symbols after lengthy calculations in angular coupling are done. The particle and hole states are generated from the three-dimensional harmonic oscillator and are described in Ref. 6 except that we now apply the model to ^{12}C on ^{12}C. The sum over intermediate states $|N\rangle$ in Eq. (2) contains sums over particle-hole states for both nuclei as well as a sum over angular momentum multipoles k as given by Eq. (1) in Ref. 6 and described therein. This sum contains a product of terms which is given by

$$\sum_k \int_0^\infty j_k(Kr)g(r)r^2dr \int_0^\infty R_{n_\Delta}^{l_\Delta}(\xi)j_k(k_\pi\xi)R_{n_h}^{l_h}(\xi)\xi^2d\xi \begin{pmatrix} l_\Delta & k & l_h \\ 0 & 0 & 0 \end{pmatrix}^2 P_k(\cos\Theta_{\pi K}) , \tag{4}$$

where the first integral is in the Born approximation with respect to the target momentum transfer $\hbar K$, and the second integral comes from the projectile Δ-hole decay matrix element with respect to the outgoing pion momentum $\hbar k_\pi$. The properties of the $3j$ symbol

$$\begin{pmatrix} l_\Delta & k & l_h \\ 0 & 0 & 0 \end{pmatrix}$$

restricts k to even/odd values if $l_\Delta+l_h=$even/odd values and severely truncates the k sum through the triangular condition $|l_\Delta-l_h|\le k\le l_\Delta+l_h$. The pion angular distribution is contained in the Legendre polynomial of order k with respect to the correlation angle $\Theta_{\pi K}$ which is the angle between the pion and target momentum transfer. In previous work,[6] we only hand calculated the differential cross section for the $k=0$ term and only calculated the square of Eq. (4) for $k=0$ plus $k=2$ to determine the relative effect of the higher-order multipole. Presently, we have a complete code which includes all k multipoles and have calculated the differential cross section as a function of the correlation angle. After including higher Δ-hole states, we discovered that the main contributions to the cross section come from the $1p_{1/2,3/2,5/2}$-Δ states and the $1p_{3/2}$-hole state in ^{12}C (Δ) for the $k=0$ and 2 multipole values. For example, when we included the Δ states for the $1d$-$2s$ shell which corresponds to $k=1,3$ and $k=1$, respectively with the $1p_{3/2}$-hole state, we found these contributions to be negligible. This occurs because the overlap between the functions in the first and second integral in Eq. (4) diminishes considerably and the value of the $3j$ symbol

$$\begin{pmatrix} l_\Delta & k & l_h \\ 0 & 0 & 0 \end{pmatrix}$$

drops. It appears that the k sum in Eq. (4) is highly convergent. Also, since we assumed equal weight particle-hole coefficients $X_{\rm ph}(P)$, they become smaller when more shell-model states are included. If these coefficients were calculated in the schematic model,[12] we would find that they are inversely proportional to the particle-hole energy difference which cuts down the contributions from higher shell-model states and justifies our truncation to lowest states. We also included the $1f$ shell and found that these results were also negligible. Therefore, the main contributor to the nonisotropic shape of the differential cross sections comes from P_2 although the actual shape is determined by the relative weighting between P_0 and P_2 in the multipole sum.

For the first time, we compared the results of this model to π^0 angular distributions in ^{12}C+^{12}C collisions at 84 MeV/nucleon transformed to the nucleon-nucleon, center-of-mass system.[13] Since we are considering small momentum transfers in the forward direction, we assume that the correlation angle $\Theta_{\pi K}$ is approximately the pion angle measured from the forward direction Θ_π. For fixed values of the pion kinetic energy and angle $(T_\pi^{\rm c.m.},\Theta_\pi^{\rm c.m.})$ in the nucleon-nucleon center of mass, we calculate the relativistically transformed set (T_π^P,Θ_π^P) for pions produced by the projectile in its own frame to obtain the Lorentz-invariant differential cross section which is further transformed to the noninvariant cross section in the center-of-mass system. A similar procedure is applied to contribution coming from the target. These two cross sections are then added so that our results contain the incoherent addition of pions coming from projectile and target; however, the Δ's are produced coherently in each nucleus. The results are shown in Figs. 1–3 for three different energy cuts. Since we are concerned primarily with shape fits, the calculations have been normalized to the data at $\cos\theta_\pi = 0.025$ since that data point has an error bar shown in Fig. 3. We do not expect to fit the data in absolute value since the calculation is exclusive whereas the data is inclusive. It is interesting to note that the theoretical shape improves at the highest-energy bin from 100–150 MeV where the peak-to-valley ratio moves towards the data. This trend is compatible with the conclusions of Braun-Munzinger and Stachel[1] where they point out that the low-energy pions are probably emitted from a local hot spot, whereas the pions produced approximately above 100 MeV cannot be fully understood by thermal models alone and may suggest the presence of a coherent mechanism in the production process. It is also interesting that our normalization factor drops by a factor of 10 from the lowest-energy cut to the highest perhaps indicating a convergence at the higher pion energies. It is also intriguing that for the lowest-energy cut, the results from the bremsstrahlung model compared to similar data[14] shows a convex trend as do our results under the condition of incoherent addition of projectile and target pions, whereas the bremsstrahlung calculations and our results improve at the higher-energy cuts.

In conclusion, for the first time, we include pion contributions coming from the target and projectile and find that these contributions are important in determining the theoretical shape of the π^0 angular distributions. We compare the present model with experimental π^0 angular distributions but do not expect to obtain absolute-value fits to the data because our calculation is exclusive whereas the data to which we compare are inclusive; however, we wanted to find out if the qualitative shape and energy trend of the theory bears some resemblance to existing experimental angular distributions before embarking on the more complicated phase of this calculation which is to include the tensor term. We find that the theoretical shape displays a forward-backward peaking similar to the shape of the data at the higher pion energies. The forward peaking in our model is due to pions coming predominantly from the projectile whereas the target predominantly contributes to the backward peaking as seen in the nucleon-nucleon center-of-mass system. We expect this qualitative result would also obtain when the tensor interaction is included. The calculations is also very sensitive to the shell-model information since the multipole sum is very convergent and not much of a washing out of the shell-model signature occurs. This was not obvious from previous work[6,7] where hard calculations were done and it was not known if a washout of the angular distribution would occur with the inclusion of higher-order multipole values. We again expect a similar result with inclusion of the tensor term and now have a tentative explanation for the forward-backward peaking seen in the data.

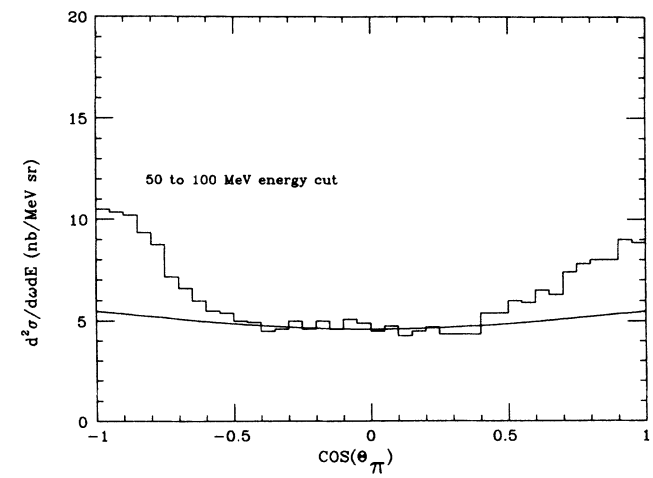

FIG. 2. Same as Fig. 1 except for the 50 to 100 MeV cut.

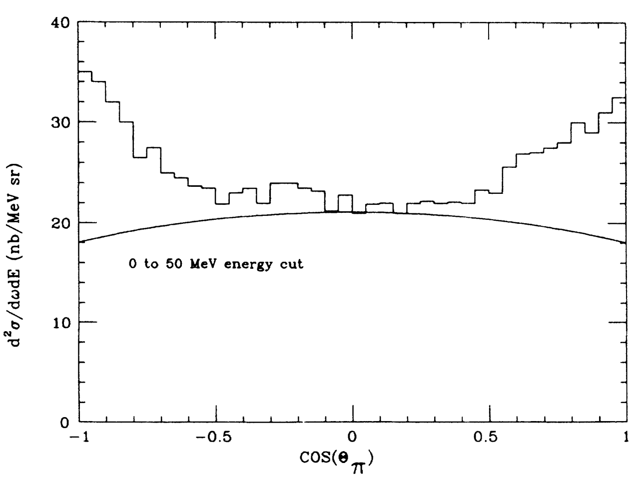

FIG. 1. Theoretical π^0 production compared to the data (Ref. 12) of C+C collisions at 84 MeV/nucleon for the 0 to 50 MeV cut.

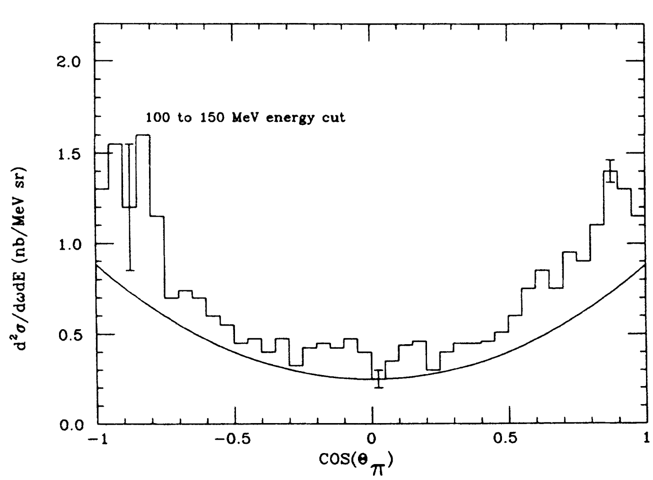

FIG. 3. Same as Fig. 1 except for the 100 to 150 MeV cut.

The model isobar-formation interaction, even though incomplete, leads to a complicated, microscopic, quantum-mechanical, many-body, angular momentum formalism. Our approach was to start with the simpler spin-spin term but include orbital-angular momentum exchanges between projectile and target nucleons as well as the scattering of the relative nucleus-nucleus system and calculate angular distributions to discover the nature of the signature for coherent production. From Ref. 8, in an examination of the momentum transfer dependence q of the central and tensor interactions, it is seen that for low momentum transfers of $q \leq 0.5$ fm^{-1}, the central term dominates over the tensor term. In fact, at $q = 0$, the tensor term is zero. At increasing values of q, the central term drops and the tensor term rises until at the critical value of $q \approx 3$ fm^{-1}, the tensor term dominates the cross section because the central term goes to zero. However, for peripheral collisions at subthreshold energies, it is likely that the lowest values of momentum transfer will be favored and that the central term might dominate these reactions. With this work, we are encouraged to include the more complicated tensor interaction and compare the roles of the central to the tensor term. In Ref. 2, where the tensor term was included, only total cross sections have been calculated for the subthreshold process using the impulse Feshbach-Zabek[15] approach. In our work, we are attempting a more fundamental, microscopic, quantum-mechanical approach. We are presently developing a heavy-ion calculation with a more realistic interaction model that includes the tensor term.

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