Gauge dependence of nonrelativistic calculations of deuteron photodisintegration

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The gauge dependence of nonrelativistic calculations of deuteron photodisintegration is investigated within simple one-pion and one-pion-one-rho exchange models with consistent exchange currents and for the realistic Paris and Bonn potentials. If no explicit exchange currents are considered but Siegert operators are used, the gauge dependence increases with energy and amounts to about 5 percent at 140 MeV. Inclusion of the dominant π- and ρ-exchange currents reduces this dependence to less than 1 percent at 140 MeV in contrast to other claims.

The conventional calculations of deuteron photodisintegration have been strongly criticized by Nagornyi *et al.*[1] They argue that the violation of gauge invariance (GI) by the use of realistic potentials in conjunction with meson exchange currents (MEC's) not completely consistent with the interaction introduces severe and uncontrollable uncertainties into the results. In fact, it seems on first sight impossible to quantify the uncertainty introduced by the violation of GI because it appears plausible that one can always find a gauge that arbitrarily increases the influence of the gauge violating part of a MEC model. However, a closer look reveals that this is an artificial construct and far from reality. Since in practice one uses only a restricted class of gauges that allow a gauge independent evaluation of the leading order of the electric multipoles in terms of the charge density in accordance with the low-energy theorems. Because of Siegert's hypothesis, i.e., no MEC contribution to the charge density in the lowest order, this leading order is also MEC-model independent. Thus the authors of Ref. 1 seem to have overlooked the crucial role of the Siegert operators in conventional calculations that just allow us to incorporate the dominant part of the MEC in a MEC-model independent way.[2,3] For the remaining part one can obtain a reasonable estimate of the GI violation by using different gauges and thus studying the gauge dependence of the results. Their variation can be taken as a quantitative measure of GI violation.

The criticism of Ref. 1 is based on the fact that in almost all calculations with realistic potentials the additional MEC $\mathbf{j}^{\mathrm{MEC}}$ does not satisfy the condition of current conservation, i.e.,

$$\nabla\cdot\mathbf{j}^{\mathrm{MEC}}+i[V,\rho]=0\ . \tag{1}$$

On the other hand, the longest-range π MEC is dominant and consistent with the long-range one-pion exchange (OPE) tail of any realistic potential. Thus the violation of (1) is restricted to the short-range part. Furthermore, the use of the Siegert operator in electric transitions incorporates MEC contributions consistently for the dominant parts. Therefore, one would be surprised to see a large uncertainty from the violation of GI.

In fact, a partial answer in support of these expectations has already been given in Refs. 2 and 4. In particular in Ref. 4 a consistent MEC has been constructed for the Paris potential[5] that fulfills (1). Only small changes in the results have been found if the consistent MEC had been replaced by a simple π- and ρ-exchange model. That seems to be at variance with recent results from Ying *et al.*,[6] where a larger gauge dependence within the given class has been found despite the expectations that the differences should be diminished when MEC is added. The authors of Ref. 6 suspect that the long-wavelength approximation used for the MEC evaluation is responsible for this persisting gauge dependence.

In order to give further support to the reliability and consistency of present nonrelativistic calculations of deuteron photodistintegration we have effectively studied the gauge dependence by using different Siergert operators for the electric transitions. We will briefly review the definition of the Siegert operators. The electric multipole matrix element of order L between intrinsic states $|\alpha\rangle$ and $|\beta\rangle$ is given by

$$\langle\beta|T_{\mathrm{el}}^{[L]}(k)|\alpha\rangle=\int d^3x\,\langle\beta|\mathbf{J}_{\mathrm{int}}(\mathbf{x})|\alpha\rangle\cdot\mathbf{A}_{\mathrm{el}}^{[L]}(\mathbf{x},k) \tag{2}$$

where

$$\mathbf{A}_{\mathrm{el}}^{[L]}(\mathbf{x},k)=\frac{1}{ik\sqrt{L(L+1)}}\nabla\times(\mathbf{x}\times\nabla)(j_L(kx)Y^{[L]}(\hat{\mathbf{x}})) \tag{3}$$

and $\mathbf{J}_{\mathrm{int}}(\mathbf{x})$ denotes the intrinsic current, i.e., relative to the center-of-mass (c.m.) motion. It fulfills current conservation with the intrinsic Hamiltonian H_{int}

$$\nabla\cdot\mathbf{J}_{\mathrm{int}}(\mathbf{x})+i[H_{\mathrm{int}},\rho_{\mathrm{int}}(\mathbf{x})]=0\ . \tag{4}$$

Since the leading order in kx of the electric multipole field is given by

$$\mathbf{A}_{\mathrm{el}}^{[L]}(\mathbf{x},k)\underset{kx\to 0}{\longrightarrow}\frac{i}{k}\frac{L+1}{\sqrt{L(L+1)}}\nabla\left[\frac{(kx)^L}{(2L+1)!!}Y^{[L]}(\hat{\mathbf{x}})\right]+\mathbf{O}^{[L]}[(kx)^{L+1}] \tag{5}$$

it is customary to separate a gradient term from $\mathbf{A}_{\mathrm{el}}^{[L]}$, i.e.,

$$\mathbf{A}_{\mathrm{el}}^{[L]}(\mathbf{x},k)=\nabla\Phi^{[L]}(\mathbf{x},k)+\mathbf{A}_{\mathrm{el}}^{\prime[L]}(\mathbf{x},k;\Phi)\ . \tag{6}$$

In principle, the scalar function $\Phi^{[L]}$ may be chosen arbitrarily reflecting the full gauge freedom. However, in view of the leading order in kx of (5) only a restricted class of gauges will be used in order to comply with the low-energy theorem. This restricted class is defined by

$$\Phi^{[L]}(\mathbf{x},k)=\frac{i}{k}\frac{L+1}{\sqrt{L(L+1)}}\varphi_L(kx)Y^{[L]}(\hat{\mathbf{x}})\ , \tag{7}$$

where

$$\varphi_L(z)=\varphi_L^0(z)+\varphi_L'(z) \tag{8}$$

and

$$\varphi_L^0(z)=\frac{z^L}{(2L+1)!!}\ , \tag{9}$$

$$\varphi_L'(z)=\mathcal{O}(z^{L+2})\ . \tag{10}$$

Within the given lowest order φ_L' remains arbitrary and thus defines the restricted gauge freedom still present.

Then one finds by partial integration and using current conservation the well-known expression

$$\begin{aligned}\langle\beta|T_{\mathrm{el}}^{[L]}(k)|\alpha\rangle&=i\int d^3x\,\langle\beta|[H_{\mathrm{int}},\rho_{\mathrm{int}}(\mathbf{x})]|\alpha\rangle\Phi^{[L]}(\mathbf{x},k)\\&\quad+\int d^3x\,\langle\beta|\mathbf{J}_{\mathrm{int}}(\mathbf{x})|\alpha\rangle\cdot\mathbf{A}_{\mathrm{el}}^{\prime[L]}(\mathbf{x},k;\Phi)\\&=i(\epsilon_\beta-\epsilon_\alpha)\int d^3x\,\rho_{\mathrm{int},\beta\alpha}(\mathbf{x})\Phi^{[L]}(\mathbf{x},k)\\&\quad+\int d^3x\,\langle\beta|\mathbf{J}_{\mathrm{int}}(\mathbf{x})|\alpha\rangle\cdot\mathbf{A}_{\mathrm{el}}^{\prime[L]}(\mathbf{x},k;\Phi)\ ,\end{aligned} \tag{11}$$

where the first term defines the Siegert operator for electric transitions. It is important to note that the energy difference in front of the Siegert operator is given by the difference of the intrinsic energies, which differs from k by c.m. energy contributions. This seems to have been overlooked in Ref. 6. The transition to the Siegert operators corresponds to a gauge transformation of the electric multipole fields, and different choices for φ_L in (8) correspond to different gauges. Thus, we will study the gauge dependence by considering the following choices:

(i) z^L gauge (z^L) $\varphi_L'(z)=0$, (12)

(ii) standard gauge (st) $\varphi_L'(z)=j_L(z)-\varphi_L^0(z)$, (13)

(iii) Partovi gauge[7] (Pa)

$$\varphi_L'(z)=\frac{1}{L+1}\left[1+z\frac{d}{dz}\right]j_L(z)-\varphi_L^0(z)\ , \tag{14}$$

(iv) Foldy[8] or Friar-Fallieros[9] gauge (FF)

$$\varphi_L'(z)=\varphi_L^0(z)[g_L(z)-1]\ , \tag{15}$$

where g_L is defined in Ref. 9.

The last gauge has been recommended strongly by Friar and Fallieros[9] as an optimal gauge on the operator level in the sense that it leaves to $\mathbf{A}_{\mathrm{el}}^{\prime[L]}$ only the magnetization density $\mathbf{x}\times\mathbf{J}(\mathbf{x})$. In this context we will be able to check whether (iv) is an optimal gauge also in the sense that the corresponding Siegert operators give results closest to the ones with MEC included. Incidentally, this gauge has been used also by Hämäläinen.[10]

We will start the discussion with a simple one-pion-exchange potential (OPEP)

$$V_\pi=\frac{g_\pi^2}{4\pi}\frac{1}{4m^2}\tau_1\cdot\tau_2[\sigma_1\cdot\mathbf{p}_1,[\sigma_2\cdot\mathbf{p}_2,J(m_\pi;|\mathbf{r}_1-\mathbf{r}_2|)]]-(m_\pi\leftrightarrow\Lambda_\pi) \tag{16}$$

with

$$J(m_\pi;r)=\frac{e^{-m_\pi r}}{r}\ . \tag{17}$$

(For the πNN coupling constant we take $g_\pi^2/4\pi\simeq15.5$; $\Lambda_\pi\simeq587$ MeV, the cutoff mass is fitted to the deuteron binding energy.) The corresponding pion-exchange current is given by

$$\begin{aligned}\mathbf{j}_\pi(\mathbf{x})=-ie\frac{g_\pi^2}{4\pi}\frac{1}{4m^2}(\tau_1\times\tau_2)_0\{&\sigma_1\delta(\mathbf{x}-\mathbf{r}_1)[\sigma_2\cdot\mathbf{p}_2,J(m_\pi;|\mathbf{x}-\mathbf{r}_2|)]-\sigma_2\delta(\mathbf{x}-\mathbf{r}_2)[\sigma_1\cdot\mathbf{p}_1,J(m_\pi;|\mathbf{x}-\mathbf{r}_1|)]\\&+\frac{i}{4\pi}[\sigma_1\cdot\mathbf{p}_1,[\sigma_2\cdot\mathbf{p}_2,J(m_\pi;|\mathbf{x}-\mathbf{r}_1|)\overleftrightarrow{\nabla}_xJ(m_\pi;|\mathbf{x}-\mathbf{r}_2|)]]\}-(m_\pi\leftrightarrow\Lambda_\pi)\ .\end{aligned} \tag{18}$$

Since (18) follows from (16) in the usual way by minimal coupling, gauge invariance

$$\nabla\cdot\mathbf{j}_\pi(\mathbf{x})+i[V_\pi,\rho(\mathbf{x})]=0 \tag{19}$$

is automatically fulfilled for the point particle charge density. Therefore, the electric multipole matrix elements in (11) will be independent of any gauge choice (10). Moreover, since the Siegert theorem is not only a helpful tool but a mathematical identity in the case of a consistent current, we could even leave the restricted class of gauges (7)–(10), and we still would have gauge independent results. This means that in the case of a consistent current any arbitrary function $\Phi^{[L]}$ can be inserted into (6). Of course, if this function does not fulfill condition (9), the contribution of $\mathbf{A}_{\mathrm{el}}^{\prime[L]}$ in (11) will not be small, of order $(kx)^2$, compared to the contribution of the Siegert operator. Thus, such a choice would not be useful in general.

One special gauge outside the restricted class (7)–(10) is the transverse gauge of the radiation field, i.e.,

(v) special Coulomb gauge (Cb): $\Phi^{[L]}\equiv0$. (20)

Using this gauge means nothing other than calculating the electric multipole matrix elements without the Siegert theorem, i.e., with explicit transverse currents only.

For the case of the OPEP, the GI is illustrated in Fig. 1, where for the differential cross section at 100 MeV photon laboratory energy are shown the relative deviations of the calculations with different gauges of the restricted class (7)–(10) from the calculation with explicit currents, i.e., Cb gauge. The deviations are of order 15 ppm, which merely reflects the numerical accuracy. In principle numerical accuracy could be strongly improved by wasting computer time, but it is sufficient for our purpose. This result is thus a convenient check for the quality of the calculation.

Next we consider the gauge dependence of the so-called "normal" part (N). This N includes the contribution of the Siegert operator and the one-body part of the current $\mathbf{J}_{\text{int}}(\mathbf{x})$ in (11) to the electric matrix elements. Concerning the magnetic matrix elements, only the contribution of the one-body current is included. Of course, a restriction to this N introduces a gauge dependence of the electric matrix elements. If a gauge of type (7)–(10) is used, the effects will be of relative order $(kx)^2$, but they may not be negligible, since the importance of MEC beyond N is known. Clearly, the Cb gauge (20) is totally unsuited when restricted to its N, because in this case the large MEC contribution would be ignored completely, whereas the gauges of the restricted class (7)–(10) include the dominant $(kx)^L$ part of the MEC contributions consistently.

Figures 2(a) and 2(b) show the N's for the OPEP at 30 and 140 MeV as ratios, where [N, gauge (x)] denotes the calculation restricted to its N corresponding to the

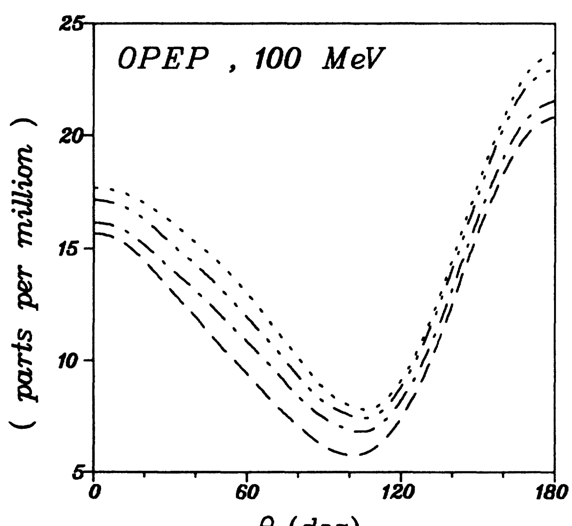


FIG. 1. Illustration of the GI of the differential cross section at 100 MeV photon laboratory energy for the OPEP with consistent currents as functions of the proton c.m. angle Θ. Shown is $1-d\sigma/d\Omega$ [gauge (i)–(iv)]$/d\sigma/d\Omega$[gauge (v)] with the following notation for gauges: dotted line (z^L), dashed line (Pa), dotted-dashed line (st), and dotted-dotted-dashed line (FF).

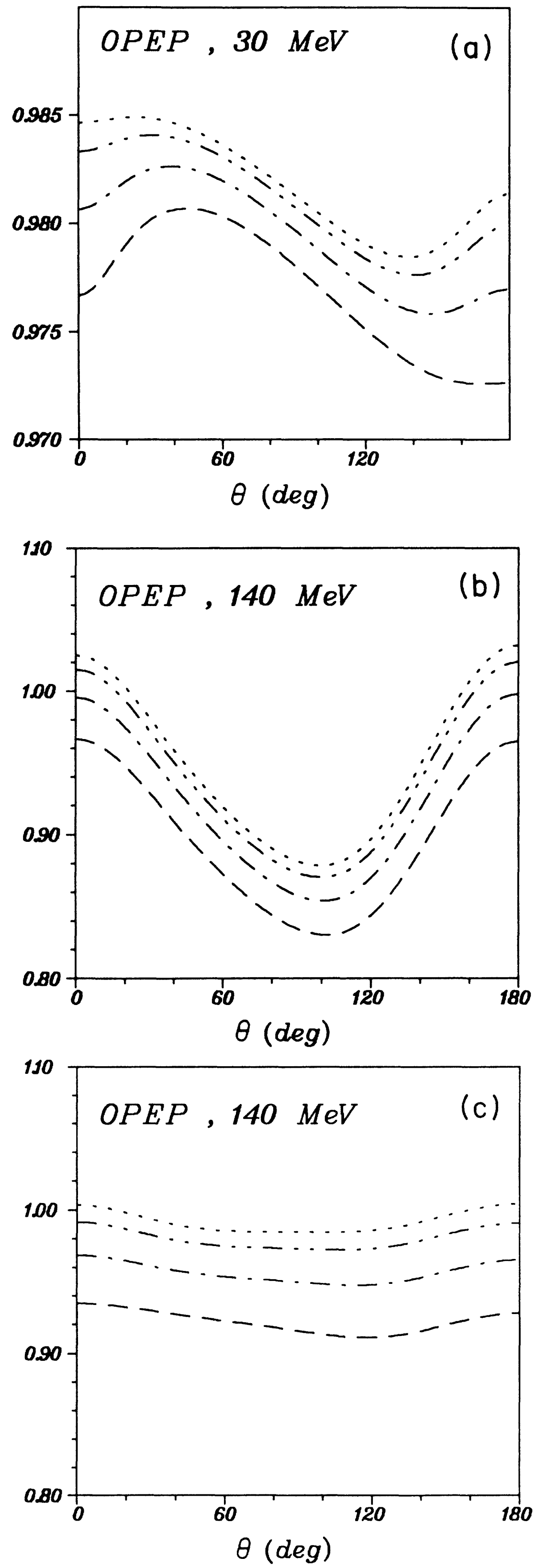


FIG. 2. Gauge dependence of N for the differential cross section at (a) 30 and (b) 140 MeV for the OPEP. Shown is $d\sigma/d\Omega$ [N, gauge (i)–(iv)]$/d\sigma/d\Omega$[gauge (v)]. In (c) the magnetic matrix elements are switched off. (Notation of gauges as in Fig. 1.)

specified gauge (x). The gauge dependence turns out to be of order 1% at 30 MeV (6% at 140 MeV), whereas the N's exhaust the full calculation up to $\sim 2\%$ at 30 MeV (10% at 140 MeV). Therefore, the neglect of that part of the π MEC not covered by the Siegert operator is reflected in this non-negligible gauge dependence of the results. Here it is not obvious that one can favor one of the gauges (i)–(iv) for N as an optimal approximation to the full calculation. For example, the purest and least sophisticated z^L gauge seems to be as good as any other choice. However, since the question of an optimal gauge refers to electric transitions only, one should leave out the magnetic contributions in this comparison. If the magnetic matrix elements are switched off completely, one obtains the results of Fig. 2(c) for 140 MeV. Now the ordering of gauges becomes clearer, and from a practical point of view one would uniquely prefer the z^L or FF gauge against the standard and Partovi gauges.

Figure 3 shows the total cross sections as ratios in the nomenclature of Figs. 2(a) and 2(b). With increasing photon energy, the insufficiency of the N's is getting stronger and this effect is accompanied by an increase of the gauge dependence. Note that at π threshold the missing π-MEC contribution in the case of the Partovi gauge is about twice the size of the one in the z^L gauge.

From this OPE model we get a qualitative impression of the gauge dependence of the cross sections as caused by the inconsistencies in the currents taken into account, in this case artificially simulated by the restriction to N. To further deepen the insight into this interplay, we will now consider a one-pion-one-rho-exchange potential (OPORP). This potential is defined by

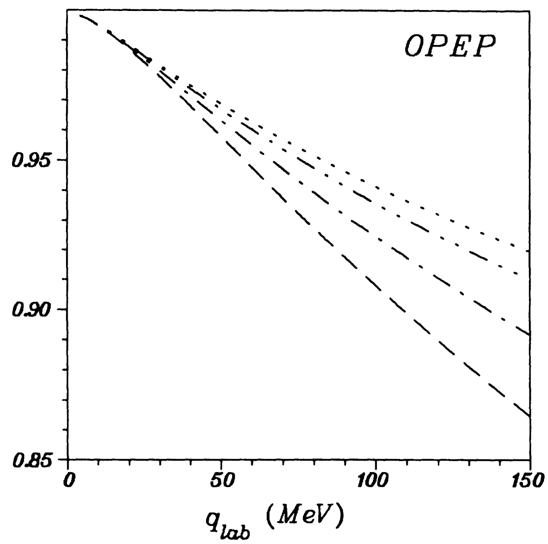


FIG. 3. Gauge dependence of N for the total cross section for the OPEP. Shown is $\sigma_{\rm tot}$ [N, gauge (i)–(iv)]/$\sigma_{\rm tot}$ [gauge (v)]. (Notation of gauges as in Fig. 1.)

$$V_{\pi+\rho}=V_\pi+V_\rho\ , \tag{21}$$

with

$$V_\rho=\frac{(g_\rho+f_\rho)^2}{4\pi}\frac{1}{4m^2}\tau_1\cdot\tau_2[(\sigma_1\times\mathbf{p}_1);[(\sigma_2\times\mathbf{p}_2),J(m_\rho;|\mathbf{r}_1-\mathbf{r}_2|)]]-(m_\rho\leftrightarrow\Lambda_\rho)\ . \tag{22}$$

We have chosen this simple form (22) because normally the ρNN vertex is dominated by its tensor part, i.e., $g_\rho \ll f_\rho$. (Here we have chosen $g_\pi^2/4\pi\simeq 15.1$, $\Lambda_\pi\simeq 730$ MeV, $(g_\rho+f_\rho)^2/4\pi\simeq 5.9$, $\Lambda_\rho=2$ GeV.) From (22) minimal coupling gives again the corresponding current

$$\begin{aligned}\mathbf{j}_\rho(\mathbf{x})=&-ie\frac{(g_\rho+f_\rho)^2}{4\pi}\frac{1}{4m^2}(\tau_1\times\tau_2)_0\\&\times\{-\delta(\mathbf{x}-\mathbf{r}_1)\sigma_1\times[(\sigma_2\times\mathbf{p}_2),J(m_\rho;|\mathbf{x}-\mathbf{r}_2|)]+\delta(\mathbf{x}-\mathbf{r}_2)\sigma_2\times[(\sigma_1\times\mathbf{p}_1),J(m_\rho;|\mathbf{x}-\mathbf{r}_1|)]\\&+\frac{i}{4\pi}[(\sigma_1\times\mathbf{p}_1);[(\sigma_2\times\mathbf{p}_2),J(m_\rho;|\mathbf{x}-\mathbf{r}_1|)\overleftrightarrow{\nabla}_x J(m_\rho;|\mathbf{x}-\mathbf{r}_2|)]]\}-(m_\rho\leftrightarrow\Lambda_\rho)\ .\end{aligned} \tag{23}$$

With this OPORP we again have a consistent model for which the continuity equation for $\mathbf{j}_\pi(\mathbf{x})+\mathbf{j}_\rho(\mathbf{x})$ is fulfilled, and the electric matrix elements (11) will again be independent of the gauge for any arbitrary $\Phi^{[L]}$ in (6).

This OPORP now allows us to examine the following problem that usually plays a critical role in any calculation with realistic potentials, if the corresponding currents are not completely consistent: What is the gauge dependence of the results, if only the dominant MEC contribution beyond N, i.e., π MEC, is consistently included but shorter-range MEC effects are still missing? The answer as given by the OPORP model will be a pessimistic estimate, since in realistic calculations one nowadays would always include at least the dominant part of the ρ MEC of (23), thus shifting the problem to the role of MEC effects beyond π and ρ MEC.

To simulate this typical problem for realistic potentials with the OPORP, we look for the gauge dependence of N and the gauge dependence of ($N+\pi$ MEC). Of course, if [$N+(\pi+\rho)$ MEC] is considered, because of its consistency no gauge dependence occurs. Figures 4(a) and 4(b) show the results for the differential cross section at 100 MeV. In Fig. 4(a) is shown $d\sigma/d\Omega$ [N, gauge (i)–(iv)]/$d\sigma/d\Omega$ [gauge (v)], where gauge (v) now stands for the full gauge independent calculation including consistent π and ρ MEC. The effects are qualitatively the same as discussed earlier for the case of the OPEP. Figure 4(b) shows $d\sigma/d\Omega$ [$N+\pi$ MEC, gauge (i)–(iv)]/$d\sigma/d\Omega$ [gauge (v)], where [$N+\pi$ MEC, gauge (x)] denotes the inclusion of consistent π MEC in the

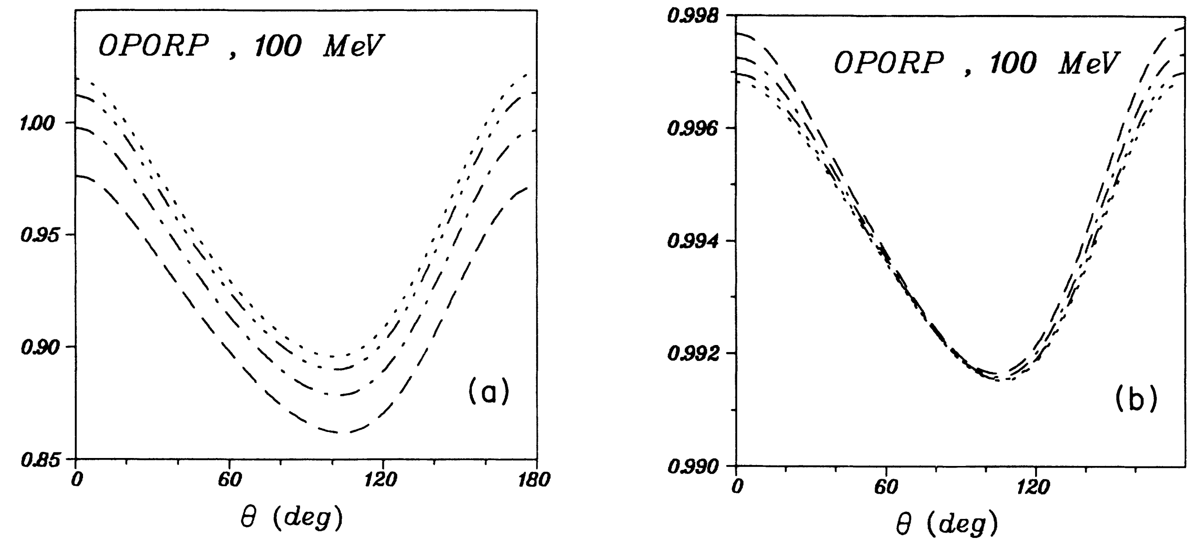


FIG. 4. Gauge dependence of the differential cross section at 100 MeV for the OPORP. In (a) is shown $d\sigma/d\Omega$ [N, gauge (i)–(iv)]/$d\sigma/d\Omega$ [gauge (v)] and in (b) $d\sigma/d\Omega$ [$N+\pi$ MEC, gauge (i)–(iv)]/$d\sigma/d\Omega$ [gauge (v)]. (Notation of gauges as in Fig. 1.)

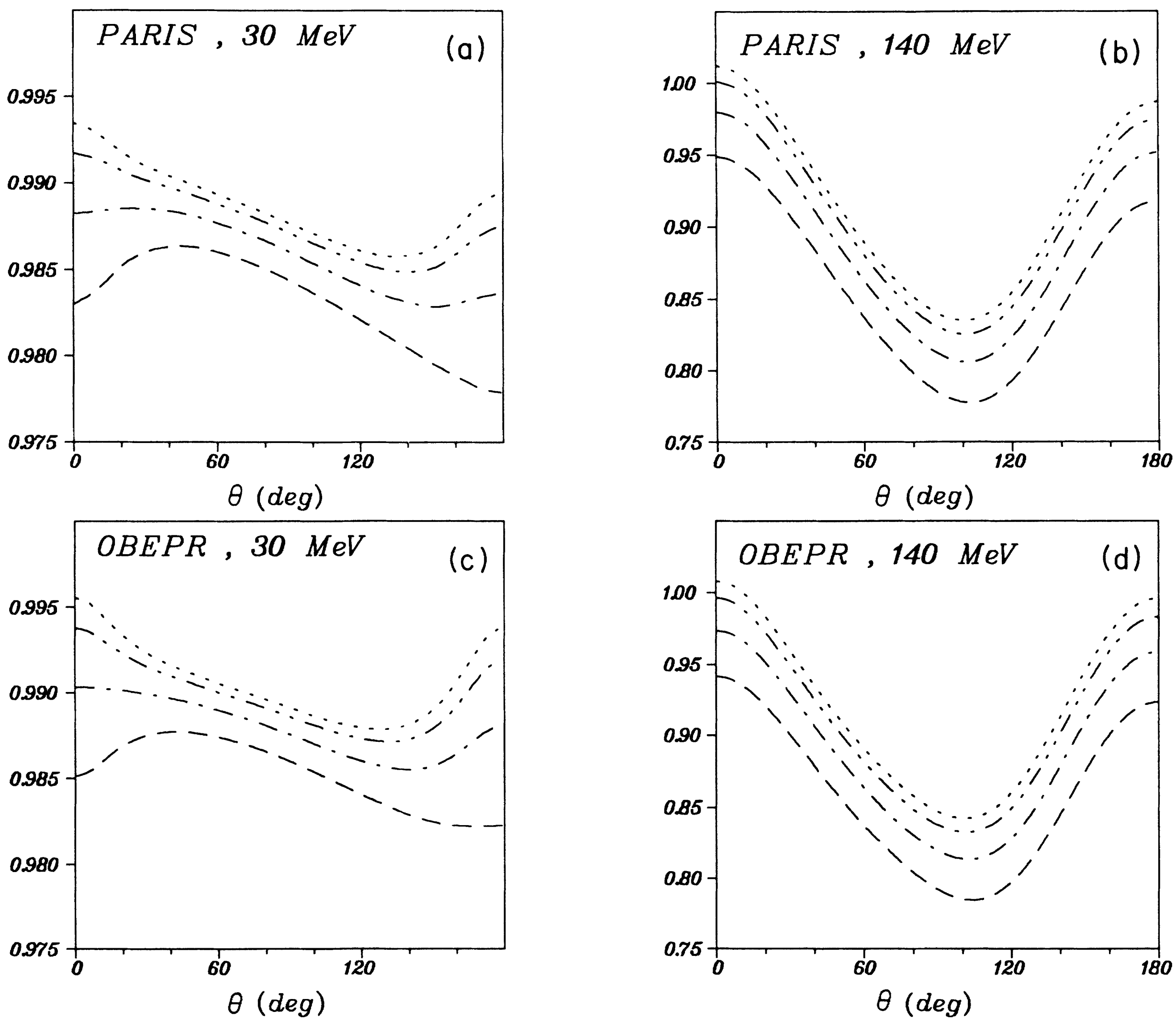


FIG. 5. Gauge dependence of N for the differential cross section at 30 and 140 MeV for the Paris potential and the OBEPR. Shown is $d\sigma/d\Omega$ [N, gauge (i)–(iv)]/$d\sigma/d\Omega$ (T). (Notation of gauges as in Fig. 1.)

magnetic matrix elements and in the $\mathbf{A}'^{[L]}_{\rm el}$ part of (11) corresponding to the gauge (x). As it can be seen, the inclusion of π MEC strongly reduces the gauge dependence (from say 5% to 0.1% in forward direction, for example). But the contribution of missing ρ MEC is still sizable, i.e., up to 0.8%, and stronger than the gauge dependence of π MEC.

Thus, several things can be learned from this simple OPORP model and will be supported later by the examination of realistic potentials:

(a) Even if not considered explicitly, the short-range ρ MEC is implicitly included in the Siegert operators up to a high degree, i.e., the calculations are accurate on the 1% level at 100 MeV, for example.

(b) The increased consistency by including π MEC in the magnetic matrix elements and in the $\mathbf{A}'^{[L]}_{\rm el}$ part of (11) in addition to N is reflected in a strongly reduced gauge dependence of the matrix elements.

(c) The lack of consistency cannot be read off the size of the gauge dependence quantitatively. For example, in Fig. 4(b) the gauge dependence becomes rather small at intermediate angles, whereas the contributions of missing ρ MEC is still relatively large.

Finally, we will consider two realistic potential models: the Paris potential[5] and the OPEPR Bonn potential.[11] First, we look again at the gauge dependence of N and of ($N+\pi$ MEC). In Fig. 5 the ratio $d\sigma/d\Omega$ [N, gauge (i)–(iv)]$/d\sigma/d\Omega(T)$ is plotted for the differential cross

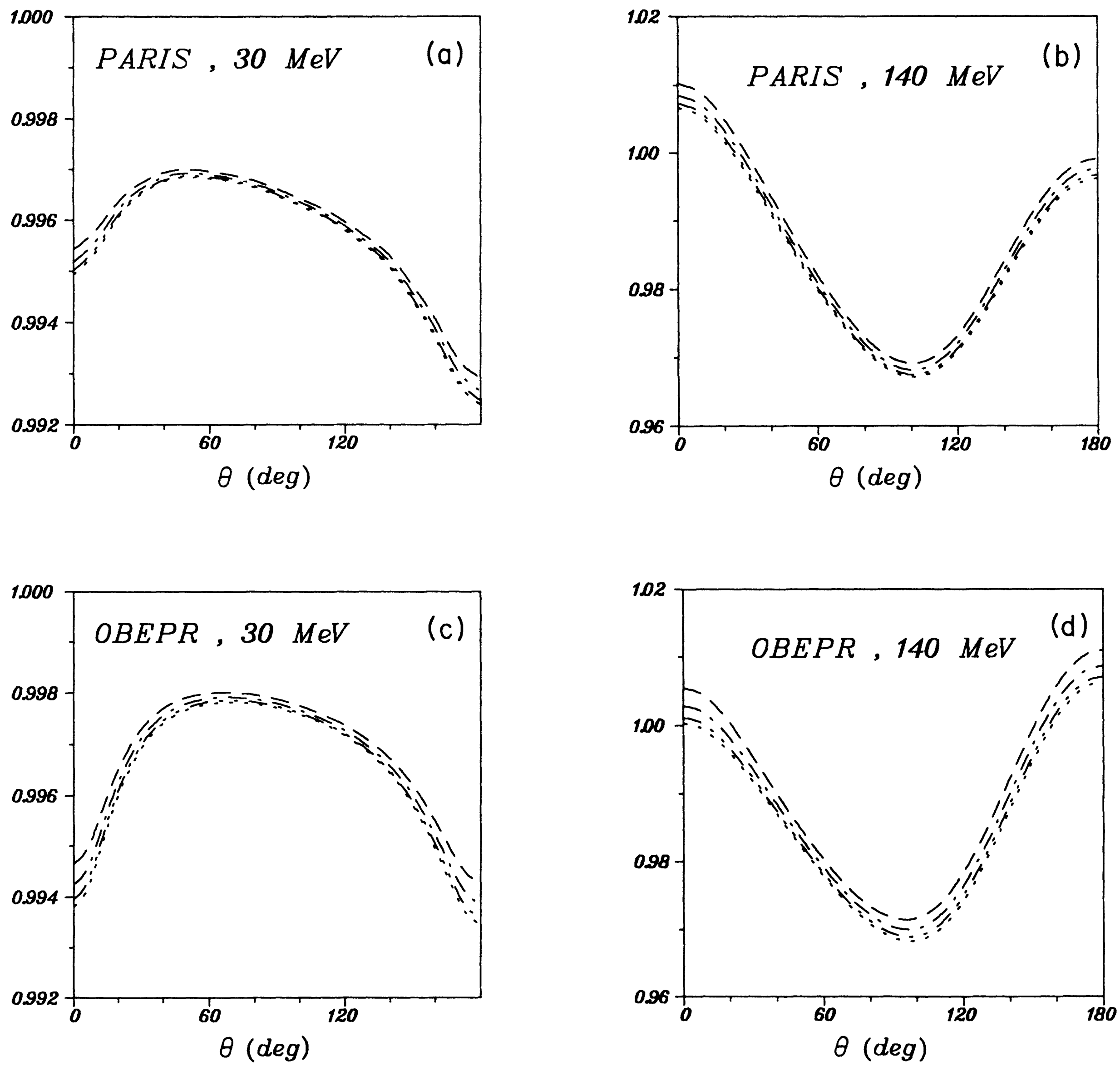
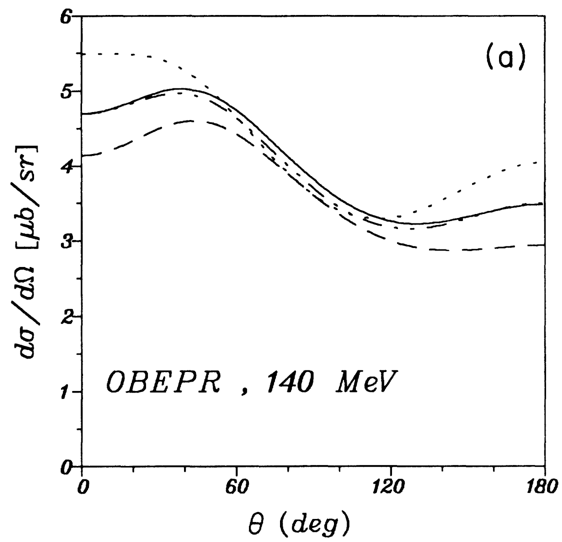


FIG. 6. Gauge dependence of ($N+\pi$ MEC) for the cases considered in Fig. 5. Shown is $d\sigma/d\Omega$ [$N+\pi$ MEC, gauge (i)–(iv)]$/d\sigma/d\Omega(T)$. (Notation of gauges as in Fig. 1.)

section at 30 and 140 MeV, where "T" refers in the case of the Paris potential to a calculation using the consistent exchange current (Paris EC) from Ref. 4 mentioned already and in the case of the OPEPR to $[N+(\pi+\rho)$ MEC, gauge (i)]. The results turn out to be rather similar for both potentials and, furthermore, agree qualitatively with those of the OPEP in Fig. 2, which is due to the dominance of π MEC corresponding to the long-range OPE tail implemented in any realistic potential.

The remaining gauge dependence after adding the π MEC is shown in Fig. 6, illustrating the most important result of our investigations that the inclusion of the π MEC beyond the Siegert operators reduces the gauge dependence within the restricted class of gauges drastically by more than one order of magnitude. Thus, the gauge dependence in deuteron photodisintegration below the pion threshold is reduced to a relative uncertainty of distinctly less than 1 percent in cross sections and also in other observables not shown here and the question of which gauge one should prefer does actually become rather unimportant.



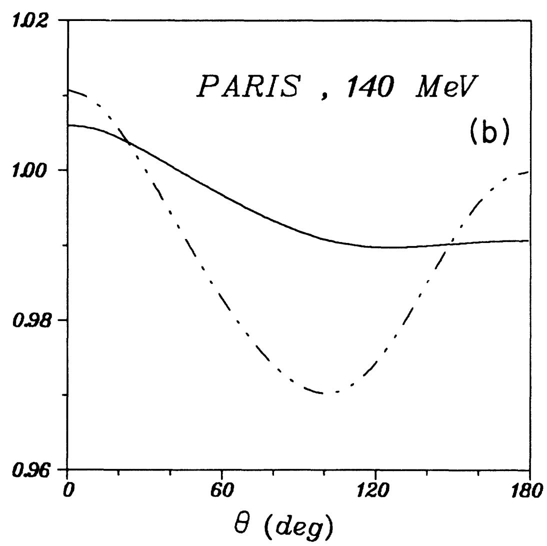


FIG. 7. Influence of the ρ MEC on the differential cross section at 140 MeV for the OBEPR and the Paris potential. Shown in (a) is $d\sigma/d\Omega$: $[N+\pi$ MEC, gauge (v)], dotted line; $[N+(\pi+\rho)$ MEC, gauge (v)], dashed line; $[N+\pi$ MEC, gauge (i)], dot-dot-dashed line; and $[N+(\pi+\rho)$ MEC, gauge (i)], solid curve, and in (b): $d\sigma/d\Omega$ $[N+\pi$ MEC, gauge (iii)]$/d\sigma/d\Omega$ (T), dot-dot-dashed line and $d\sigma/d\Omega$ $[N+(\pi+\rho)$ MEC, gauge (iii)]$/d\sigma/d\Omega$ (T), solid curve.

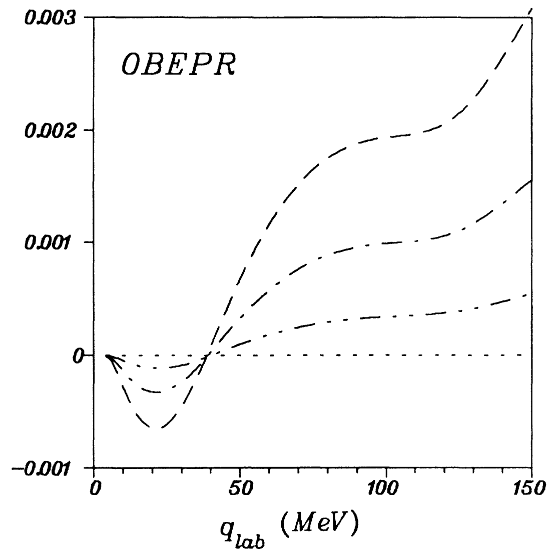


FIG. 8. Gauge dependence of the forward differential cross section for the OBEPR. Shown is $1-d\sigma/d\Omega$ $[N+(\pi+\rho)$ MEC, gauge (i)–(iv)]$|_{0°}/d\sigma/d\Omega$ $[N+(\pi+\rho)$ MEC, gauge (i)]$|_{0°}$. (Notation of gauges as in Fig. 1.)

In Fig. 7 we demonstrate the crucial role of Siegert operators in incorporating exchange current contributions beyond π MEC, if realistic potentials are used. To this end differential cross sections at 140 MeV are plotted in Fig. 7(a) for the OPEPR calculated with Siegert operators, where z^L gauge was chosen and with Siegert operators corresponding to the Cb gauge. The curves show the different influence of the ρ MEC in both approaches, which gets reduced from a 20% effect at extreme angles in the nonSiegert approach to a 2% effect in the Siegert approach. In Fig. 7(b) the ρ MEC influence in the Siegert approach is shown more quantitatively for the Paris potential again for the cross section at 140 MeV by using a relative representation referring to the consistent Paris EC result. Because of this analysis of ρ MEC, it seems reasonable to assume that the dominant contribution of other short-range exchange currents beyond π and ρ MEC, although not known explicitly, is also included in the Siegert operators to a large extent. Whereas in a non-Siegert approach rather large contributions, for example, corresponding to the difference between the full and dashed curves in Fig. 7(a) would be completely ignored.

Concluding the discussion we may state that our re-

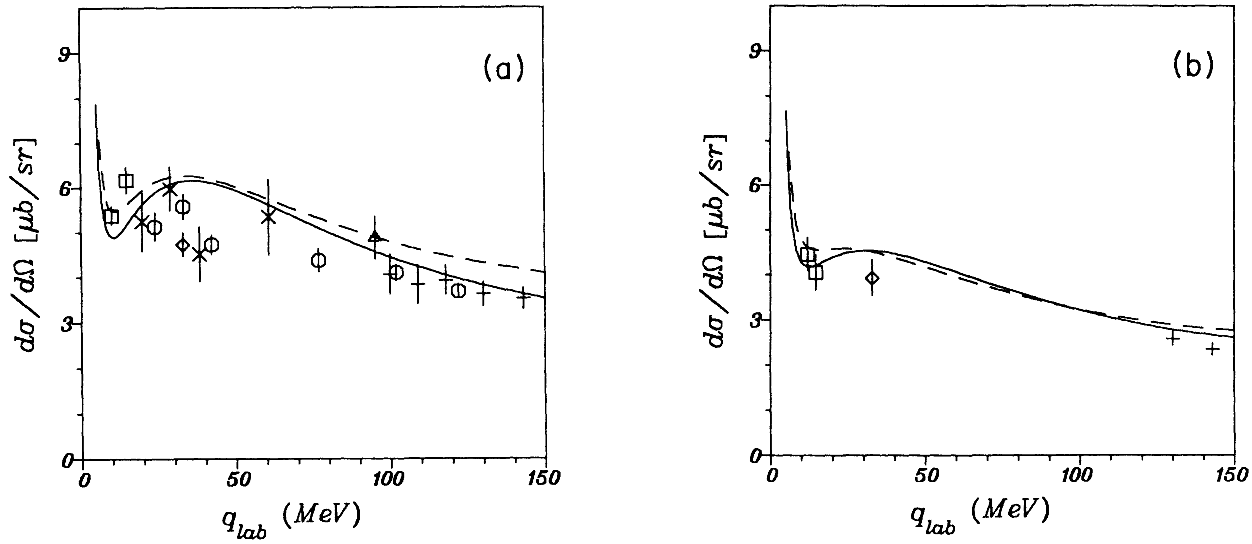


FIG. 9. Differential cross section at (a) forward and (b) backward direction for the OBEPR (solid curve) and the Paris potential (dashed curve) including [$N+(\pi+\rho)$ MEC], isobar configurations and the relativistic spin-orbit current. Data points are from Ref. 12: ○, Ref. 13: △, Ref. 14: ◇, Ref. 15: ×, Ref. 16: □, Ref. 17: +.

sults clearly contradict Ref. 6. There a rather strong gauge dependence in the forward differential cross section for the OPEPR was found, which remained almost unchanged within the order of some percent after adding MEC. For comparison with their results we show the gauge dependence of this quantity again for the OPEPR in Fig. 8. The variation inside the restricted class of gauges is less than 0.3% over the whole energy range. We note that the curves in Fig. 8 also include the ρ MEC, however, its influence concerning the gauge dependence is very small compared to the π MEC. Thus, it seems indeed that the approximate evaluation of the MEC in Ref. 6 is poor and responsible for the gauge dependence of their results.

Our main conclusion is that the conventional theory provides a reliable framework for the calculation of deuteron photodisintegration if the dominant π- and ρ-exchange currents are included in conjunction with the Siegert operators. Then the gauge dependence is negligible even if the exchange current is not consistent beyond π and ρ exchange. Also, the neglect of explicit ρ MEC beyond the Siegert operator does not increase the gauge dependence. Thus, the critique of Nagornyi et al.[1] is not well founded. As an illustration of the present state of the conventional theory in comparison to the experiment we show in Fig. 9, for the differential cross section at 0° and 180° presently available experimental data and theoretical results for the Paris and Bonn potentials, where besides MEC also the relativistic spin-orbit current and isobar configurations are included. In view of the still rather large experimental uncertainties the agreement is quite satisfactory.

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[1]S. I. Nagornyi, Yu. A. Kasatkin, E. V. Inopin, and I. K. Kirichenko, Yad. Fiz. **44**, 1171 (1986) [Sov. J. Nucl. Phys. **44**, 760 (1986)].
[2]H. Arenhövel, Z. Phys. A **302**, 25 (1981).
[3]M. Gari and H. Hebach, Phys. Rep. **72C**, 1 (1981).
[4]A. Buchmann, W. Leidemann, and H. Arenhövel, Nucl. Phys. **A443**, 726 (1985).
[5]M. Lacombe, B. Loiseau, J. M. Richard, R. Vinh Mau, J. Coté, P. Pires, and R. de Tourreil, Phys. Rev. C **21**, 861 (1980).
[6]S. Ying, E. M. Henley, and G. A. Miller, Phys. Rev. C **38**, 1584 (1988).
[7]F. Partovi, Ann. Phys. (N.Y.) **27**, 79 (1964).
[8]L. L. Foldy, Phys. Rev. **92**, 178 (1953).
[9]J. L. Friar and S. Fallieros, Phys. Rev. C **29**, 1645 (1984); **34**, 2029 (1986).
[10]R. M. K. Hämäläinen, Ann. Acad. Sci. Fennicae A VI **125**, 35 (1962).
[11]R. Machleidt, K. Holinde, and Ch. Elster, Phys. Rep. **149**, 1 (1987).
[12]R. J. Hughes, A. Zieger, H. Wäffler, and B. Ziegler, Nucl. Phys. **A267**, 329 (1976).
[13]H. O. Meyer, J. R. Hall, M. Hugi, H. J. Karwowski, R. E. Pollock, and P. Schwandt, Phys. Rev. Lett. **52**, 1759 (1984).
[14]A. Ninane, C. Dupont, P. Leleux, P. Lipnik, and P. Macq, Phys. Rev. C **35**, 402 (1987).
[15]G. Matone et al., in *Proceedings of the International School of Intermediate Energy Nuclear Physics (Verona), 1985,* edited by R. Bergère, S. Costa, and C. Schaerf (World Scientific, Singapore, 1986), p. 505.
[16]A. De Graeve, Ph.D. thesis, Rijksúniversiteit Gent, 1988.
[17]P. Levi Sandri et al., Phys. Rev. C **39**, 1701 (1989).