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Energy shifts and widths of kaonic atoms

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Microscopic calculations of the energy shifts and widths of kaonic atoms are carried out using a low-energy kaon-nucleon T matrix which is parametrized by a single Yukawa function. The $\Lambda(1405)$ resonance is not included explicitly but only via its influence on the T matrix at threshold. The experimental values for the shifts and widths are approximately reproduced using a range of 0.66 fm for both the real and imaginary parts of the T matrix although the use of different ranges for the two parts may result in an improvement of the fit. Medium corrections are negligibly small.

The energy shifts and widths of antiprotonic atoms were recently[1,2] systematically investigated in a microscopic model. In general the results agree with measurements within the experimental errors. This successful calculation uses nuclear charge distributions obtained from electron-scattering experiments and a phenomenological nucleon-antiproton interaction. The optical potential in the lowest order approximation (folding the free T matrix with the nuclear density) together with the Schrödinger equation then gives the antiproton atomiclike bound states. Medium corrections and relativistic effects are unimportant.

In this paper we shall study kaonic atoms using the same approximations. The data are nicely collected[3] and analyzed using phenomenological models[4] by Batty. The pure zero-range approximation for the T matrix needs an average kaon-nucleon scattering length of $\bar{a}=(0.34+i0.84)$ fm which differs considerably from values calculated from the free two-body scattering lengths obtained from fits to low-energy two-body scattering data, i.e., $(-0.15+i0.62)$ fm (Ref. 5) or $(-0.31+i0.61)$ fm (Ref. 6). In the case of antiprotonic atoms the inclusion of a finite range is essential to establish the proper connection between energy shifts and widths on the one hand and the basic two-body interaction on the other[2,7] and for this reason we shall attempt to do the same for kaonic atoms with $Z \geq 4$.

For simplicity we assume that the central part of the K^--nucleon T matrix can be approximated by a single Yukawa function for each isospin I

$$t_I = \frac{t_0^I m^2}{m^2+q^2} . \tag{1}$$

It would, of course, be possible to use several Yukawa functions with different ranges but at this stage we prefer to keep our model as simple as possible and use only a single term. The approximation in terms of Yukawa functions is particularly convenient since the Fourier transform of the T matrix to configuration space also has a simple form. Note that we will only need the two-body T matrix and not the potential, so we do not have to make any assumptions about the two-body potential. The scattering lengths[5,6] determine the two complex numbers t_0^I, $I=0,1$ in the region very close to threshold. In the case of antiprotonic atoms it is the T matrix in this region which determines the energy shifts and widths and we assume that the same is true for kaonic atoms. For this reason it is not necessary to introduce any energy dependence into the constants t_0^I over the energy range of interest to us although this is, of course, necessary in other situations. Independent of any assumptions about energy dependence, some information on the range parameter m can be obtained from either very low energy p-wave scattering data or from the integrated elastic cross section very close to threshold. The latter constrains the quantity $1+4k^2/m^2$ to differ from unity by less than about 20% at the lowest measured kaon laboratory momentum of $\frac{3}{2}k \approx 100$ MeV/c. Thus, we must have a range of less than about 0.70 fm.

The charge distributions are obtained from electron-scattering experiments[8] and the neutron distributions are assumed to have the same form, the related point distributions being found by unfolding. Using a Yukawa function for the nucleon form factor is very convenient in our case, since it simply corresponds to an added zero-range interaction in Eq. (1), as is easily verified by folding the T matrix and the nuclear point distributions.

The kaon-nucleus optical potential is now defined in lowest order. This is sufficient for our purpose (see later), as corrections are small in the nuclear tail where the change of the wave function of the kaonic atom from the pure Coulomb form is determined. For example, the kaon optical potential is of the same absolute value as that for antiprotons where corrections were negligibly small.[7] The Schrödinger equation is then used with this first-order potential together with the ordinary Coulomb interaction. Relativistic effects are vanishingly small for shifts and widths of antiprotons,[1,2] and we shall therefore also ignore them in the present case. Note that our approach differs from previous attempts like Ref. 9, where the potential was used in the folding procedure.

This procedure only includes the $\Lambda(1405)$ resonance at 27 MeV below threshold indirectly via its influence on the scattering amplitude very close to threshold. Thus the possibility of a stronger and perhaps more complicated energy dependence of the T matrix is not considered here, although it may provide a link with various alternative many-body calculations.[10] For the antiprotonic atoms, as

mentioned above, it is apparently sufficient to use an optical potential determined by the threshold value of the two-body T matrix, although using the value of the T matrix in an energy range of up to about ± 20 MeV above threshold is equally acceptable.[1,2] In the case of kaonic atoms this energy interval should be somewhat smaller due to the smaller mass of the kaon. In the following, we shall numerically investigate the consequences of our assumptions.

The energy shifts and widths calculated using the two-body scattering lengths of Ref. 5 are shown in Fig. 1 as functions of the two-particle range parameter $1/m$. Both quantities increase with the range for all three nuclei considered. The curves are almost parallel for ^{113}In and ^{238}U and are much steeper than those for ^{12}C. The folding of a finite range interaction with the nuclear density amounts to averaging the latter locally over distances equal to the range. The tail of the density is roughly equal for all nuclei, only depending on the distance from the nuclear radius. As the shifts and widths are given by the overlap between the strong-interaction optical potential and the kaon wave function, their range dependence should be a function of the distance between the overlap region and the nuclear radius.

The nonlinear relation between these quantities makes a simple estimate rather difficult. However, the main effects can be easily seen by use of a Coulomb wave function and the exponential tail of the optical potential. The maximum overlap radius R_S is then crudely estimated[2] to be

$$R_S = \frac{2da_K n^2}{2d + na_K}, \tag{2}$$

where $a_K = \hbar^2/(\mu Ze^2)$ is the Bohr radius of the kaon, n is the principal quantum number, and d is the diffuseness parameter of the potential. Reasonable values of d around 1 fm then places R_S closer to the nuclear surface for ^{12}C than for In and U. This explains the characteristic, relative behavior of the curves in Fig. 1. The experimental points indicate a range between 0.5 and 0.8 fm, although the use of different ranges for the real and imaginary parts of the interaction may result in a better fit.

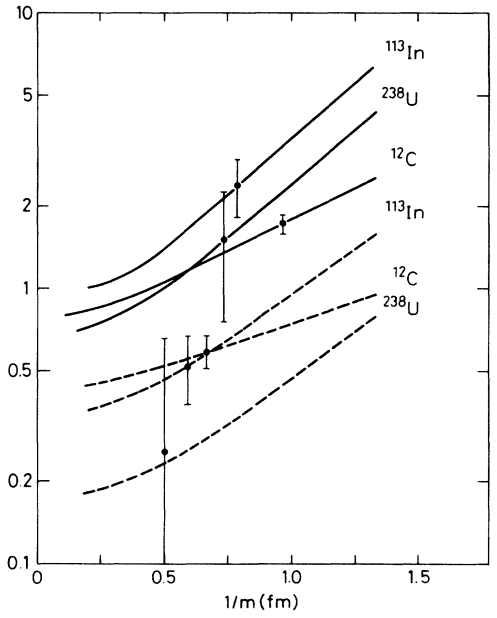

FIG. 1. The absolute value of the energy shift (dashed lines) and widths (solid lines) as function of the kaon-nucleon range parameter $1/m$ for three nuclei. The solid circles are measured values displayed with corresponding error bars. The unit on the energy scale is keV. The scattering lengths are from Ref. 5.

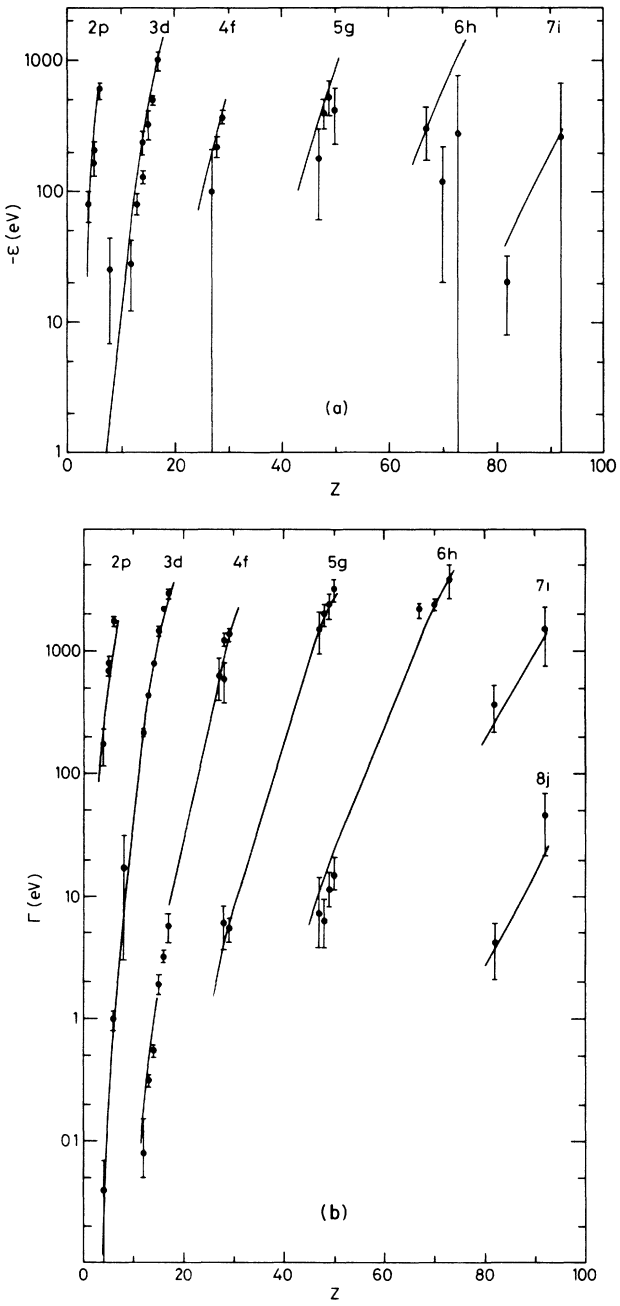

FIG. 2. (a) The energy shifts as function of the nuclear proton number along the β-stability line. The solid circles are measured values with the corresponding error bars. The principal quantum numbers and the l values are given on top of the figure. (b) The same as (a) for the widths.

For simplicity we maintain the same range for both parts of the T matrix. The constraint from the cross section combined with the results of Fig. 1 indicate a range of less than or about 0.7 fm. In Fig. 2 we show the shifts and widths computed for a series of nuclei using $1/m = \frac{2}{3}$ fm. The agreement with the available experimental values is very good. The calculated shifts are systematically above the measured values and a somewhat smaller range is preferred but this would, however, lead to a systematic underestimate of the widths.

The real part of the scattering length is apparently less accurately determined by the low-energy scattering data than the imaginary part.[5,6] In Table I we show shifts and widths for three nuclei obtained using the two recent sets of two-body scattering lengths of Refs. 5 and 6. The widths decrease and the absolute values of the (negative) shifts increase systematically from the first to the second set. Thus the deviation from the measured values discussed above will increase on average when using the second set. The first set clearly leads to a better agreement with experiments.

It is obvious that different ranges for the real and imaginary parts will improve the agreement. Since a smaller range may be compensated by a larger strength, we might as well consider the results obtained using the second set of scattering lengths as corresponding to those of the first set in the case when a smaller range is used for the real T matrix. Using the information of Table I and Fig. 1, we then conclude that a smaller range of the imaginary T matrix for the first set is able to decrease the absolute values of the shifts while roughly keeping the widths unchanged and thereby improve the agreement with the experimental data shown in Fig. 2. For the second interaction a larger range of the real T matrix is presumably necessary.

The calculations rely on the neglect of medium corrections and the accuracy of the assumed nuclear densities. The dependence on the latter was tested for light nuclei by comparing results for harmonic oscillators and Fermi functions. They are identical for all practical purposes over a wide range of values of the range parameter. For ^{113}In and ^{238}U we tried a larger neutron radius given by the droplet model.[11] The shifts remained unchanged and the widths increased by a factor of 1.3 in the worse case of uranium. Such variations are smaller than the experimental uncertainties. The importance of medium corrections was tested in the case of the $3d$ level in ^{32}S which has a relatively small R_S value and is, therefore, expected to be especially sensitive to such effects. The crude scattering length approximation[7] which is an extreme upper limit, changes the shift by 9% and the width by 18%. For our purpose these effects are negligible.

TABLE I. Shifts and widths for $1/m = \frac{2}{3}$ fm for two sets of K^--nucleon scattering lengths (Refs. 5 and 6). The measured values are also given with the related uncertainties below.

(ε,Γ)	^{12}C	^{113}In	^{238}U
I	(−590,1272)	(−576,1872)	(−286,1315)
II	(−676,1058)	(−745,1601)	(−422,1133)
Expt.	(−590,1730) (±80, ±150)	(−530,2380) (±150, ±570)	(−260,1500) (±400, ±750)

In conclusion, the energy shifts and widths of kaonic atoms are computed microscopically by use of a single Yukawa function which reproduces the kaon-nucleon T matrix very close to threshold. The scattering lengths of Ref. 5 are preferred over those of Ref. 6, if we require the widths, shifts, and cross section to be simultaneously reproduced, the range of the interaction being required to be slightly smaller than 0.70 fm. The available experimental data are, in general, consistent with the choice of this range, although improvements seem to be found using a smaller range of the imaginary part of the T matrix. This range is, interestingly enough, close to the range of the uncorrelated two-pion exchange term which is the lowest mass exchange allowed in the t channel. This is maybe not so surprising since the effect of the strong interaction on the kaonic atoms is determined by the local r dependence of the tail of the optical potential where it is the long-range uncorrelated two-pion exchange term which is determining the r dependence even though other exchange terms are more important at shorter distances. A more detailed study using several Yukawa terms of differing ranges would provide more information on this problem.

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