Deuteron Compton scattering

M. Weyrauch
Department of Physics and Mathematical Physics, University of Adelaide, G.P.O. Box 498, Adelaide, South Australia 5001, Australia
(Received 13 September 1989)

The theory of deuteron Compton scattering developed previously [Phys. Rev. C **38**, 611 (1988)] is reviewed and corrected in some minor points. Numerical results are presented and compared with other theoretical approaches. In particular, I now find reasonable agreement with a previous dispersion theoretical analysis of this process. I also get good agreement with low-energy parameters calculated elsewhere. I present predictions for angular distribution cross sections at a number of energies. These are of particular interest, since experiments at these energies are presently under way. I also investigate the influence of the spin-orbit correction to the current operator on the Compton amplitude.

I. INTRODUCTION

Recently I presented a theory of deuteron Compton scattering below pion production threshold.[1] In this calculation the requirements of gauge invariance are implemented in terms of a generalized Siegert's theorem.[2] As a consequence the low-energy theorem (LET) is satisfied and the bulk part of meson exchange effects is taken care of. Furthermore constraints imposed by unitarity are correctly implemented. From the imaginary part of the Compton amplitude the correct photoabsorption cross section is obtained, and the different multipoles of the imaginary part are related to the corresponding nucleon-nucleon (*NN*) scattering partial wave amplitudes by Watson's theorem.

At this stage I restrict the possible intermediate states to *NN* intermediate states only, i.e., I assume that the deuteron can only be excited to *NN* scattering states. This approximation affects only the real part of the Compton amplitude below pion production threshold and it should be a fairly good approximation for incident photon energies below about 100 MeV.

With this restriction the dynamics considered in this paper can be characterized by the graphs in Fig. 1. Figures 1(a) and 1(b) depict the Born terms for the direct and crossed processes, respectively. These terms correspond to free intermediate propagation of the *NN* states. Figures 1(c) and 1(d) are the rescattering terms, which involve the full off-shell *NN* scattering *T* matrix. The two-photon amplitude is graphically represented by Fig. 1(e). Consideration of this term is crucially important to render the Compton amplitude gauge invariant. It has been found in Ref. 1 that the two-photon amplitude together with the Born terms [Figs. 1(a) and 1(b)] are the most important terms numerically.

Only Figs. 1(a) and 1(c) contribute to the imaginary part. This part is related to photoabsorption via the optical theorem. My calculation of the imaginary part is, therefore, equivalent to the complete analysis by Partovi[3] for the photoabsorption cross section. The real part draws contributions from all graphs of Fig. 1. In Ref. 1 a very large real part was obtained. This large real part comes as a surprise, since it does not agree with expectations obtained using dispersion relations.[4] It turns out that this large real part is indeed erroneous and it is one purpose of this paper to correct for this error. I find that the results of the theory developed in Ref. 1 actually are in quite good agreement with dispersion theoretical predictions obtained previously.

I furthermore present here results for the low-energy parameters α_E (static electric polarizability), and β_M (paramagnetic susceptibility), and compare them with values obtained by other authors.[5] I moreover study the influence of some operators of relativistic order, which have been found to be of importance in studies of photoabsorption.[6]

In Sec. II I review the theory developed in Ref. 1 and take the opportunity to formulate the theory somewhat more generally and straightforwardly. The specific structure of the current, charge, and two-photon operators is discussed in Sec. III along with a brief account of the nucleon-nucleon *T* matrix to be used. In Sec. IV I analyze the numerical results in some detail. I present low-energy parameters, and total and differential cross sections. As far as I am aware my results are in agreement with the (little) experimental and theoretical knowledge obtained until now on deuteron Compton scattering.

At present an experiment to determine the angular distributions at 50 and 70 MeV incident photon energy is under way at the University of Illinois Nuclear Physics Laboratories.[7] Although my results indicate that these cross sections are more or less model independently determined by the Born terms [Figs. 1(a) and 1(b)], I am awaiting these results with anticipation.

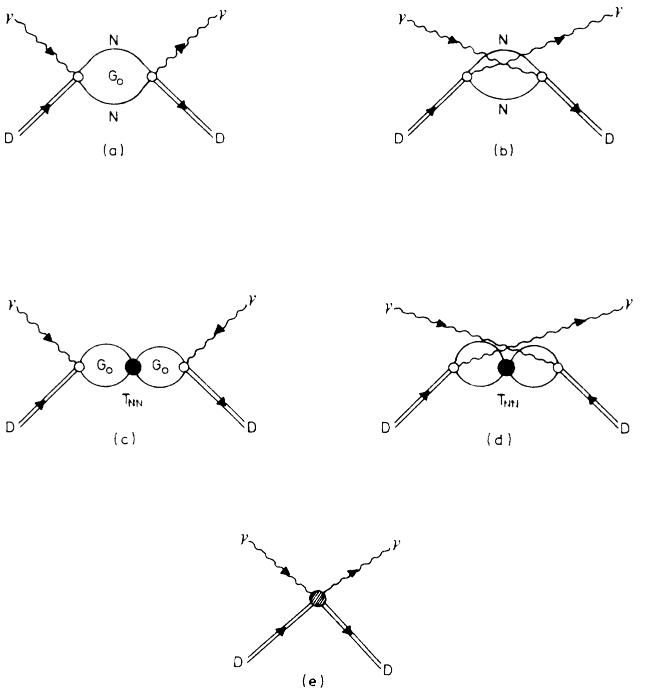

FIG. 1. The deuteron Compton scattering amplitude: Born terms (a) and (b), rescattering terms (c) and (d), two-photon amplitude (e). D and γ stand for deuteron and photon, respectively, N for nucleon, G_0 denotes the free NN propagator, and T_{NN} the full off-shell NN scattering amplitude.

II. THE COMPTON SCATTERING AMPLITUDE

In this section I review the essential elements of the formalism from Ref. 1 correcting a number of minor errors. The formulas given here are more general than those in Ref. 1 since separability of the full off-shell NN T matrix is not assumed.

As is explained in more detail in Ref. 1, the deuteron Compton amplitude can be separated into a resonance amplitude $R_{\lambda'\lambda}$ [Figs. 1(a)–1(d)] and a two-photon amplitude $B_{\lambda'\lambda}$ [Fig. 1(e)]:

$$T_{\lambda'\lambda}(-\mathbf{k}',\mathbf{k})=R_{\lambda'\lambda}(-\mathbf{k}',\mathbf{k})+B_{\lambda'\lambda}(-\mathbf{k}',\mathbf{k}) \ . \tag{1}$$

The resonance amplitude describes intermediate excitation of the deuteron, and I restrict here to virtual NN excitations only. According to the graphs in Fig. 1, I furthermore split the resonance amplitude into a Born term $A_{\lambda'\lambda}$ [Figs. 1(a) and 1(b)] and a rescattering term $C_{\lambda'\lambda}$ [Figs. 1(c) and 1(d)]

$$R_{\lambda'\lambda}(-\mathbf{k}',\mathbf{k})=A_{\lambda'\lambda}(-\mathbf{k}',\mathbf{k})+C_{\lambda'\lambda}(-\mathbf{k}',\mathbf{k}) \ . \tag{2}$$

The structure of these terms can be immediately read off the graphs in Fig. 1,

$$A_{\lambda'\lambda}(-\mathbf{k}',\mathbf{k})=\sum_{s\nu}\int\frac{d^3q}{(2\pi)^3}\langle 1m_d'|\epsilon_{\lambda'}'^{*}\cdot\mathbf{j}(-\mathbf{k}')|\mathbf{q};s\nu\rangle G_0(E,q)\langle\mathbf{q};s\nu|\epsilon_\lambda\cdot\mathbf{j}(\mathbf{k})|1m_d\rangle+\left[\begin{matrix}\mathbf{k}\leftrightarrow-\mathbf{k}'\\ \epsilon_\lambda\leftrightarrow\epsilon_{\lambda'}'^{*}\\ E\leftrightarrow\bar{E}\end{matrix}\right] \ . \tag{3}$$

The rescattering term involves the full off-shell NN T-matrix T_{NN},

$$C_{\lambda'\lambda}(-\mathbf{k}',\mathbf{k})=\sum_{s'\nu's\nu}\int\frac{d^3q'}{(2\pi)^3}\int\frac{d^3q}{(2\pi)^3}\langle 1m_d'|\epsilon_{\lambda'}'^{*}\cdot\mathbf{j}(-\mathbf{k}')|\mathbf{q}',s'\nu'\rangle$$

$$\times G_0(E,q')\langle\mathbf{q}',s'\nu'|T_{NN}|\mathbf{q},s\nu\rangle G_0(E,q)\langle\mathbf{q},s\nu|\epsilon_\lambda\cdot\mathbf{j}(\mathbf{k})|1m_d\rangle+\left[\begin{matrix}\mathbf{k}\leftrightarrow-\mathbf{k}'\\ \epsilon_\lambda\leftrightarrow\epsilon_{\lambda'}'^{*}\\ E\leftrightarrow\bar{E}\end{matrix}\right] \ . \tag{4}$$

For the free NN propagator I take the nonrelativistic form

$$G_0(E,q)=\frac{1}{E-q^2/2\mu+i\epsilon} \tag{5}$$

with $\mathbf{q}$ the relative NN momentum and μ the reduced two-nucleon mass. The deuteron initial and final states are denoted by $|1m_d\rangle$, and the intermediate plane wave states $|\mathbf{q},s\nu\rangle$ carry spin s and spin projection ν. (Note that I neglect the deuteron state as a possible intermediate state.)

The photon momentum and polarization is described by the vectors $\mathbf{k},\epsilon_\lambda$ and $\mathbf{k}',\epsilon_{\lambda'}$ for the incoming and scattered photons, respectively. They satisfy the transversality condition $\epsilon_\lambda\cdot\mathbf{k}=0=\epsilon_{\lambda'}'\cdot\mathbf{k}'$. The energies E and $\bar{E}$ are found to be

$$E=\frac{k^2}{2M}+k-\epsilon_d,\quad \bar{E}=-\frac{k^2}{2M}-k-\epsilon_d \ , \tag{6}$$

in the photon deuteron center-of-mass frame with ϵ_d the deuteron binding energy, M the deuteron mass, and $k=|\mathbf{k}|$.

The current and charge operators $\mathbf{j},\rho$ will be specified later as will the two-photon operator $B_{l'l}$ in terms of which the two-photon amplitude reads

$$B_{\lambda'\lambda}(-\mathbf{k}',\mathbf{k})=\langle 1m_d|\sum_{l'l}\int d^3x'\int d^3x e^{-i\mathbf{k}'\cdot\mathbf{x}'}e^{i\mathbf{k}\cdot\mathbf{x}}\epsilon'_{\lambda' l'}B_{l'l}(\mathbf{x}',\mathbf{x})\epsilon_{\lambda l}|1m_d\rangle \ . \tag{7}$$

I just mention here that these operators are not independent but are related by the gauge conditions,

$$\nabla\cdot\mathbf{j}(\mathbf{x})=i[\rho(\mathbf{x}),H] \ ,$$
$$\sum_{l'}\frac{\partial}{\partial x_{l'}}B_{l'l}(\mathbf{x}',\mathbf{x})=i[\rho(\mathbf{x}'),j_l(\mathbf{x})] \ , \tag{8}$$

where H is the Hamiltonian describing the NN interaction.

It is convenient to write the Compton amplitude in terms of the generalized polarizabilities P_J,

$$T_{\lambda'\lambda}(-\mathbf{k}',\mathbf{k})=(-)^{1-m_d}\sum_{\substack{L'M'LMJ\\ v',v=0,1}}(-)^{L'+L}(2J+1)\begin{bmatrix}1 & J & 1\\ -m'_d & m_J & m_d\end{bmatrix}\begin{bmatrix}L & L' & J\\ M & M' & -m_J\end{bmatrix}$$
$$\times\lambda'^{v'}\lambda^{v}P_J(M^{v'}L',M^{v}L,k)D^{(L')}_{M',-\lambda'}(R')D^{(L)}_{M,\lambda}(R) \ . \tag{9}$$

One particular polarizability $P_J(M^{v'}L',ML,k)$ describes the response of the deuteron, if a photon with angular momentum L scatters into a photon with angular momentum L' while transferring an angular momentum J to the deuteron. The parity of the photons is characterized by $M^0=E=$electric and $M^1=M=$magnetic.

Since the deuteron is a spin-1 particle and I consider only elastic scattering here, only the following polarizabilities contribute: scalar ($J=0$), vector ($J=1$), and tensor ($J=2$) polarizabilities. The polarizabilities are separated into resonance and two-photon pieces as was the amplitude in Eq. (1),

$$P_J=R_J+B_J \ . \tag{10}$$

It is fairly straightforward to derive explicit expressions for the resonance and two-photon polarizabilities, and I will give only results here. More details can be found in Ref. 1. In deriving the expressions I will make use of the generalized Siegert's theorem proven in Ref. 2. I would like to emphasize that employing Siegert's theorem is not an approximation but a useful exploitation of the gauge conditions (8).

The resonance polarizabilities ($\hat{L}=\sqrt{2L+1}$)

$$R_J(M^{v'}L',M^{v}L,k)=2\pi\hat{L}'\hat{L}(-)^L\sum_{l'ljs}\begin{Bmatrix}1 & L' & j\\ L & 1 & J\end{Bmatrix}[F^{L'v'Lv}_{l'lsj}(k)+G^{L'v'Lv}_{l'lsj}(k)]+\mathcal{P}_J(M^{v'}L',M^{v}L,k) \tag{11}$$

are written in terms of the Born terms F and the rescattering terms G. The expression $\mathcal{P}_J(M^{v'}L',M^{v}L,k)$ is exactly cancelled by a counterterm arising in the expression for the two-photon polarizabilities. An exact proof of this consequence of the generalized Siegert's theorem and the explicit expression of $\mathcal{P}_J$ is given in Ref. 2.

The Born term F is found to be

$$F^{L'v'Lv}_{l'lsj}(k)=-\delta_{l'l}\frac{1}{(2\pi)^3}\int_0^\infty dq\, q^2\langle 1\|\Omega^{[L']}_{v',+}(k,E)\|R_l(qr);(ls)j\rangle G_0(E,q)$$
$$\times\langle R_l(qr);(ls)j\|\Omega^{[L]}_{v,-}(k,E)\|1\rangle+(-)^{L'+L+J}\begin{Bmatrix}E\leftrightarrow\bar{E}\\ (L'v')\leftrightarrow(Lv)\end{Bmatrix} \ , \tag{12}$$

in terms of the radial function $R_l(qr)=i^l4\pi j_l(qr)$. The multipole operators are

$$\Omega^{[L]}_{0,\pm}(k)=\beta(E)M^{[L]}(k)\pm T^{[L]}(k) \ , \tag{13}$$

$$\Omega^{[L]}_{1,+}(k)=\Omega^{[L]}_{1,-}=\int d^3x\, \mathbf{j}(\mathbf{x})\cdot\mathbf{a}^{[L]}(M;\mathbf{x},k) \ , \tag{14}$$

with the Coulomb and current multipoles

$$M^{[L]}(k)=\int d^3x\, \rho(\mathbf{x})c(\mathbf{x},k) \ , \tag{15}$$

$$T^{[L]}(k)=\int d^3x\, \mathbf{j}(\mathbf{x})\cdot\mathbf{a}^{[L]}(E;\mathbf{x},k) \ . \tag{16}$$

Expressions for the multipole fields $\mathbf{a}^{[L]}(M^v;\mathbf{x},k)$ and $c(\mathbf{x},k)$ are given in the Appendix. [The separation of the electric multipole into a Coulomb and current term in Eq. (13) is not unique, and I follow here the convention adopted by Partovi.[3]] The factor $\beta(E)$ is determined to be

$$\beta(E)=1+\frac{k}{2M}, \quad \beta(\bar{E})=-1-\frac{k}{2M} \ , \tag{17}$$

in the case of the deuteron.

The q integration in Eq. (12) can be performed analytically,

$$\int_0^\infty dq\, q^2 j_l(qr')j_l(qr)G_0(E,q)=\begin{cases}-\pi i\lambda\mu j_l(\lambda r_<)h_l^{(1)}(\lambda r_>), & E>0\ ,\\ -2\lambda\mu i_l(\lambda r_<)k_l(\lambda r_>), & E<0\ ,\end{cases} \tag{18}$$

with $\lambda^2=|2\mu E|$, $r_<=\min(r',r)$, $r_>=\max(r',r)$, and $h_l^{(1)}(x)=j_l(x)+iy_l(x)$ the Hankel functions of the first kind. Furthermore, i_l and k_l are modified spherical Bessel functions. In Eq. (18) unfortunately a crucial sign error occurred in Ref. 1 for the case $E<0$.

Analogously, the rescattering terms are obtained as

$$G_{l'lsj}^{L'\nu'L\nu}(k)=-\frac{1}{(2\pi)^6}\int_0^\infty dq'q'^2\int_0^\infty dq\,q^2\langle 1\|\Omega_{\nu',+}^{[L']}(k,E)\|R_{l'}(q'r');(l's)j'\rangle \times G_0(E,q')T_{l'l}^{js}(q',q)G_0(E,q)\langle R_l(qr);(ls)j\|\Omega_{\nu,-}^{[L]}(k,E)\|1\rangle +(-)^{L'+L+J}\begin{bmatrix} E\leftrightarrow\bar{E}\\ (L'\nu')\leftrightarrow(L\nu)\end{bmatrix}, \tag{19}$$

involving the full off-shell NN T matrix. In principle the above double integral can be evaluated numerically once the multipole matrix elements and the off-shell NN T matrix is given on a suitable mesh of integration points. If T_{NN} is constructed in a separable potential model, then T_{NN} itself is separable and the above double integral separates into a product of two single integrals. Explicit expressions for this case are given in Ref. 1.

The two-photon polarizabilities [Fig. 1(e)] can be conveniently split into a center-of-mass (c.m.) part and an intrinsic part

$$B_J=B_J^{\text{c.m.}}+B_J^{\text{in}} \tag{20}$$

corresponding to a separation of the NN Hamiltonian into a c.m. and intrinsic piece

$$H=\frac{P^2}{2M}+H^{\text{in}} \tag{21}$$

with P the c.m. momentum of the two-nucleon system. I find

$$B_J^{\text{c.m.}}(M^{\nu'}L',M^{\nu}L,k)=-\frac{e^2}{M}\frac{\sqrt{\pi}}{2}\frac{(\hat{L}'\hat{L})^2}{\hat{J}}(-)^{L'}\sum_{\substack{l'l\\ \lambda'\lambda}}\lambda'^{\nu'}\lambda^{\nu}\begin{pmatrix}1 & l' & L'\\ -\lambda' & 0 & \lambda'\end{pmatrix}\begin{pmatrix}1 & l & L\\ \lambda & 0 & \lambda\end{pmatrix}\begin{pmatrix}l' & l & J\\ 0 & 0 & 0\end{pmatrix} \times\begin{Bmatrix}L' & L & J\\ l' & l & 1\end{Bmatrix}(-)^{(l'+l)/2}(\hat{l}'\hat{l})^2\langle 1\|j_{l'}(kr/2)j_l(kr/2)Y^{[J]}(\hat{r})\|1\rangle \tag{22}$$

for the c.m. polarizabilities and

$$B_J^{\text{in}}(M^{\nu'}L',M^{\nu}L,k)=2\pi(-)^{L+J}\frac{\hat{L}'\hat{L}}{\hat{J}}\langle 1\|\sum_{l'l}\int d^3x'\int d^3x[a_{l'}^{[L']}(M^{\nu'};k',\mathbf{x}')\times a_l^{[L]}(M^{\nu};k,\mathbf{x})]^{[J]}B_{l'l}^{\text{in}}(\mathbf{x}',\mathbf{x})\|1\rangle -\mathcal{P}_J(M^{\nu'}L',M^{\nu}L,k) \tag{23}$$

for the intrinsic polarizabilities. As already mentioned the expression $\mathcal{P}_J$ cancels against a corresponding term in the resonance polarizabilities. This is the advantage of using the generalized Siegert's theorem proven in Ref. 2.

I finally want to mention that the only polarizability contributing at $k=0$ is the $E1E1$ $J=0$ polarizability, which is explicitly given by

$$P_0(E1E1,\ k=0)=\frac{3e^2}{M}=B^{\text{c.m.}}(E1E1,\ k=0) \tag{24}$$

in accordance with the low-energy theorem.

III. SPECIFIC MODEL

The formulation of deuteron Compton scattering presented in the preceding section is essentially model independent; the major restriction is the disregard of all but NN intermediate states. Of course, in order to make numerical predictions I must specify the current, charge, and two-photon operators as well as the full off-shell NN T matrix. These are the essential dynamical quantities, which will be probed in a deuteron Compton scattering experiment at energies below about 100 MeV.

While in principle it would be no problem to evaluate the Compton amplitude with any given NN T matrix (e.g., from the Bonn or Paris potentials), the numerical work is simplified tremendously if a separable potential model is employed. Therefore, in this work I will take a separable NN T matrix.[9] The details of this T matrix have been discussed in some detail in Ref. 1. It turns out, that the Compton amplitude is only marginally influenced by the rescattering terms which involve the NN T matrix. Therefore, effects explicitly depending on the off-shell NN dynamics are hardly visible in the Compton cross sections at the energies considered here. This justifies taking a separable potential model at this stage.

In this work I will consider the standard single body current density of nonrelativistic point nucleons

$$\mathbf{j}^{(1)}(\mathbf{x})=\frac{e}{2m}\sum_{i=1}^{2}[e_i\{\delta(\mathbf{x}-\mathbf{r}_i),\pi_i\}+\mu_i\nabla_x\times\sigma_i\delta(\mathbf{x}-\mathbf{r}_i)]\ , \tag{25}$$

with e_i,μ_i the nucleon charge and magnetic moment operators, respectively, m the nucleon mass and π_i its momentum. For the charge density I take

$$\rho^{(1)}(\mathbf{x})=e\sum_{i=1}^{2}e_i\delta(\mathbf{x}-\mathbf{r}_i)\ . \tag{26}$$

Normalizations are such that $e^2=\frac{1}{137}$.

Of course, this current and charge density do not satisfy current conservation

$$\nabla\cdot\mathbf{j}^{(1)}(\mathbf{x})=-i[H,\rho^{(1)}(\mathbf{x})]\ , \tag{27}$$

if the NN Hamiltonian is nonlocal, as is the case here. To be strictly consistent, one would need to add a nonlocal current $\mathbf{j}^{(2)}(\mathbf{x})$ and/or a nonlocal charge density to (25) and (26), so that charge conservation is obtained. Siegert's hypothesis suggests that to nonrelativistic order only the current density gets nonlocal contributions, and in fact, these can be constructed straightforwardly.[10] These nonlocal currents would influence the magnetic multipoles as well as the higher-order electric multipoles $T^{[L]}(k)$ [Eq. (16)]. Those contributions are neglected here. It has been shown in calculations of photodisintegration that these effects are small at the energies considered here, and the bulk part of nonlocal and exchange current effects is taken care of by using Siegert's theorem.

Analogously, the two-photon operator gets nonlocal contributions $B_{l'l}^{(2)}$ in order that the gauge conditions (8) are fulfilled. Such effects have been studied in the one-pion-exchange model, and they turn out to be extremely small.[4] Accordingly they are neglected here. Consequently, while the current, charge, and two-photon operators used in this work do not strictly satisfy the gauge conditions (8), the neglected terms are expected to be very small. This is mainly due to the fact that Siegert's theorem has been used in evaluating the electric multipole operators.

I finally would like to discuss a relativistic correction to the current density operator. It has been shown recently by several authors[6] that relativistic corrections to the charge and current operators do play a significant role in photo- and electrodisintegration of the deuteron at relatively low energies. While a comprehensive discussion of such effects is beyond the scope of the present paper, it can be said that the spin-orbit correction to the current operator produces the most significant effect numerically.[6] I, therefore, only consider this term here leaving a more complete analysis for later work. The spin-orbit current operator is given by

$$\mathbf{j}^{\text{so}}(\mathbf{x})=\frac{i}{4m^2}\sum_{i=1}^{2}\Big\{[H^{\text{in}},(e_i+2\kappa_i)\delta(\mathbf{x}-\mathbf{r}_i)\pi_i\times\sigma_i]+\frac{1}{4m}(e_i+2\kappa_i)\Delta_x\delta(\mathbf{x}-\mathbf{r}_i)\pi_i\times\sigma_i\Big\}\ . \tag{28}$$

While our results confirm the findings of other authors,[6] that the imaginary part of the Compton amplitude is somewhat reduced by the above correction, the real part is hardly influenced. This will be discussed further in the following section.

IV. NUMERICAL RESULTS AND DISCUSSION

In this section I present numerical results of the present "direct" calculation of the deuteron Compton amplitude and compare them with a previous dispersion theoretical analysis.[4] I furthermore give results for the low-energy parameters α_E and β_M, i.e., the (static) electric polarizability and the magnetic susceptibility. The good agreement between theory and experiment for these values suggests that the present formulation of deuteron Compton scattering is essentially correct. Predictions are made for differential scattering cross sections at several energies at which experiments are presently under way.

At very low energies, i.e., below deuteron breakup threshold, the deuteron Compton amplitude is purely real, and can be characterized for energies much below threshold by a few low-energy parameters. In particular it can be shown[2] that for the deuteron

$$\begin{aligned}P_0(E1E1)&=3\left[\frac{e^2}{M}-\left[\alpha_E^{(0)}+\frac{e^2}{3M}\langle r^2\rangle-\frac{\bar{\mu}^2}{3M}\right]k^2\right]\ ,\\ P_0(M1M1)&=-3\left[\chi_P^{(0)}+\chi_D^{(0)}-\frac{\langle\mathbf{D}^2\rangle}{2M}\right]k^2\ ,\end{aligned} \tag{29}$$

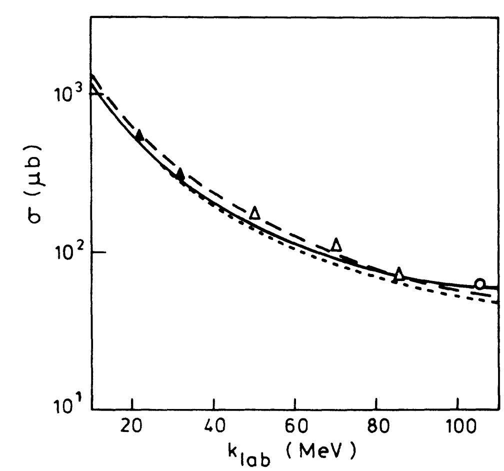

FIG. 2. The total photoabsorption cross section: Born terms only (dashed) and complete calculation (solid line). Dotted lines include spin-orbit current effects. The experimental data are from Ref. 11 (solid triangle), Ref. 12 (open triangle), and Ref. 13 (open circle).

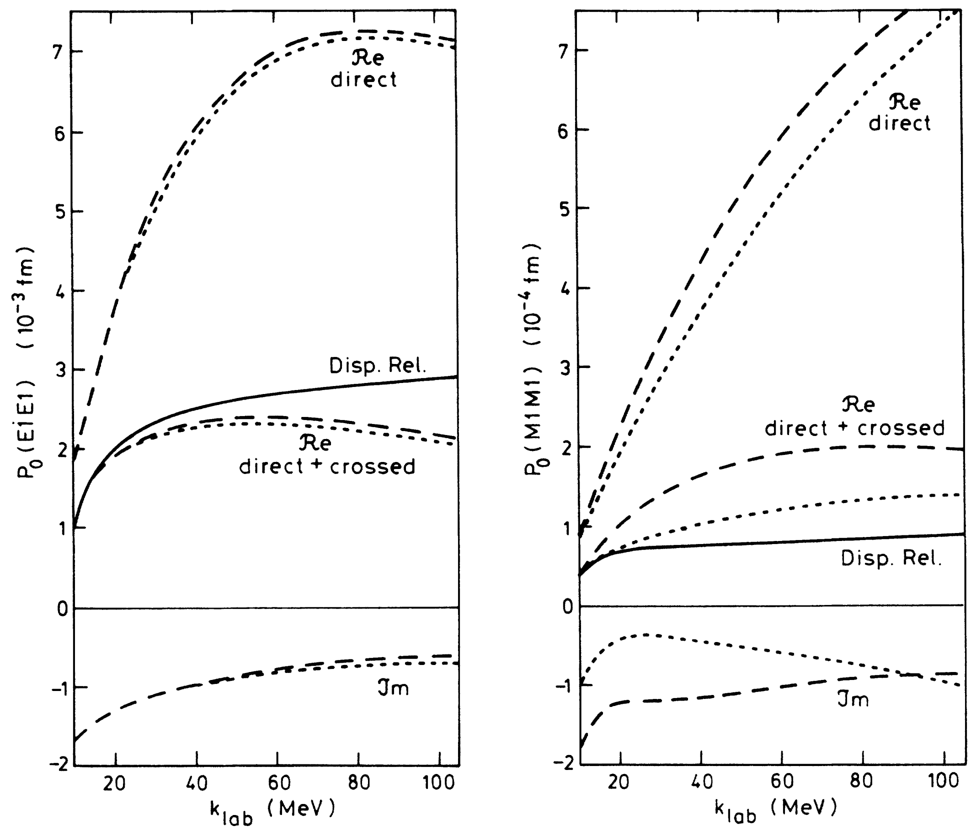

FIG. 3. Scalar polarizabilities: $P_0(E1E1)$ and $P_0(M1M1)$. Solid lines represent the dispersion theoretical results of Ref. 2. Dashed lines represent the Born approximation results [Figs. 1(a) and 1(b)]. Dotted lines include rescattering effects.

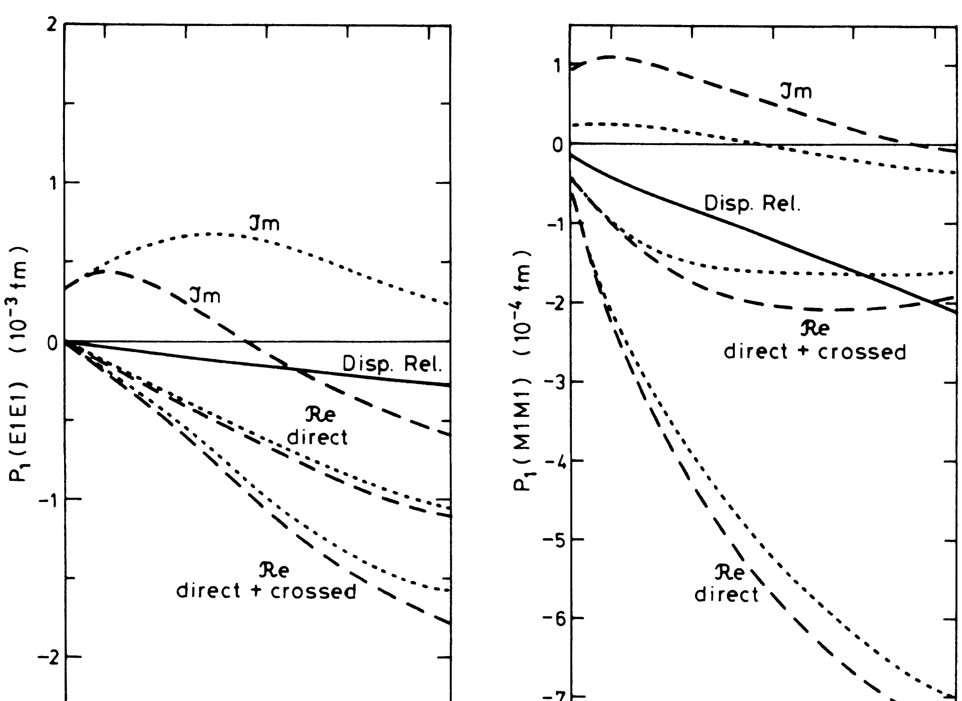

FIG. 4. Vector polarizabilities: $P_1(E1E1)$ and $P_1(M1M1)$. Notation as in Fig. 3.

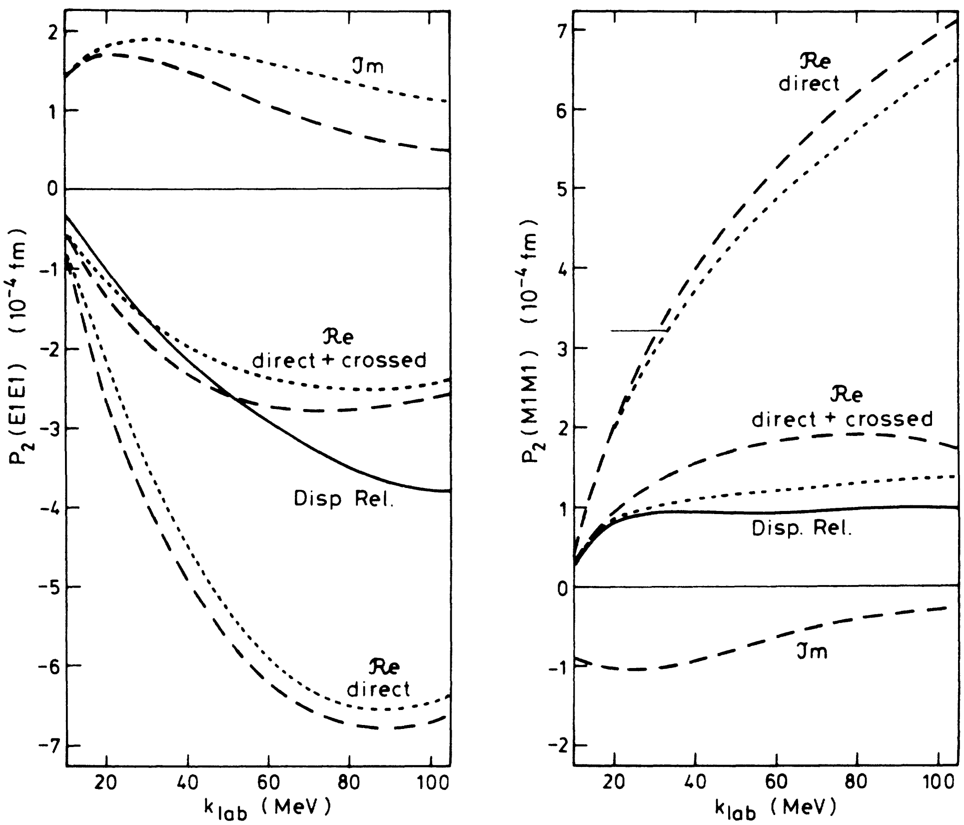

FIG. 5. Tensor polarizabilities: $P_2(E1E1)$ and $P_2(M1M1)$. Notation as in Fig. 3.

up to order k^2 in the photon energy. A more comprehensive discussion of the low-energy structure of the Compton amplitude is contained in Refs. 2 and 8. The structure constants entering the above expressions are the "static" electric polarizability $\alpha_E^{(0)}$, the paramagnetic susceptibility $\chi_P^{(0)}$, and the diamagnetic susceptibility $\chi_D^{(0)}$. The term determined by the root-mean-square radius $\langle r^2 \rangle$ and $\chi_D^{(0)}$ have their origin in the two-photon amplitude, while $\alpha_E^{(0)}$ and $\chi_P^{(0)}$ are determined by the resonance terms [Figs. 1(a)–1(d)]. Furthermore, there are recoil corrections containing the magnetic moment $\bar{\mu}$ and the electric dipole-operator **D**. The latter two corrections are neglected in the present calculation, as we do not consider a recoil correction to the current operators.

Numerically I find the following values: $\alpha_E = 0.627$ fm^3, $\chi_p = 0.055$ fm^3; $e^2/2M\langle r^2 \rangle = 0.001$ fm$^3 = -\chi_D$. Experimentally α_E has been determined[5] to be 0.61 ± 0.04, in excellent agreement with the theoretical prediction. Other calculations[5] produced $\alpha_E = 0.615$ fm^3 and 0.628 fm^3, while χ_p has been determined to be 0.065 fm^3, if one excludes explicit meson exchange effects. As will be discussed below, $P_0(M1M1)$ and consequently χ_p are more sensitive to the details of the chosen *NN* interaction, and obviously the separable potential model used here does predict a somewhat low value for χ_p. The static electric polarizability and magnetic susceptibility of the nucleon are $\alpha + \chi \approx 0.003$ fm^3 indicating that the internal nucleon structure plays a minor role in the process under consideration here. The above results and considerations suggest that the formalism proposed here is consistent with previous knowledge of the Compton amplitude at low energies.

Above deuteron breakup threshold the Compton amplitude develops an imaginary part, which is related to the total photoabsorption cross section via the optical theorem,

$$\bar{\sigma}_{\mathrm{abs}}(k) = -\frac{4\pi}{k}\frac{1}{\sqrt{3}}\sum_L \frac{(-)^L}{\hat{L}}\mathrm{Im}[P_0(ELEL) + P_0(MLML)] \ . \qquad (30)$$

Only Figs. 1(a) and 1(c) have imaginary parts and therefore contribute to absorption. Results for the total photoabsorption cross section are presented in Fig. 2 and compared to experimental data. As is obvious the Born terms alone give the essential structure, while rescattering terms are minor corrections. In agreement with other authors,[6] I find that the spin-orbit correction to the current operator reduces the total cross section quite substantially.

The interesting prediction of the present calculation is the real part of the deuteron Compton amplitude. This part has been calculated previously only by a dispersion theoretical method.[4] In Figs. 3–5 I present results for the imaginary and real parts of selected polarizabilities and compare with the dispersion theoretical results. Overall, it can be said, that the dispersion theoretical re-

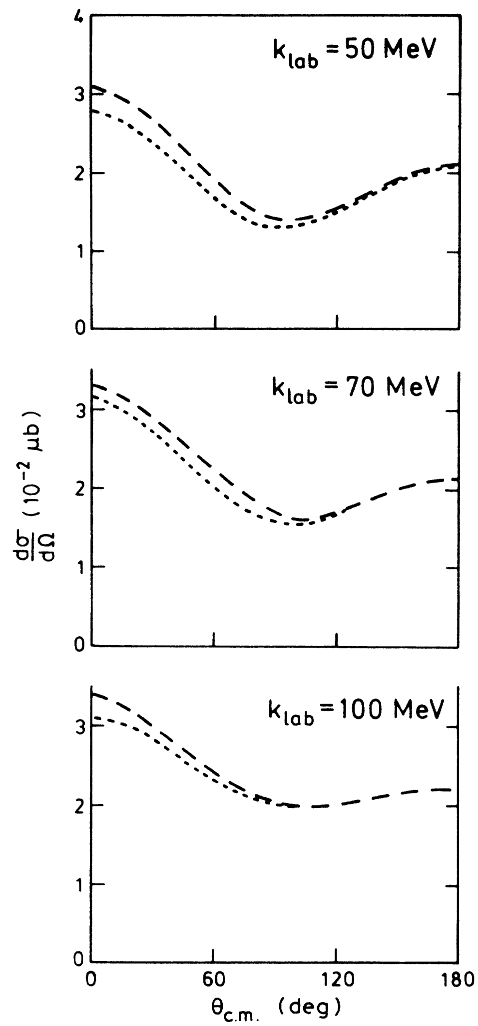

FIG. 6. Differential cross sections for $k_{lab}=50$, 70, and 100 MeV. Born terms only (dashed) and rescattering terms included (dotted).

sults are in quite good agreement with the present "direct" calculation. This contrasts with the conclusion reached in Ref. 1. This can be traced mainly to an error in the relative sign between direct and crossed terms [see Eq. (18)].

The $P_0(E1E1)$ and $P_0(M1M1)$ polarizabilities are shown in Fig. 3. Obviously the $P_0(E1E1)$ polarizability is dominant. I compare the Born approximation and the full calculation (i.e., including rescattering effects) with the previous dispersion theoretical result.[4] I note that rescattering effects are very small for $P_0(E1E1)$. Furthermore there is reasonable agreement between the previous dispersion calculation and the present "direct" calculation over the whole energy range considered here. It is obvious that both the direct and crossed graphs in Fig. 1 give significant contributions and cancel each other to a large extent. Unfortunately, the strong dominance of the Born term indicates that it would be extremely difficult to obtain information on the off-shell NN interaction from deuteron Compton scattering experiments. In the scalar $M1M1$ polarizability rescattering effects play a somewhat stronger role. But they do not show up significantly in the cross sections due to the relative smallness of this multipole. Also for this multipole I observe a reasonable agreement with the previous dispersion theoretical analysis. From this analysis one also expects that in the $P_0(M1M1)$ multipole explicit exchange effects play a somewhat more important role than in the scalar $E1E1$ polarizability. I do not consider those in this paper.

For the vector $E1E1$ and $M1M1$ polarizabilities shown in Fig. 4, I find a relatively strong real part dominated by the Born terms. This agrees with the previous dispersion theory result and indicates a strong dynamical optical activity of the deuteron. The electric term, however, is somewhat bigger than found previously. The tensor $E1E1$ and $M1M1$ polarizabilities shown in Fig. 5 also agree quite well with the dispersion relation results. Again the Born terms dominate, in particular in the electric polarizability.

I have not included effects of the spin-orbit current (28) in Figs. 3–5. It turns out that the imaginary parts are quite significantly affected at higher energies, but unfortunately the real parts are not. Since the real part is dominating at higher energies, spin-orbit current effects do not show up significantly in the differential deuteron Compton cross sections.

Polarizabilities with $L',L > 1$ play only a minor role in the energy range considered here. In my calculations of the cross sections, I include all polarizabilities up to $L',L=2$, but I do not discuss them in more detail here. Numerical results for the differential cross sections at 50, 70, and 100 MeV are given in Fig. 6. Again I compare the full calculation with the Born approximation and it becomes obvious that it would be very difficult to observe effects depending on the NN interaction.

In conclusion, I note that just as the deuteron photoabsorption cross section is essentially model independently predicted (at lower energies), so is the deuteron Compton amplitude, if one employs Siegert's theorem in calculating the electric multipoles. Explicit exchange effects as well as rescattering effects are small and difficult to detect experimentally. It appears at this stage that inelastic photon scattering off the deuteron offers more kinematic flexibility and would be more sensitive to dynamical effects in certain kinematic regions. This will be analyzed in detail in a forthcoming publication.

ACKNOWLEDGMENTS

I would like to thank Alan Nathan for a stimulating discussion and his hospitality during my visit at the University of Illinois at Urbana-Champaign. My work is supported by the Australian Research Grants Council.

APPENDIX

Here I collect a number of well-known and useful formulas for calculating the different multipole operators. The multipole fields entering Eqs. (13)–(16) and (23) are given by

$$\mathbf{a}^{[L]}(E;\mathbf{x},k)=\frac{1}{\sqrt{L(L+1)}}i^{L+1}k\mathbf{x}j_L(kx)Y_M^{[L]}(\hat{x})\ ,$$

$$\mathbf{a}^{[L]}(M;\mathbf{x},k)=\frac{1}{\sqrt{L(L+1)}}i^{L}j_L(kx)[\mathbf{L}Y_M^{[L]}(\hat{x})]\ ,$$

where $\mathbf{L}$ is the angular momentum operator. Furthermore,

$$c(\mathbf{x},k)=\frac{i^L}{\sqrt{L(L+1)}}\left[1+x\frac{d}{dx}\right]j_L(kx)Y_M^{[L]}(\hat{x})\ .$$

[1]M. Weyrauch, Phys. Rev. C **38**, 611 (1988).
[2]H. Arenhövel and M. Weyrauch, Nucl. Phys. **A457**, 573 (1986).
[3]F. Partovi, Ann. Phys. (N.Y.) **27**, 79 (1964).
[4]M. Weyrauch and H. Arenhövel, Nucl. Phys. **A408**, 425 (1983).
[5]J. L. Friar, S. Fallieros, E. L. Tomusiak, D. Skopik, and E. G. Fuller, Phys. Rev. C **27**, 1364 (1983); H. Arenhövel, W. Fabian, Nucl. Phys. **A292**, 429 (1977); N. L. Roding, L. D. Knutson, W. G. Lynch, and M. B. Tsang, Phys. Rev. Lett. **49**, 909 (1982).
[6]A. Cambi, B. Mosconi, and P. Ricci, Phys. Rev. Lett. **48**, 462 (1982); J. L. Friar, B. F. Gibson, and G. L. Payne, Phys. Rev. C **30**, 441 (1984).
[7]A. Nathan, private communication.
[8]J. L. Friar, Ann. Phys. (N.Y.) **95**, 170 (1975).
[9]T. R. Mongan, Phys. Rev. **175**, 1260 (1968).
[10]R. K. Osborne and L. L. Foldy, Phys. Rev. **79**, 795 (1950).
[11]M. Bosmans *et al.*, Phys. Lett. **82B**, 212 (1979).
[12]J. A. Galey, Phys. Rev. **117**, 763 (1960).
[13]E. A. Whalin, B. D. Shiever, and A. D. Hanson, Phys. Rev. **101**, 377 (1956).