

s -wave partial cross sections for the reaction $p + p \rightarrow p + p + \pi^0$

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We have studied the reaction $p + p \rightarrow p + p + \pi^0$, near threshold, at five energies between 320 and 500 MeV, by detecting the decay γ rays of the π^0 in coincidence in two large NaI crystals. The π^0 energy and angular distributions were measured from which we determine that the s -wave contribution to the total cross section is $\sigma_{11}^s(\mu\text{b}) = (15.2 \pm 3.0)\eta_0^2$. This is somewhat lower than results of previous experiments.

The pion, even 40 years after its experimental discovery remains somewhat of an enigma. Because of its low mass, it exhibits chiral symmetry properties which have been utilized in the traditional soft-pion calculations. With corrections for chiral symmetry breaking, calculations can be made near threshold for s -wave scattering or production. Recently, pion nucleon and pion nucleus scattering near threshold have been intensively studied, yet many problems still exist.^{1,2} Related studies on $\pi\pi$ scattering³ are limited by the difficulty of doing the experiments. Recent results in the photomeson reaction $\gamma p \rightarrow \pi^0 p$ have shown that previous experiments overestimated the s -wave production.^{4,5} We have reinvestigated the related nucleon-nucleon reaction $p + p \rightarrow p + p + \pi^0$, and have found a similar situation. Because the cross section is very small, and π^0 detection is difficult, this channel has been neglected for many years. The present study has obtained far more information than the earlier experiments, and is thus better able to eliminate the contribution from higher partial waves. Our results have relevance to many topics, especially the nucleon-nucleon interaction, because this reaction is one of the four basic isospin transitions, i.e., σ_{11} (the transition where both the initial and final isotopic spins of the two-nucleon system are one). This transition contributes to the pion production in neutron-proton reactions, and thus is related to pion absorption on $T=1$ pairs in nuclei. In addition there are auxiliary uses, for example, a recent application of this reaction was to light axion production in nucleon-nucleon collisions.⁶

The cross section for the reaction $p + p \rightarrow p + p + \pi^0$ can be broken down into several "intensity classes" in terms of the angular momentum of the final two-nucleon system and the angular momentum of the pion.⁷ It has been shown that near threshold $l \geq 2$ could be neglected, and the final-state partial waves can be classified in order of decreasing intensity as Sp , Ss , Pp , and Ps . The first letter of an intensity class (S, P) refers to the relative angular momentum of the two nucleons in the final state, while the second letter (s, p) refers to the angular momentum of the pion with respect to the center of mass of the two nucleons. For the reaction $pp \rightarrow pp\pi^0$ the Sp component which plays a major role in other meson production reactions is forbidden due to spin and parity con-

siderations. The absence of the Sp class is responsible for the relatively small cross sections near the threshold for this reaction. Also it turns out that it is impossible to explain the experimental excitation function by invoking Ss alone.⁷ Therefore, for σ_{11} only, reactions of classes Pp and Ps must be considered, even near threshold.

Differential energy spectra of the pions produced in the $NN \rightarrow NN\pi$ reaction have been first calculated by Gell-Mann and Watson.⁸ For the reaction $pp \rightarrow pp\pi^0$ at low energies the energy spectra for the pion are

$$\left(\frac{d\sigma_{11}}{dT} \right)_{Ss} \propto \eta \frac{(T_0 - T)^{1/2}}{T_0 - T - B'}, \quad (1)$$

$$\left(\frac{d\sigma_{11}}{dT} \right)_{Pp} \propto \eta^3 (T_0 - T)^{3/2}, \quad (2)$$

$$\left(\frac{d\sigma_{11}}{dT} \right)_{Ps} \propto \eta (T_0 - T)^{3/2}, \quad (3)$$

where η is the pion momentum in units of $M_{\pi c}$ (140 MeV/c), T is the kinetic energy of the π^0 , T_0 is the maximum kinetic energy available for the π^0 in the center-of-mass system, and B' is the energy of the two nucleons in a virtual 1S_0 state. B' is considered to be essentially zero (≈ 60 keV). Integrating (1), (2), and (3), the total cross sections near threshold take the form

$$(\sigma_{11})_{Ss} \propto \eta_0^2, \quad (\sigma_{11})_{Pp} \propto \eta_0^8, \quad (\sigma_{11})_{Ps} \propto \eta_0^6, \quad (4)$$

where η_0 is the maximum pion momentum. Using the contributions from the three classes, the total cross section can thus be described by an excitation function of the form

$$\sigma_{11} = B_1 \eta_0^2 + B_2 \eta_0^6 + B_3 \eta_0^8, \quad (5)$$

where B_1 , B_2 , and B_3 are empirical parameters which provide a measure of the partial cross sections from the three classes Ss , Ps , and Pp .

Early attempts to determine these parameters used only total-cross-section data. Stallwood *et al.*,⁹ who studied the reaction $pp \rightarrow pp\pi^0$ from 346 to 437 MeV, obtained

$$\sigma_{11}(\mu\text{b}) = (19 \pm 6)\eta_0^2 + (62 \pm 15)\eta_0^6. \quad (6)$$

Dunaitsev and Prokoshkin¹⁰ performed a similar experiment in the proton energy range of 313–665 MeV. For energies less than 400 MeV they found as their best fit

$$\sigma_{11}(\mu\text{b}) = (32 \pm 7)\eta_0^2 + 40\eta_0^6 + 47\eta_0^8. \quad (7)$$

Thus, the existing experimental values of the B parameters are somewhat inconsistent.

Our experiment was performed in the proton beam line 1B at TRIUMF. The reaction $pp \rightarrow pp\pi^0$ was studied by bombarding a liquid-hydrogen target with the external proton beam, and detecting the two γ rays from the π^0 decay, in coincidence, in a two-armed NaI spectrometer. By varying the position of the arms of the spectrometer, the energy and the angular distribution of the π^0 were measured. The experiment was run at five proton energies 320, 350, 403, 450, and 497 MeV. Details of the experiment are given in Ref. 11 and will be published in a later paper.

The partial cross sections of the three classes were determined by fitting the measured π^0 energy spectra with functions derived from the center-of-mass energy spectra of Gell-Mann and Watson [Eqs. (1)–(3)]. The “theoretical” laboratory spectra for each of the three classes were created using a Monte Carlo simulation. For each class, π^0 mesons were randomly generated with an isotropic center-of-mass angular distribution; for the Pp class an additional $\cos^2\theta$ distribution was added. The energy and the direction of the mesons were transformed from the center-of-mass frame to the laboratory frame, and the energy spectra at the experimental laboratory angles were determined. These laboratory spectra were then corrected for the detection efficiency and the response of the spectrometer. These theoretical energy spectra, along with background terms, were then fitted to the experimental energy spectra. This was done using the CERN minimization routine MINUIT.

Now, if we separate the isotropic and $\cos^2\theta$ distributions of the Pp class, then Eq. (5) can be written as

$$\sigma_{11} = B_1\eta_0^2 + B_2\eta_0^6 + B_3\eta_0^8 + \frac{B_4}{3}\eta_0^8, \quad (8)$$

where $B_3\eta_0^8$ and $B_4\eta_0^8$ are the contributions from the isotropic and $\cos^2\theta$ angular distributions of the Pp class.

In the analysis two types of fits were performed. (1) Simultaneous fits to all the spectra of a single proton energy (individual energy fits) and (2) simultaneous fits to all the spectra at all proton energies (global fits). The B_i ($i=1,2,3,4$) parameters of the excitation function can be deduced from both types of fits. The results obtained

from individual energy fits are listed in Table I and it is clear that there are significant variations. Low energies are insensitive to classes Ps and Pp , whereas the high energies are insensitive to the class Ss . This clearly shows that one cannot determine the B_i parameters unambiguously using only one proton energy. Thus we decided to fit all the data simultaneously. The only disadvantage is that we now rely on the validity of the energy dependence [Eq. (8)].

There were 46 spectra, so this global fit contained 96 parameters most of which are related to energy calibration and to background. In order to simplify the fit and to obtain an unambiguous result, all the spectra for each proton energy were first fitted individually, and then, in the final fits, all the parameters related to the calibration and the background were fixed at the values obtained from the individual fits. This left only the four B_j parameters to be fitted. Since Gell-Mann and Watson’s functions are valid only at low energies, but it is difficult to specify the range precisely, two fits were performed, one for the region 319 to 402 MeV, and the other from 319 to 450 MeV.

From the global fit for 319–402 MeV, we obtained

$$\sigma_{11} = (15.2 \pm 0.4)\eta_0^2 + (22.8 \pm 1.8)\eta_0^6 + (57.0 \pm 2.7)\eta_0^8, \quad (9)$$

from the global fit for 319–450 MeV, we obtained

$$\sigma_{11} = (15.2 \pm 0.4)\eta_0^2 + (11.4 \pm 1.2)\eta_0^6 + (74.4 \pm 1.6)\eta_0^8. \quad (10)$$

Although both global fits give identical values for the parameter B_1 , the values obtained for B_2 and B_3 do not overlap within errors in the two fits. This is an indication that our data are unable to completely distinguish between Ps and Pp classes. Both the energy spectrum and the energy dependence are similar. Hence, using the two sets of values given in Eqs. (9) and (10), we have obtained an average set for B_2 and B_3 . They have been assigned a larger error to cover the results from both fits. The error of B_1 was determined by estimating its sensitivity to the constraints from the high-energy data. With these, the overall result obtained from our experiment is estimated to be

$$\sigma_{11}(\mu\text{b}) = (15.2 \pm 3.0)\eta_0^2 + (17 \pm 8)\eta_0^6 + (66 \pm 11)\eta_0^8. \quad (11)$$

The energy dependence of the total cross sections of the

TABLE I. Integrated contributions from the three classes analyzed independently at each energy.

Proton Energy (MeV)	η_0	σ_i^s/η_0^2 (B_1)	σ_i^p/η_0^6 (B_2)	σ_i^{pp}/η_0^8 (B_3)	σ_i^{pp}/η_0^8 (B_4)
496	1.333	18.7 ± 10.7	0.0 ± 0.0	60.1 ± 1.7	6.9 ± 0.7
450	1.157	0.0 ± 0.0	0.0 ± 0.0	84.5 ± 1.8	12.2 ± 0.9
402	0.957	0.0 ± 0.0	14.8 ± 1.8	76.7 ± 2.8	15.0 ± 1.1
349	0.699	8.8 ± 2.3	43.7 ± 25.7	147 ± 60	17 ± 12
319	0.517	8.6 ± 1.5	283 ± 283	1 ± 630	16 ± 262

three classes obtained from the above excitation function is shown in Fig. 1. It is clear that near threshold the *Ss* transition is the dominant cross section, but as the energy increases its contribution becomes less important due to the rapid increase of *Pp*. The contribution from *Ps* is less important and is around 20%. It is important to emphasize that our result of $(15.2 \pm 3.0) \mu\text{b}$ relies on the validity of the assumptions made by Gell-Mann and Watson.⁸ It is possible that rescattering corrections would slightly modify their findings. The experimental results are not sufficiently extensive nor precise to test the functional form of the pion energy spectrum. (Of course, all previous experiments also based their analyses on these approximations.)

To compare the individual energy and the global fits it was decided to refit the 319 and 349 MeV data, but this time constraining the *p* waves to the values obtained from the global fits. These fits gave the following *s*-wave total cross sections which are in good agreement with those obtained from global fits.

$$319 \text{ MeV: } \sigma_{\pi^0}^{Ss}(\mu\text{b}) = (14.8 \pm 0.6)\eta_0^2, \quad (12)$$

$$349 \text{ MeV: } \sigma_{\pi^0}^{Ss}(\mu\text{b}) = (14.1 \pm 1.4)\eta_0^2. \quad (13)$$

This demonstrates the consistency of the data at 319 and 349 MeV and indicates that these two energies are the ones that provide the information which constrain the *Ss* contribution.

An alternative approach for estimating the contributions of the three classes is to fit the excitation function (5) directly to the total cross sections alone. This method was used in the past because the pion energy spectra were not available, but it ignores the valuable information

available in the energy spectra. Nevertheless, to compare our results with the previous measurements, a fit was performed to the total cross sections only using (5). The results of this fit for the total cross sections from 319–450 MeV gave

$$\sigma_{\pi^0} = (15.6 \pm 6.1)\eta_0^2 + (0.0 \pm 13.4)_{0.0}^{\eta_0^6} + (79.8 \pm 5.5)_{5.5}^{\eta_0^8}. \quad (14)$$

In order to compare our fits with the results of the previous measurements, total-cross-section fits were performed to the measurements of Dunaitsev and Prokoshkin¹⁰ and also of Stallwood *et al.*⁹ in the energy region below 450 MeV, and the results obtained were

$$\text{Dunaitsev: } \sigma_{\pi^0}(\mu\text{b}) = (23.9 \pm 10.4)_{8.2}^{\eta_0^2} + (82 \pm 11)_{50}^{\eta_0^6} + (0 \pm 37)_{0}^{\eta_0^8}, \quad (15)$$

$$\text{Stallwood: } \sigma_{\pi^0}(\mu\text{b}) = (22.6 \pm 14.9)_{19.1}^{\eta_0^2} + (21 \pm 10)_{21}^{\eta_0^6} + (38 \pm 38)_{38}^{\eta_0^8}. \quad (16)$$

The *Ss* contributions given by these fits, though of very poor quality, are in agreement with our measurements. Although our fit to the data of Dunaitsev and Prokoshkin gives the *Ss* parameter as (23.9 ± 10.4) , the number given by the authors is 32 ± 7 . They obtained this parameter by assuming the Mandelstam model¹² and fixing the *p*-wave parameters at the values given by the model. Hence the value given by them clearly depends on the validity of Mandelstam's model and cannot be compared with our result. In Fig. 2 the excitation function obtained from the global fit to our data [Eq. (11)] as well as from the fits to the data of Dunaitsev and Prokoshkin and of Stallwood *et al.*

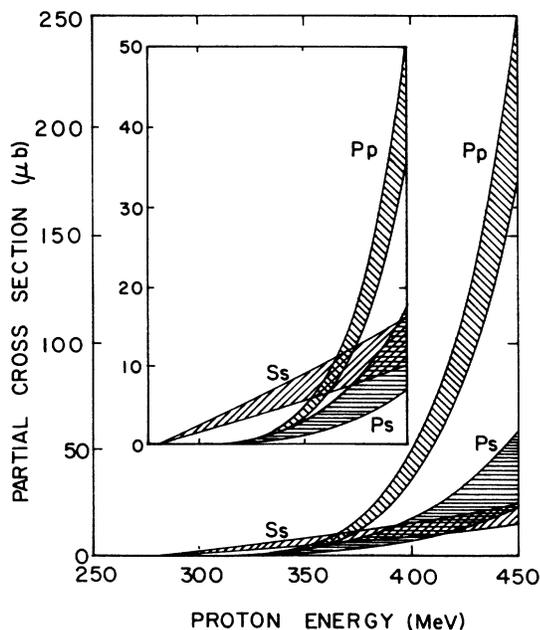


FIG. 1. The energy dependence of the *Ss*, *Ps*, and *Pp* partial cross sections for the reaction $pp \rightarrow pp\pi^0$ from Eq. (11). The shaded areas represent the uncertainties. The inset enlarges the energy region below it.

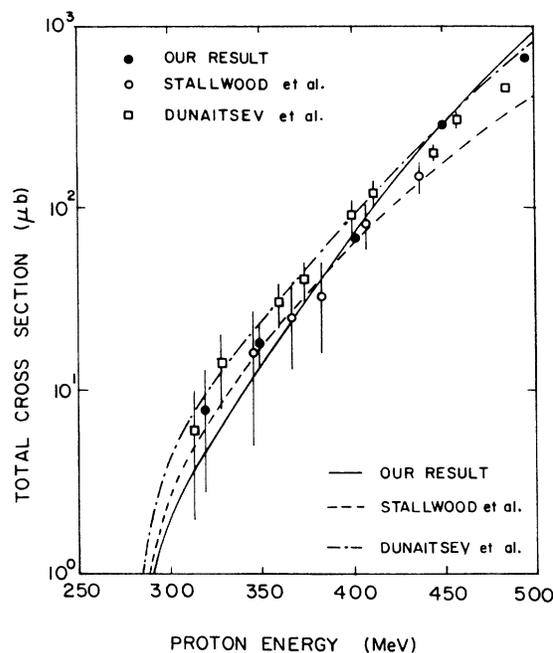


FIG. 2. The total cross section for the reaction $pp \rightarrow pp\pi^0$. Our fits [Eq. (11)] and those of Stallwood *et al.* (Ref. 9) and Dunaitsev and Prokoshkin (Ref. 10) are also shown.

al. are shown. The fits obtained from earlier experiments appear to deviate significantly from ours, but it should be remembered that their fits have large uncertainties, both statistical and systematic.

It is interesting to compare the s -wave production of pions in the reaction $pp \rightarrow pp\pi^0$ with other reactions. For the reaction $pp \rightarrow \pi^+d$ (or $np \rightarrow \pi^0d$) the equivalent relation is $\sigma(pp \rightarrow \pi^+d) = \alpha\eta + \beta\eta^3$, with α being the s -wave contribution, which has the value of $\sim 200 \mu\text{b}$ (Ref. 13), much higher than in our reaction, mainly because of re-scattering effects. For $np \rightarrow \pi^-pp$ there is evidence only from Handler,¹⁴ the s -wave cross section is clearly small, but not clearly separated from other contributions. As mentioned before, recent results for $\gamma p \rightarrow \pi^0p$ also find a lower s -wave contribution⁴ than had been found previously.

Only a few of the theoretical calculations make specific predictions for the reaction $pp \rightarrow pp\pi^0$. The s -wave soft

pion calculations done by Efrosinin *et al.*¹⁵ predict B_1 to be about $10 \mu\text{b}$. The nonrelativistic calculations of Koltun and Reitan¹⁶ for s -wave pion production near threshold found B_1 to be $17 \mu\text{b}$. The field theoretical model calculations of Hachenberg and Pirner¹⁷ gave the s -wave parameter as $B_1 = 65 \mu\text{b}$, which is about four times larger than our experimental result, but the improvements suggested by Efrosinin *et al.* reduces the value to $11\text{--}18 \mu\text{b}$, which is in better agreement with experiment. Our results should act as a stimulus to further study of the threshold production of pions.

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¹V. P. Efrosinin and D. A. Zaikin, *Fiz. Elem. Chastits At. Yadra* **16**, 1330 (1985) [*Sov. J. Part. Nucl.* **16**, 593 (1985)].

²W. Kluge, in *Pion-Nucleus Physics: Future Directions and New Facilities at LAMPF*, Proceedings of the Los Alamos Conference on Pion-Nucleus Physics, AIP Conf. Proc. No. 163, edited by R. J. Peterson and D. D. Stroitman (AIP, New York, 1987).

³C. D. Roberts, R. T. Cahill, and J. Praschifka, *Ann. Phys. (N.Y.)* **188**, 20 (1988).

⁴E. Mazzucato *et al.*, *Phys. Rev. Lett.* **57**, 3144 (1986).

⁵L. M. Nath and S. K. Singh, *Phys. Rev. C* **39**, 1207 (1989).

⁶K. Choi, K. Kang, and J. E. Kim, *Phys. Rev. Lett.* **62**, 849 (1989).

⁷A. H. Rosenfeld, *Phys. Rev.* **96**, 139 (1954).

⁸M. Gell-Mann and K. M. Watson, *Annu. Rev. Nucl. Sci.* **4**, 219 (1954).

⁹R. A. Stallwood, R. B. Sutton, P. H. Fields, J. G. Fox, and J. A. Kane, *Phys. Rev.* **109**, 1716 (1958).

¹⁰A. F. Dunaitsev and Yu. D. Prokoshkin, *Zh. Eksp. Teor. Fiz.* **36**, 1656 (1959) [*Sov. Phys. JETP* **9**, 1179 (1959)].

¹¹S. Stanislaus, Ph.D. thesis, University of British Columbia, 1988 (unpublished).

¹²S. Mandelstam, *Proc. R. Soc. London Ser. A* **244**, 491 (1958).

¹³D. A. Hutcheon *et al.*, *Phys. Rev. Lett.* **64**, 176 (1990).

¹⁴R. Handler, *Phys. Rev. B* **138**, 1230 (1965).

¹⁵V. P. Efrosinin *et al.*, *Yad. Fiz.* **42**, 950 (1985) [*Sov. J. Nucl. Phys.* **42**, 604 (1985)]; see also *Z. Phys. A* **322**, 573 (1985).

¹⁶D. S. Koltun and A. Reitan, *Phys. Rev.* **141**, 1413 (1966).

¹⁷F. Hachenberg and H. J. Pirner, *Ann. Phys. (N.Y.)* **112**, 401 (1978).