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Occupation probabilities and hole-state strengths in nuclear matter

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The interpretation of the nucleon spectral function $P(\mathbf{k},E)$ in terms of two parts, associated with one- and many-particle emission processes, is discussed in connection with the analytical structure of the nucleon momentum distribution $n(\mathbf{k})$. The integrals over E of the one-particle emission part of $P(\mathbf{k},E)$ yield the one-hole strengths. The estimated strengths in the lead region agree fairly well with recent $(e,e'p)$ data.

The quenching of the shell model occupation probabilities with respect to the predictions of the mean-field theory represents one of the cleanest signatures of correlation effects in nuclei. The strong NN interactions give rise to virtual scattering of nucleons to states of energy larger than the Fermi energy, producing a depletion of the normally fully occupied states of the Fermi sea.

Empirical evidence of such a depletion has been obtained from elastic e-nucleus scattering experiments, whose results show that the difference between the charge densities of ^{206}Pb and ^{205}Tl (Ref. 1), as well as the magnetic form factors of ^{205}Tl and ^{207}Pb (Ref. 2), are sizably suppressed with respect to the results of mean-field calculations. Valuable complementary information on the quenching of single-particle occupation probabilities can also be extracted from inelastic processes[3,4] and from the coincidence $(e,e'p)$ cross sections,[5] which yield a direct measure of the nucleon spectral function.[6]

An $(e,e'p)$ experiment on ^{208}Pb with high missing energy resolution has recently been carried out at the Nationaal Instituut voor Kernfysica en Hoge-Energiefysica (NIKHEF).[7] The interpretation of the integrated strengths obtained from this reaction in terms of occupation probabilities $n(e)$ of the states in the vicinity of the Fermi surface leads to a significant disagreement with the theoretical calculations performed by Pandharipande *et al.*,[8] in which short- and long-range NN correlations as well as surface vibrations are taken into account. The experimental results, plotted as a function of the separation energy e, turn out to be similar in shape to the theoretical $n(e)$, but display an extra quenching of more than 20%.[7]

In this paper we show that the experimental data of Ref. 7 should be compared with the hole strengths $Z(e)$ rather than with the occupation probabilities $n(e)$, and that there is indeed good agreement between $Z(e)$ estimated from a fully microscopic nuclear matter calculation and the data.

In plane wave impulse approximation the $(e,e'p)$ cross sections are proportional[6] to the spectral function

$$P(\mathbf{k},E)=\sum_n |\langle \bar{n}|a_k|\bar{0}\rangle|^2\delta(E-E_n+E_0)\,, \qquad (1)$$

where $\mathbf{k}$ and E are the missing momentum and energy in the reaction, $|\bar{0}\rangle$ is the ground state of the target nucleus, $|\bar{n}\rangle$ are eigenstates of the residual $(A-1)$-particle system and E_0 and E_n are the corresponding energy eigenvalues.

The leading contributions to $P(\mathbf{k},E)$ at relatively small values of k are provided by the intermediate states $|\bar{n}\rangle$ which are close to one-hole states of the target nucleus. In fact, for a given momentum $\mathbf{k}$, the experimental $P(\mathbf{k},E)$ shows several peaks, corresponding to the energy eigenvalues of single-particle states. However, other intermediate states, representing two and more nucleon emission processes contribute to $P(\mathbf{k},E)$: they give rise to a background contribution which results to be 2–3 orders of magnitude smaller and very much spread out in energy. Its energy integral measures the existing difference between $n(e)$ and $Z(e)$.

Nuclear matter is a suitable system to estimate such a difference, since the total momentum is a good quantum number. It follows that for $k\le k_F$, the only peak of $P(\mathbf{k},E)$ is the one located at $E=-e(k)$, with $e(k)$ being the excitation energy of the quasihole state $|\bar{\mathbf{k}}\rangle$, and one can separate it from the background tail.

The quasihole strength is given by

$$Z[e(k)]=|\langle \bar{\mathbf{k}}|a_{\mathbf{k}}|\bar{0}\rangle|^2\,, \qquad (2)$$

whereas the occupation probability is the energy integral of the full $P(\mathbf{k},E)$ and therefore results to be

$$n[e(k)]=\sum_n |\langle \bar{n}|a_{\mathbf{k}}|\bar{0}\rangle|^2=\langle \bar{0}|a^\dagger_{\mathbf{k}}a_{\mathbf{k}}|\bar{0}\rangle\,. \qquad (3)$$

Recently $P(\mathbf{k},E)$ has been calculated[9] for a symmetrical nuclear matter at the equilibrium density $\rho=0.16$ fm^{-3}, using the orthogonalized version of the correlated basis function (CBF) theory.[10] In such a theory a set of realistic correlated states having the form

$$|n)=\frac{S\left[\prod_{i<j}F(ij)\right]|n]}{[n|S\left[\prod_{i<j}F^{\dagger}(ij)\right]S\left[\prod_{i<j}F(ij)\right]|n]^{1/2}} \tag{4}$$

is first constructed,[10–12] where $|n]$ are Fermi gas states, $F(ij)$ is a state-dependent two-body correlation operator including spin-isospin and tensor components, and S is the symmetrization operator. The correlation operator has been determined variationally, by minimizing the expectation value of the bare Hamiltonian on $|0)$ for the Urbana $v_{14}+TNI$ model interaction.[13] The Schmidt and Löwdin transformations are then used to generate a set of orthonormal correlated states $|n\rangle$ which are *sufficiently close* to $|n)$ to preserve, in the thermodynamical limit, the diagonal matrix elements of the Hamiltonian $(n|H|n)$ calculated variationally, namely $H_{nn}=\langle n|H|n\rangle=(n|H|n)+O(1/A)$. The orthonormal set $|n\rangle$ is used in a perturbative scheme having an unperturbed Hamiltonian H_0 which has $|n\rangle$ as eigenstates. Such a theory, in conjunction with the Urbana $v_{14}+TNI$ model interaction, has been used in a number of calculations, providing a very realistic description of various properties of nuclear matter, such as the equation of state,[14] the momentum distribution,[15] the nucleon optical potential,[16] and the response functions.[17,18]

We have calculated the hole strengths $Z(e)$ and the occupation probabilities $n(e)$ by integrating over the energy the proper contributions [see Eqs. (2) and (3)] to $P(\mathbf{k},E)$ evaluated in Ref. 9. However, for the sake of brevity, our discussion will be limited here to the terms appearing at the zeroth and the second order of the perturbative expansion.

Let us first discuss the zeroth order of the theory. The two-body breakup contribution to $P(\mathbf{k},E)$ is obtained by taking $|n\rangle=|\mathbf{k}\rangle$ as the only intermediate state, namely

$$P^{(0)}(\mathbf{k},E)=|\Phi_{\mathbf{k}}(\mathbf{k})|^2\delta[E+e^{v}(k)]\theta(k-k_F)\,, \tag{5}$$

where $\Phi_n(\mathbf{k})=\langle n|a_{\mathbf{k}}|0\rangle$ and $e^{v}(k)=H_{00}-H_{\mathbf{kk}}$. As a consequence, $Z^{(0)}[e(k)]=|\Phi_{\mathbf{k}}(\mathbf{k})|^2$, which amounts to take $|\bar{\mathbf{k}}\rangle=|\mathbf{k}\rangle$ in Eq. (2). The following comments are in order: (i) short-range correlations are already included at the zeroth order of the theory; in fact, $Z^{(0)}(e)$ is substantially quenched with respect to unity (see Table I) and $e^{v}(k)$ is already in satisfactory agreement with the experimental data on the nucleon optical potential;[16] (ii) the cluster expansion of $\Phi_{\mathbf{k}}(\mathbf{k})$[9] reveals interesting similarities with that of the variational momentum distribution[15] $n^{v}(k)$, which is given by the sum of a *discontinuous* part $n_d(k)\theta(k-k_F)$ and a second part $n_c(k)$ which is *continuous* at $k=k_F$. It turns out that $|\Phi_{\mathbf{k}}(\mathbf{k})|^2$ coincides with $n_d(k)$, which therefore has to be interpreted as the variational (or zeroth order) estimate of $Z[e(k)]$. In fact, $|\Phi_{\mathbf{k}}(\mathbf{k})|^2$ equals the discontinuity of the momentum distribution across the Fermi surface and this, according to the Migdal theorem,[19,20] coincides with the strength $Z(e_F)$.

The processes with $2h\,1p$ intermediate states $|\mathbf{p}_i\mathbf{h}_i\mathbf{h}_i'\rangle$, which are forbidden in an uncorrelated system, give rise to the background contribution to $P^{(0)}(\mathbf{k},E)$. This contribution is nonvanishing for $k>k_F$, indicating the possibility of knocking out nucleons from normally empty states outside the Fermi sea, and extends over a wide range of missing energy. As a consequence, in a mean-field-type calculation very large single-particle basis are required to include the effects of short-range correlations in a realistic way.

The analysis of $P(\mathbf{k},E)$ in terms of the one-hole state part and a background can also be carried out at the second order of the CBF expansion. Second-order perturbation theory is needed to correct the long-range behavior of the correlation $F(ij)$ which is poorly determined by the variational calculation. In fact, long-range correlations do not contribute significantly to the expectation value of the Hamiltonian H_{00}; nevertheless, they play an important role, like for instance, explaining the behavior of $n(e)$ around the Fermi energy e_F.[15] Since short-range correlations are already correctly embedded at the zeroth order, by means of a Fermi-hypernetted-chain treatment which includes all the important many-body cluster terms, second-order CBF theory is expected to be a reliable approximation to evaluate $P(\mathbf{k},E)$.

The inclusion of $2p\,2h$ admixtures into $|0\rangle$ leading to

$$|\bar{0}\rangle\sim\frac{\left[|0\rangle+\frac{1}{4}\sum_i\alpha_0(i)\,|\mathbf{p}_i\mathbf{p}_i'\mathbf{h}_i\mathbf{h}_i'\rangle\right]}{\left[1+\frac{1}{4}\sum_i|\alpha_0(i)|^2\right]^{1/2}}\,, \tag{6}$$

where

$$\alpha_0(i)=\langle\mathbf{p}_i\mathbf{p}_i'\mathbf{h}_i\mathbf{h}_i'|H|0\rangle/(H_{00}-H_{ii})$$

gives rise to the so-called ground-state corrections $\delta P_{\rm gr}(\mathbf{k},E)$. The corresponding terms involving the $1h$ intermediate state $|\mathbf{k}\rangle$ give a contribution for $k\le k_F$ only, whereas terms with $2h\,1p$ intermediate states provide a correction at $k>k_F$ which is all background. The explicit expressions of $\delta P_{\rm gr}(\mathbf{k},E)$ are given in Ref. 9 and will not be repeated here. The integration of $\delta P_{\rm gr}(\mathbf{k},E)$ over the missing energy E exactly yields the perturbative corrections to $n^{v}(k)$ originally derived in Ref. 15:

$$\delta n_{\rm gr}(k<k_F)=\delta Z_{\rm gr}(k)=-\tfrac{1}{2}\sum_i|\alpha_0(\mathbf{p}_i\mathbf{p}_i'\mathbf{h}_i\mathbf{k})|^2|\Phi_{\mathbf{k}}(\mathbf{k})|^2-\tfrac{1}{2}\sum_i[\alpha_0(\mathbf{p}_i\mathbf{p}_i'\mathbf{h}_i\mathbf{k})\langle\mathbf{p}_i\mathbf{p}_i'\mathbf{h}_i\mathbf{k}|a_{\mathbf{k}}^{\dagger}|\mathbf{k}\rangle\Phi_{\mathbf{k}}(\mathbf{k})\ \text{c.c.}]\,, \tag{7}$$

$$\delta n_{\rm gr}(k>k_F)=\tfrac{1}{2}\sum_i|\alpha_0(\mathbf{p}_i\mathbf{k}\mathbf{h}_i\mathbf{h}_i')|^2|\langle\mathbf{p}_i\mathbf{k}\mathbf{h}_i\mathbf{h}_i'|a_{\mathbf{k}}^{\dagger}|\mathbf{p}_i\mathbf{h}_i\mathbf{h}_i'\rangle|^2-\tfrac{1}{2}\sum_i[\alpha_0(\mathbf{p}_i\mathbf{k}\mathbf{h}_i\mathbf{h}_i')\langle\mathbf{p}_i\mathbf{k}\mathbf{h}_i\mathbf{h}_i'|a_{\mathbf{k}}^{\dagger}|\mathbf{p}_i\mathbf{h}_i\mathbf{h}_i'\rangle\Phi_{\mathbf{p}_i\mathbf{h}_i\mathbf{h}_i'}(k)+\text{c.c}]\,. \tag{8}$$

The intermediate state corrections $\delta P_{\rm int}(\mathbf{k},E)$ are due to the coupling

$$\alpha_{\mathbf{k}}(i)=\langle\mathbf{p}_i\mathbf{h}_i\mathbf{h}_i'|H|\mathbf{k}\rangle/(H_{\mathbf{kk}}-H_{ii})$$

between $|\mathbf{k}\rangle$ and the $2h1p$ states $|\mathbf{p}_i\mathbf{h}_i\mathbf{h}_i'\rangle$. One can easily verify that, up to quadratic terms in α, the $2h1p$ admixtures do not contribute to $\sum_n |\langle n|a_{\mathbf{k}}|0\rangle|^2$, but the depletion of the one-hole strength

$$\delta Z_{\rm int}(\mathbf{k}) = |\langle\bar{\mathbf{k}}|a_{\mathbf{k}}|0\rangle|^2 - |\langle\mathbf{k}|a_{\mathbf{k}}|0\rangle|^2$$

is given by

$$\delta Z_{\rm int}(\mathbf{k}) = -\tfrac{1}{2}\sum_i |a_{\mathbf{k}}(i)|^2 |\Phi_{\mathbf{k}}(\mathbf{k})|^2 + \tfrac{1}{2}\sum_i [a_{\mathbf{k}}(i)\langle 0|a_{\mathbf{k}}^\dagger|\mathbf{p}_i\mathbf{h}_i\mathbf{h}_i'\rangle\Phi_{\mathbf{k}}(\mathbf{k}) + \text{c.c.}] . \quad (9)$$

The remaining background contribution $\delta n_b(\mathbf{k}) = -\delta Z_{\rm int}(\mathbf{k})$ turns out to be the analytical continuation of $\delta n_{\rm gr}(k>k_F)$ for $k<k_F$, namely $\delta n_{\rm gr}(k_F^+) = -\delta Z_{\rm int}(k_F^-)$. As a consequence, the hole-state strength $Z(\mathbf{k})$ can be written as

$$Z(\mathbf{k}) = |\Phi_{\mathbf{k}}(\mathbf{k})|^2 + \delta Z_{\rm gr}(\mathbf{k}) + \delta Z_{\rm int}(\mathbf{k}) , \quad (10)$$

and the nucleon momentum distribution results to be given by the sum of two terms: $Z(\mathbf{k})$ which vanishes for $k>k_F$ and $n_c(\mathbf{k})$, associated with the background part of $P(\mathbf{k},E)$, which is continuous at $k=k_F$.

It should be pointed out that $Z(\mathbf{k})^{-1}$ can be expressed in terms of the *so-called-E* mass,[20] namely

$$Z(\mathbf{k})^{-1} = m_E = [1 - \partial\,\mathrm{Re}\Sigma(\mathbf{k},E)/\partial E]_{E=-e(k)} , \quad (11)$$

$\Sigma(\mathbf{k},E)$ being the proper self-energy. There are formal similarities between the CBF corrections to $Z(\mathbf{k})$ given here and those provided[20-22] by G-matrix theory. An important feature of the present approach is that $1-|\Phi_{\mathbf{k}}(\mathbf{k})|^2$ as well as the second term on the right-hand side of Eqs. (7)-(9) include short-range correlation effects which are taken into account only at high perturbative orders in G-matrix theory. Recent calculations[23] performed in nuclear matter at the second order in G-matrix give values much too high for the effective mass m^* and the background as compared to those found in the present work.

The breakdown of the various terms contributing to $Z(e)$ is reported in Table I, whereas in Fig. 1 $Z(e)$ and $n_c(e)$ resulting from our calculations are displayed together with the full occupation probability distribution $n(e)$. It clearly appears that $Z(e)$ and $n(e)$ appreciably differ, particularly in the vicinity of e_F.

The difference between $Z(e)$ and $n(e)$ points to the problem of the interpretation of the $(e,e'p)$ data in ^{208}Pb

TABLE I. Breakdown of the one-hole strength $Z(k)$ defined in Eq. (8). All quantities are multiplied by a factor of 100.

k (fm^{-1})	$1-|\Phi_k(k)|^2$	$\delta Z_{\rm gr}(k)$	$\delta Z_{\rm int}(k)$	$Z(k)$
0.892	12.66	−5.2	−1.4	80.7
0.986	12.56	−5.8	−4.1	77.5
1.072	12.45	−6.4	−6.9	74.3
1.152	12.33	−7.3	−9.0	71.4
1.226	12.21	−8.4	−9.9	69.5
1.296	12.08	−9.9	−8.4	69.6

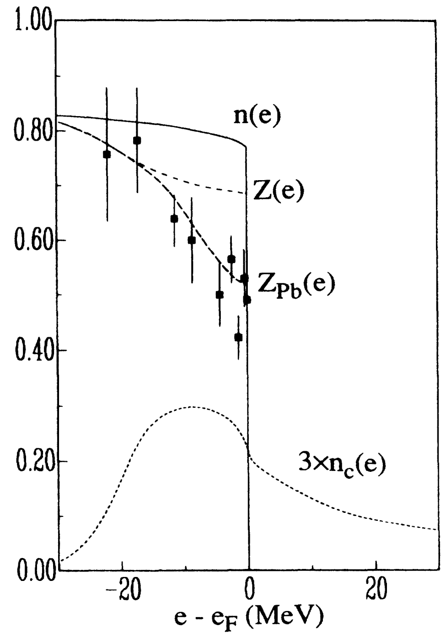

FIG. 1. Occupation probabilities $n(e)$ and hole-state strengths $Z(e)$. The nuclear matter results $n(e)$, $n_c(e)$, and $Z(e)$ have been obtained at $k_F=1.33$ fm^{-1}. The dashed line gives the estimated $Z(e)$ in ^{208}Pb. Experimental data are taken from Ref. 7.

in terms of shell model occupation probabilities. The fact that the observed integrated strengths are a factor ~0.8 smaller than the theoretical occupation probabilities could be regarded as a possible signature of a modification of the elementary eN cross section in the nuclear medium.[7] However, because of its magnitude and shape, it is unlike-

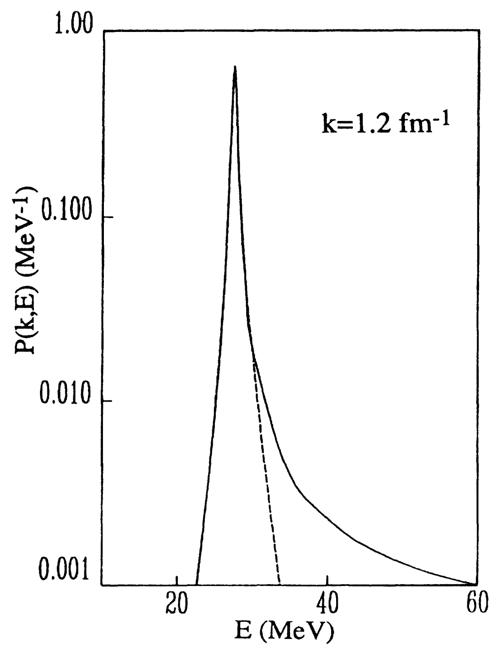

FIG. 2. Spectral function of nuclear matter at $k=1.2$ fm^{-1}. The dashed line represents the two-body breakup contribution.

ly that the background contribution has been included into the observed integrated strengths in ^{208}Pb. Moreover, we have shown that the energy integral of the background in nuclear matter is ~ 0.1 in the vicinity of the Fermi momentum, and this value roughly accounts for the discrepancy between the observed strengths and the theoretical estimates of $n(e)$.[8,9] To better stress this point in Fig. 2, the spectral function at $k = 1.2$ fm^{-1} is displayed: the area below the dashed curve, representing the two-body breakup contribution to $P(\mathbf{k},E)$, is 0.7, whereas $n(k) = 0.82$.

A fully realistic calculation of $Z(e)$ in ^{208}Pb should include a consistent treatment of surface effects. An estimate of the coupling between single-particle states and surface vibrations[24] within the present approach has been obtained by modifying the imaginary part of the CBF self-energy[9] in such a way to reproduce the measured widths[7] of the states around the Fermi surface. The E behavior of the CBF self-energy around e_F weakly depends upon k and has been described using the empirical parametrization $\mathrm{Im}\Sigma \sim W_0(e-e_F)^2$, with $W_0 = 0.024$ MeV^{-1} for $|e-e_F| < 10$ MeV. Figure 1 shows that there is a fair agreement between the theoretical estimates of $Z(e)$ and the experimental data of Ref. 7. Although a fully quantitative comparison with the data would require a more accurate treatment of finite size effects, two main conclusions can be drawn from the results of our calculations: (i) the relatively low missing energy spectra measured at NIKHEF, corresponding to single nucleon knock-out processes, provide a clean evidence of correlation effects; (ii) the empirical integrated strengths do not include the background contribution and therefore have to be regarded as a measure of hole strengths rather than occupation probabilities.

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