Relation among theories of inclusive breakup reactions

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The relation among various recently proposed theories for inclusive breakup reactions is clarified from a unified viewpoint. Udagawa and Tamura's prior form formalism, Ichimura, Austern, and Vincent's post form formalism, and Hussein and McVoy's formalism are understood on a common basis by referring to a three-body model of Austern *et al.*

I. INTRODUCTION

A variety of theories have been proposed for inclusive breakup reactions

$$a + A \rightarrow b + \text{anything} , \tag{1.1}$$

where the ejectile b is a definite fragment emitted from the projectile a ($=b+x$). A prior form formalism was proposed by Udagawa and Tamura[1] (UT), while a post from DWBA formalism was given by Austern and Vincent[2] and elaborated by Kasano and Ichimura.[3] This formalism was carefully investigated by Ichimura, Austern, and Vincent[4] and thus it is usually referred to as the IAV formalism. A surface approximation method of Baur *et al.*[5] belongs to this kind. Hussein and McVoy[6] (HM) introduced a different formalism through a somewhat intuitive way and Austern *et al.*[7] presented a formalism based on a three-body model (3B).

The differential cross section of the process (1.1) is often decomposed into elastic breakup (EB) and nonelastic breakup (NEB) parts,

$$\frac{d^2\sigma^{\text{inc}}}{d\Omega_b dE_b} = \frac{d^2\sigma^{\text{EB}}}{d\Omega_b dE_b} + \frac{d^2\sigma^{\text{NEB}}}{d\Omega_b dE_b} . \tag{1.2}$$

The former represents the process $a + A \rightarrow b + x + A$ where the target A stays in its ground state, and the latter is the rest which is sometimes called inelastic breakup[5] or breakup fusion.[1]

Methods of calculating the EB part are on a common basis in all the theories, but the NEB part is calculated in very different ways. Very interestingly the theories all have a common formal structure

$$\frac{d^2\sigma^{\text{NEB}}}{d\Omega_b dE_b} = -\frac{2}{v_a}\rho(E_b)\langle \Psi_x | W_{xA} | \Psi_x \rangle , \tag{1.3}$$

but the definition of Ψ_x in this expression differs very much from theory to theory. Here v_a is the velocity of the projectile a and W_{xA} is the imaginary part of the optical potential of x on A, U_{xA}, and $\rho(E_b)$ is the state density of the observed ejectile b with energy E_b. That is to say that Ψ_x of UT, IAV, HM, and 3B, are, respectively, given by the formulas (2.37), (2.27), (2.29), and (2.20), which will be rederived in the next section. The differences sometimes give rise to controversy.[4,9–15]

The main aim of this paper is to understand these formalisms in a unified manner, to clarify the meaning of the differences, and to consider their merits and demerits. In the following argument the three-body model[7] plays a key role. A preliminary report was given in a Calcutta conference.[16]

In Sec. II we review the theories in a way that comparison among the theories is as transparent as possible. We clarify the relation among them in Sec. III and discussions are given in Sec. IV.

II. REVIEW OF THE THEORIES

Let us consider a system with a Hamiltonian

$$H = K_b + K_x + H_A + V_{bx} + V_{xA} + V_{bA} , \tag{2.1}$$

where we assume that b and x are structureless and A is infinitely massive. Here K_b and K_x are the kinetic energy operators of b and x, respectively, and H_A is the internal Hamiltonian of A and V_{ij} is the interaction between particles i and j.

In the post interaction form the T matrix for the process $a + A \rightarrow b + c$ (c being a state of the $x + A$ system) is given by

$$T_c = \langle \chi_b^{(-)} \Psi_{xA}^{c,(-)} | V_{bx} + V_{bA} - U_{bA} | \Psi^{(+)} \rangle , \tag{2.2}$$

where $\Psi^{(+)}$, $\Psi_{xA}^{c,(-)}$, and $\chi_b^{(-)}$ are the wave functions of the total system with the incident wave in the $a + A$ channel, of the state c of the system ($x + A$) and the distorted wave of b, respectively, and U_{bA} is the optical potential of b on A. They obey the equations,

$$H\Psi^{(+)} = E\Psi^{(+)} , \tag{2.3}$$

$$H_{xA}\Psi_{xA}^{c,(-)} = E_c \Psi_{xA}^{c,(-)}, \quad H_{xA} = K_x + V_{xA} + H_A , \tag{2.4}$$

and

$$(K_b + U_{bA}^{\dagger} - E_b)\chi_b^{(-)} = 0 . \tag{2.5}$$

A spectator model commonly used in all the theories mentioned above assumes that particle b travels simply in the complex mean field (the optical potential) U_{bA} and it replaces V_{bA} by U_{bA}. The inclusive breakup cross section is then given by

$$\frac{d^2\sigma^{\rm inc}}{d\Omega_b dE_b}=\frac{2\pi}{v_a}\rho(E_b)\sum_c |T_c|^2\delta(E_b+E_c-E)$$
$$=-\frac{2}{v_a}\rho(E_b)\mathrm{Im}\langle\Psi^{(+)}|V_{bx}|\chi_b^{(-)}\rangle G_{xA}\langle\chi_b^{(-)}|V_{bx}|\Psi^{(+)}\rangle\ , \tag{2.6}$$

with

$$G_{xA}=\frac{1}{E-E_b-H_{xA}+i\epsilon}\ , \tag{2.7}$$

where the completeness of $\Psi_{xA}^{c,(-)}$ is used.

A. Three-body (3B) model of Austern *et al.*

In order to make the presentation of the theories clearer, let us disturb chronological order and start with the latest one, the three-body model of Austern *et al.*[7] In this model the structure of the target A is frozen to its ground state and its excitation is implicitly included through the complex potential U_{bA} and U_{xA}. Then, the wave function of the total system $\Psi^{(+)}$ is approximated as

$$\Psi^{(+)}\approx\psi_{xb}^{3\mathrm{B}(+)}\Phi_A^0\ . \tag{2.8}$$

Here $\psi_{xb}^{3\mathrm{B}(+)}$ obeys the three-body model equation

$$(K_b+K_x+V_{bx}+U_{xA}+U_{bA})\psi_{xb}^{3\mathrm{B}(+)}=E\psi_{xb}^{3\mathrm{B}(+)}\ , \tag{2.9}$$

and Φ_A^0 is the ground state wave function of the target subject to the equation

$$H_A\Phi_A^n=E_A^n\Phi_A^n\ , \tag{2.10}$$

where $n=0$ denotes the ground state and we set $E_A^0=0$.

The first key equation is the *exact* optical reduction[2]

$$G_{xA}^{\rm opt}=(\Phi_A^0|G_{xA}|\Phi_A^0)=\frac{1}{E-E_b-K_x-U_{xA}+i\epsilon}\ , \tag{2.11}$$

where the optical potential U_{xA} is defined by

$$U_{xA}=(\Phi_A^0|\hat{U}_{xA}|\Phi_A^0)\ , \tag{2.12}$$

and

$$\hat{U}_{xA}=PV_{xA}P+PV_{xA}Q(E_x-QH_{xA}Q+i\epsilon)^{-1}QV_{xA}P\ , \tag{2.13}$$

with $P=|\Phi_A^0\rangle\langle\Phi_A^0|$ and $Q=1-P$.

Inserting Eq. (2.8) into Eq. (2.6), noting that V_{bx} does not excite Φ_A^0 and using Eq. (2.11), we obtain the expression

$$\frac{d^2\sigma^{\rm inc,3B}}{d\Omega_b dE_b}=-\frac{2}{v_a}\rho(E_b)\mathrm{Im}\langle\psi_{xb}^{3\mathrm{B}(+)}|V_{bx}|\chi_b^{(-)}\rangle G_{xA}^{\rm opt}\langle\chi_b^{(-)}|V_{bx}|\psi_{xb}^{3\mathrm{B}(+)}\rangle\ . \tag{2.14}$$

The second key identity[1,3] is

$$\mathrm{Im}G_{xA}^{\rm opt}=(1+G_{xA}^{\rm opt\dagger}U_{xA}^\dagger)[\mathrm{Im}G_0](1+U_{xA}G_{xA}^{\rm opt})+G_{xA}^{\rm opt\dagger}W_{xA}G_{xA}^{\rm opt}$$
$$=-\pi\sum_{\mathbf{k}_x}|\chi_x^{(-)}(\mathbf{k}_x)\rangle\delta\left[E-E_b-\frac{k_x^2}{2m_x}\right]\langle\chi_x^{(-)}(\mathbf{k}_x)|+G_{xA}^{\rm opt\dagger}W_{xA}G_{xA}^{\rm opt}\ , \tag{2.15}$$

where G_0 is the free Green's function of x, $G_0=(e-E_b-K_x+i\epsilon)^{-1}$. The distorted wave of x, $\chi_x^{(-)}(\mathbf{k}_x)$, is defined by

$$\chi_x^{(-)}(\mathbf{k}_x)=(\Phi_A^0|\Psi_{xA}^{c0,(-)}\rangle \tag{2.16}$$

and is subject to the equation

$$(K_x+U_{xA}^\dagger-E_x)\chi_x^{(-)}(\mathbf{k}_x)=0\ ,$$
$$E_x=E-E_b=\frac{k_x^2}{2m_x}\ , \tag{2.17}$$

where $c0$ denotes the elastic breakup channel.

Employing Eq. (2.15), the inclusive breakup cross section (2.14) is decomposed into EB and NEB parts as

$$\frac{d^2\sigma^{\rm EB,3B}}{d\Omega_b dE_B}=\frac{2\pi}{v_a}\rho(E_b)\sum_{\mathbf{k}_x}|\langle\chi_b^{(-)}\chi_x^{(-)}(\mathbf{k}_x)|V_{bx}|\psi_{xb}^{3\mathrm{B}(+)}\rangle|^2\delta\left[E-E_b-\frac{k_x^2}{2m_x}\right]\ , \tag{2.18}$$

and

$$\frac{d^2\sigma^{\rm NEB,3B}}{d\Omega_b dE_b}=-\frac{2}{v_a}\rho(e_b)\langle\Psi_x^{3\mathrm{B}}|W_{xA}|\Psi_x^{3\mathrm{B}}\rangle\ , \tag{2.19}$$

with

$$\Psi_x^{3\mathrm{B}}=G_{xA}^{\rm opt}(\chi_b^{(-)}|V_{bx}|\psi_{xb}^{3\mathrm{B}(+)}\rangle\ . \tag{2.20}$$

They have now a manageable form. Using the expression of the T matrix for EB

$$T^{\rm EB}=\langle\chi_b^{(-)}\Psi_{xA}^{c0,(-)}|V_{bx}|\Psi^{(+)}\rangle\ , \tag{2.21}$$

and Eqs. (2.8) and (2.16), we can identify Eq. (2.18) with the EB cross section.

Rewriting V_{bx} in Eq. (2.20) as

$$V_{bx}=[(V_{bx}+K_b+K_x+U_{bA}+U_{xA}-E) -(K_b+K_x+U_{bA}+U_{xA}-E)] , \quad (2.22)$$

and using the Schrödinger equations for $\psi_{xb}^{3B(+)}$ and $\chi_b^{(-)}$, Austern *et al.*[7] derived an interesting identity

$$\Psi_x^{3B}=(\chi_b^{(-)}|\psi_{xb}^{3B(+)}\rangle \quad (2.23)$$

which is very useful for later discussion.

B. Post-form DWBA (Ichimura-Austern-Vincent formula) (IAV)

In the post form DWBA, $\Psi^{(+)}$ in Eq. (2.2) is approximated as

$$\Psi^{(+)}\approx\chi_a^{(+)}\phi_a\Phi_A^0 , \quad (2.24)$$

where ϕ_a and $\chi_a^{(+)}$ are the bound state internal wave function and the distorted wave of a respectively.

Following the same procedure used in Sec. II A, we obtain the formulas for the cross sections by simply replacing $\psi_{xb}^{3B(+)}$ by $\chi_a^{(+)}\phi_a$ as

$$\frac{d^2\sigma^{EB,IAV}}{d\Omega_b dE_b}=\frac{2\pi}{v_a}\rho(E_b)\sum_{\mathbf{k}_x}|\langle\chi_b^{(-)}\chi_x^{(-)}(\mathbf{k}_x)|V_{bx}|\chi_a^{(+)}\phi_a\rangle|^2\delta\left[E-E_b-\frac{k_x^2}{2m_x}\right] \quad (2.25)$$

and

$$\frac{d^2\sigma^{NEB,IAV}}{d\Omega_b dE_b}=-\frac{2}{v_a}\rho(E_b)\langle\Psi_x^{IAV}|W_{xA}|\Psi_x^{IAV}\rangle , \quad (2.26)$$

with

$$\Psi_x^{IAV}=G_{xA}^{opt}(\chi_b^{(-)}|V_{bx}|\chi_a^{(+)}\phi_a\rangle . \quad (2.27)$$

where Eq. (2.20) is used.

C. Hussein-McVoy formula (HM)

A way[14,15] to derive their formula is to replace $\psi_{xb}^{3B(+)}$ by $\chi_a^{(+)}\phi_a$ as was done in IAV, but in the expression (2.23) instead of in (2.20). Thus the EB cross section of HM is the same as that of IAV, but the NEB one is given by

$$\frac{d^2\sigma^{NEB,HM}}{d\Omega_b dE_b}=-\frac{2}{v_a}\rho(E_b)\langle\Psi_x^{HM}|W_{xA}|\Psi_x^{HM}\rangle , \quad (2.28)$$

with

$$\Psi_x^{HM}=(\chi_b^{(-)}|\chi_a^{(+)}\phi_a\rangle . \quad (2.29)$$

In the original paper[6] they derived these formulas in an intuitive way and considered that they gave the full inclusive breakup cross section, but later it turned out that Eq. (2.28) with (2.29) only represents the NEB part.[14]

D. Udagawa-Tamura formula (UT)

UT started from the total reaction cross section

$$\sigma^{rec}=-\frac{2}{v_a}\rho(E_b)\mathrm{Im}\langle\chi_a^{(+)}\phi_a\Phi_A^0|(V_{xA}+U_{bA}^{\dagger}-U_{aA}^{\dagger})\frac{1}{E-H+i\epsilon}(V_{xA}+U_{bA}-U_{aA})|\chi_a^{(+)}\phi_a\Phi_A^0\rangle , \quad (2.30)$$

and claimed that the inclusive breakup contribution can be singled out[1] by replacing $(E-H-i\epsilon)^{-1}$ by a "Green's function of the breakup channel"

$$G_d=[E-(K_b+U_{bA}+K_x+\hat{U}_{xA}+H_A)+i\epsilon]^{-1} . \quad (2.31)$$

The point is that this Green's function does not contain V_{bx} and thus the bound state of $b+x$ system does not appear in the asymptotic region of the final states.

This still allows processes in which b is a participant. To discard processes in which b is absorbed and to fix the outgoing momentum of b, UT developed a more elaborate manipulation,[1] with some additional approximations. Then they reached an approximate expression for the inclusive breakup cross section

$$\frac{d^2\sigma^{inc,UT}}{d\Omega_b dE_b}=-\frac{2}{v_a}\rho(E_b)\mathrm{Im}\langle\chi_a^{(+)}\phi_a\Phi_A^0|(V_{xA}+U_{bA}^{\dagger}-U_{aA}^{\dagger})|\chi_b^{(-)}) \times(E-E_b-K_x-\hat{U}_{xA}-H_A+i\epsilon)^{-1}(\chi_b^{(-)}|(V_{xA}+U_{bA}-U_{aA})|\chi_a^{(+)}\phi_a\Phi_A^0\rangle . \quad (2.32)$$

Further if one makes the replacement

$$V_{xA}\rightarrow\hat{U}_{xA} , \quad (2.33)$$

the excitation of the target A by the interaction V_{xA} does not occur any more and one can set H_A by $E_A^0=0$ and arrive

at the expression

$$\frac{d^2\sigma^{\mathrm{inc,UT}}}{d\Omega_b dE_b}=-\frac{2}{v_a}\rho(E_b)\mathrm{Im}\langle\chi_a^{(+)}\phi_a|(U_{xA}^{\dagger}+U_{bA}^{\dagger}-U_{aA}^{\dagger})|\chi_b^{(-)})G_{xA}^{\mathrm{opt}}(\chi_b^{(-)}|(U_{xA}+U_{bA}-U_{aA})|\chi_a^{(+)}\phi_a\rangle\ . \tag{2.34}$$

Using Eq. (2.15), one obtains the UT formula

$$\frac{d^2\sigma^{\mathrm{EB,UT}}}{d\Omega_b dE_b}=\frac{2\pi}{v_a}\rho(E_b)\sum_{\mathbf{k}_x}|\langle\chi_b^{(-)}\chi_x^{(-)}(\mathbf{k}_x)|(U_{xA}+U_{bA}-U_{aA})|\chi_a^{(+)}\phi_a\rangle|^2\delta\left[E-E_b-\frac{k_x^2}{2m_x}\right], \tag{2.35}$$

$$\frac{d^2\sigma^{\mathrm{NEB,UT}}}{d\Omega_b dE_b}=-\frac{2}{v_a}\rho(E_b)\langle\Psi_x^{\mathrm{UT}}|W_{xA}|\Psi_x^{\mathrm{UT}}\rangle\ , \tag{2.36}$$

with

$$\Psi_x^{\mathrm{UT}}=G_{xA}^{\mathrm{opt}}(\chi_b^{(-)}|(U_{xA}+U_{bA}-U_{aA})|\chi_a^{(+)}\phi_a\rangle\ . \tag{2.37}$$

The replacement (2.33) means[4] that this formalism does not include breakup with simultaneous excitation of the target A by the interaction V_{xA}. Therefore the expression (2.36) only represents the process, $a+A\to b+x+A\to b+c$, which UT named elastic breakup fusion (EBF).

III. RELATION AMONG THE THEORIES

In the preceding section, the 3B-model, IAV, and HM formalisms are derived in a unified manner. The relation between IAV and UT formalisms was first discussed in IAV.[4] In this section we clarify the relation among them in a more explicit and/or simpler way.

A. Elastic breakup (EB)

As was shown, the EB formulas if IAV and HM are equivalent and are obtained just by the replacement $\psi_{xb}^{3\mathrm{B}(+)}\to\chi_a^{(+)}\phi_a$ in the 3B-model formula. The equivalence of the EB formulas of UT and IAV was shown by IAV.[4] A detailed proof is the following.

The prior-post symmetry of the DWBA amplitudes is expressed as

$$\langle\chi_b^{(-)}\Psi_{xA}^{c0,(-)}|V_{xA}+U_{bA}-U_{aA}|\chi_a^{(+)}\phi_a\Phi_A^0\rangle=\langle\chi_b^{(-)}\Psi_{xA}^{c0,(-)}|V_{bx}|\chi_a^{(+)}\phi_a\Phi_A^0\rangle\ . \tag{3.1}$$

Using Eq. (2.16) and noting V_{bx} does not affect Φ_A^0, the right hand side (rhs) of Eq. (3.1) becomes

$$\mathrm{rhs}=\langle\chi_b^{(-)}\chi_x^{(-)}(\mathbf{k}_x)|V_{bx}|\chi_a^{(+)}\phi_a\rangle\ . \tag{3.2}$$

On the other hand, the left hand side (lhs) is rewritten as

$$\mathrm{lhs}=\langle\chi_b^{(-)}\chi_x^{(-)}(\mathbf{k}_x)|(U_{xA}+U_{bA}-U_{aA})|\chi_a^{(+)}\phi_a\rangle\ , \tag{3.3}$$

by use of the identity

$$\langle\Psi_{xA}^{c0,(-)}|V_{xA}|\Phi_A^0\rangle=\langle\chi_x^{(-)}(\mathbf{k}_x)|U_{xA}\ , \tag{3.4}$$

which is proved by writing $\Psi_{xA}^{c0,(-)}$ as

$$\Psi_{xA}^{c0,(-)}=(P+Q)\Psi_{xA}^{c0,(-)}=\left[1+Q\frac{1}{E_x-QH_{xA}Q-i\epsilon}QV_{xA}\right]P\Psi_{xA}^{c0,(-)}\ , \tag{3.5}$$

and using Eqs. (2.12), (2.13), and (2.16). Thus the equivalence between Eqs. (2.25) and (2.35) is proved.

B. Nonelastic breakup (NEB)

Recently Frederico, Mastroleo, and Hussein[15] (FMH) discussed the relation among these theories by using the Faddeev equations for the three-body model. Here we present similar argumentation in a somewhat different manner.

The three-body model wave function $\psi_{xb}^{3\mathrm{B}(+)}$ is expressed in terms of the wave function, $\chi_a^{(+)}\phi_a$, used in DWBA as

$$\psi_{xb}^{3\mathrm{B}(+)}=\chi_a^{(+)}\phi_a+G_{3\mathrm{B}}(U_{xA}+U_{bA}-U_{aA})\chi_a^{(+)}\phi_a\ , \tag{3.6}$$

where $G_{3\mathrm{B}}$ is the Green's function of the three-body model

$$G_{3\mathrm{B}}=\frac{1}{E-(K_b+U_{bA}+K_x+U_{xA}+V_{bx})+i\epsilon}\ . \tag{3.7}$$

Inserting Eq. (3.6) into Eq. (2.20), we obtain the relation

$$\Psi_x^{3\mathrm{B}}=\Psi_x^{\mathrm{IAV}}+\delta\Psi_x^{3\mathrm{B}}\ , \tag{3.8}$$

with

$$\delta\Psi_x^{3\mathrm{B}}=G_{xA}^{\mathrm{opt}}(\chi_b^{(-)}|V_{bx}G_{3\mathrm{B}}(U_{xA}+U_{bA}-U_{aA})|\chi_a^{(+)}\phi_a\rangle\ . \tag{3.9}$$

On the other hand inserting Eq. (3.6) into Eq. (2.23), we get

$$\Psi_x^{3\mathrm{B}}=\Psi_x^{\mathrm{HM}}+(\chi_b^{(-)}|G_{3\mathrm{B}}(U_{xA}+U_{bA}-U_{aA})|\chi_a^{(+)}\phi_a\rangle\ . \tag{3.10}$$

Using the identity

$$(\chi_b^{(-)}|G_{3\mathrm{B}}=G_{xA}^{\mathrm{opt}}(\chi_b^{(-)}|+G_{xA}^{\mathrm{opt}}(\chi_b^{(-)}|V_{bx}G_{3\mathrm{B}}\ , \tag{3.11}$$

the second term of Eq. (3.10) turns out to be

$$\Psi_x^{\mathrm{UT}}+G_{xA}^{\mathrm{opt}}(\chi_b^{(-)}|V_{bx}G_{3\mathrm{B}}(U_{xA}+U_{bA}-U_{aA})|\chi_a^{(+)}\phi_a\rangle \ . \tag{3.12}$$

Thus we get the relation

$$\Psi_x^{3\mathrm{B}}=\Phi_x^{\mathrm{HM}}+\Psi_x^{\mathrm{UT}}+\delta\Psi_x^{3\mathrm{B}} \ . \tag{3.13}$$

Equations (3.8) and (3.13) lead to a previously known relation[8,4,15]

$$\Psi_x^{\mathrm{IAV}}=\psi_x^{\mathrm{UT}}+\Psi_x^{\mathrm{HM}} \ . \tag{3.14}$$

From Eqs. (3.8) and (3.14), we get relations among the NEB cross sections

$$\frac{d^2\sigma^{\mathrm{NEB,3B}}}{d\Omega_b dE_b}=\frac{d^2\sigma^{\mathrm{NEB,IAV}}}{d\Omega_b dE_b}-\frac{2}{v_a}\rho(E_b)[2\,\mathrm{Re}\langle\Psi_x^{\mathrm{IAV}}|W_{xA}|\delta\Psi_x^{3\mathrm{B}}\rangle+\langle\delta\Psi_x^{3\mathrm{B}}|W_{xA}|\delta\Psi_x^{3\mathrm{B}}\rangle] \tag{3.15}$$

and

$$\frac{d^2\sigma^{\mathrm{NEB,IAV}}}{d\Omega_b dE_b}=\frac{d^2\sigma^{\mathrm{NEB,HM}}}{d\Omega_b dE_b}+\frac{d^2\sigma^{\mathrm{NEB,UT}}}{d\Omega_b dE_b}-\frac{4}{v_a}\rho(E_b)\mathrm{Re}\langle\Psi_x^{\mathrm{HM}}|W_{xA}|\Psi_x^{\mathrm{UT}}\rangle \ . \tag{3.16}$$

The latter relation was first derived by IAV. The first term of the lhs (the HM term) is often called the nonorthogonality term.

Within the framework of the three-body model, IAV and HM are approximations in which the replacement $\psi_{xb}^{3\mathrm{B}(+)}\rightarrow\chi_a^{(+)}\phi_a$ is made in Eq. (2.20) and Eq. (2.23), respectively. The same approximate replacement in *formally identical* but different expressions can lead to different results. The difference between the IAV and HM cross sections is explicitly given in Eq. (3.16).

IV. DISCUSSIONS

Here we compare these formalisms and discuss their merits and demerits, problems in derivation and in application, etc.

A. 3B model vs DWBA

In general the three-body model is more advanced than DWBA. However it should be noted that the distorted wave approximation, $\Psi^{(+)}\rightarrow\chi_a^{(+)}\phi_a$, is not necessarily within the three-body model, because U_{aA} could implicitly include some processes which are not in the three-body model. For instance, the 3B model does not include the process in which the target A is excited by V_{bA} or V_{xA} then V_{bx} acts and then A is deexcited by V_{bA} or V_{xA}, because U_{xA} and U_{bA} are used instead of V_{bA} and V_{xA} and thus A is always in the ground state when V_{bx} works. On the other hand, the optical potential U_{aA} implicitly includes such a process.

B. IAV vs HM formalisms

As was discussed, the IAV and HM formulas are derived by inserting the same replacement (2.33) in formally equivalent but different expressions, (2.20) and (2.23), of the 3B model. Therefore it is hard to discuss which is better. However, it is worth pointing out that to calculate Ψ_x^{IAV} one needs a good approximation for the wave function only for small separation r_{bx} between b and x, because of the presence of V_{bx}, while to calculate Ψ_x^{HM} one needs a good wave function in the whole region of r_{bx}. We may use zero range approximation in IAV (e.g., in the case of deuteron breakup) but a full finite range calculation is needed in HM.

According to numerical analysis by Mastroleo, Udagawa, and Tamura[13] (MUT), the NEB cross section of HM is very close to that of IAV for $^{58}\mathrm{Ni}(\alpha,p)$ but the former is much smaller (about a factor of 2) than the latter for $^{62}\mathrm{Ni}(d,p)$. This suggests that IAV and HM give similar results for a tightly bound projectile like α, but they are different for a loosely bound one like d.

It is interesting to note that the second term of Eq. (3.6) involves both the bound and unbound states of the $b+x$ system, but the first only contains the bound state ϕ_a. Therefore the second term may be relatively more important for a loosely bound projectile than for a tightly bound one.

Further, one may expect that in the loosely bound case the unbound components have large effects on overlaps (2.23) of the HM type but not so much on Ψ_x of the IAV type (2.20), because in the latter the wave function $\psi_{xb}^{3\mathrm{B}(+)}$ is only used at small separations of b and x. This observation seems consistent with the results of the MUT calculation. Therefore the IAV method seems preferable in practical use.

C. Meaning of UT formalism

The interaction $(V_{xA}+U_{bA}-U_{aA})$ in Eq. (2.32) reminds us of the prior form DWBA, which has the t matrix

$$T_c^{pr-DW}=\langle\chi_b^{(-)}\Psi_{xA}^{c,(-)}|V_{xA}+U_{bA}-U_{aA}|\chi_a^{(+)}\phi_a\Phi_A^0\rangle \ , \tag{4.1}$$

instead of Eq. (2.2). The equation corresponding to Eq. (2.6) is found to be

$$\frac{d^2\sigma^{\text{inc,prDW}}}{d\Omega_b dE_b} = -\frac{2}{v_a}\rho(E_b)\text{Im}\langle \chi_a^{(+)}\phi_a\Phi_A^0 | V_{xA}+U_{bA}^{\dagger}-U_{aA}^{\dagger}|\chi_b^{(-)}\rangle G_{xA}(\chi_b^{(-)}|V_{xA}+U_{bA}-U_{aA}|\chi_a^{(+)}\phi_a\Phi_A^0\rangle \ . \tag{4.2}$$

Note that here V_{xA} appears in the Green's function G_{xA}, whereas Eq. (2.32) contains $\hat{U}_{xA}$.

However, the replacement (2.33) in the interaction again reduces Eq. (4.2) to the UT formula (2.34), because $\hat{U}_{xA}$ does not excite Φ_A^0 and hence G_{xA} is sandwiched by Φ_A^0. The physical meaning of the UT approximations is now clearer. Namely, the UT formula for NEB represents the cross section of the EBF process in the prior form DWBA via the interaction $(\hat{U}_{xA}+U_{bA}-U_{aA})$,

$$\frac{d^2\sigma^{\text{NEB,UT}}}{d\Omega_b dE_b} = \frac{2\pi}{v_a}\rho(E_b)\sum_{c\neq c0}|\langle \chi_b^{(-)}\Psi_{xA}^{c,(-)}|\hat{U}_{xA}+U_{bA}-U_{aA}|\chi_a^{(+)}\phi_a\Phi_A^0\rangle|^2\delta(E_b+E_c-E) \ . \tag{4.3}$$

It is worth commenting that the procedure $(E-H+i\epsilon)^{-1}\rightarrow G_d=(E-K_b-K_x-\hat{U}_{xA}-U_{bA}-H_A+i\epsilon)^{-1}$ in the step from Eq. (2.30) to Eq. (2.32) is replaced in Eq. (4.2) by the weaker approximation $(E-H+i\epsilon)^{-1}\rightarrow(E-K_b-K_x-V_{xA}-U_{bA}-H_A+i\epsilon)^{-1}$.

On the other hand, starting from Eq. (4.2) and eliminating V_{xA} by *exact* manipulation, IAV (Ref. 4) derived the relation (3.16) between the UT and IAV formalisms and also proved that the prior-form DWBA cross section is given by the rhs of Eq. (3.16), that is,

$$\frac{d^2\sigma^{\text{NEB,IAV}}}{d\Omega_b dE_b} = \text{rhs of (3.16)}$$

$$= \frac{2\pi}{v_a}\rho(E_b)\sum_{c\neq c0}|\langle \chi_b^{(-)}\Psi_{xA}^{c,(-)}|V_{xA}+U_{bA}-U_{aA}|\chi_a^{(+)}\phi_a\Phi_A^0\rangle|^2\delta(E_b+E_c-E) \ , \tag{4.4}$$

as is required by the post-prior symmetry of the DWBA formalisms. Comparing Eqs. (4.4) and (4.3), the difference between the two methods is very transparent. The interaction V_{xA} can excite the target A but $\hat{U}_{xA}$ does not.

The replacement $V_{xA}\rightarrow\hat{U}_{xA}$ used by UT (Refs. 1 and 11) holds only in the matrix element (3.4) but not in general, as is proved below Eq. (3.4). Therefore this procedure of UT is an additional approximation which only picks up the EBF process. On the contrary, a complete treatment of V_{xA} in the interaction and in the Green's function leads to the IAV formulas (4.4) and (3.16). To remedy the defect of missing the process $a+A\rightarrow b+x+A^*$ in which the target is simultaneously excited with breakup, UT proposed a CCBA-type approximation.[9] The IAV formalism can also be extended to a CCBA version.[10]

If one considers Ψ_x^{IAV} to be the leading order of Ψ_x^{3B}, $\delta\Psi_x^{\text{3B}}$ represents higher order processes. On the other hand if one starts from HM, $\Psi_x^{\text{UT}}+\delta\Psi_x^{\text{3B}}$ corresponds to the higher order. FMH call this a three-body contribution and conclude that the UT formula only takes into account higher order processes and misses the leading part. However, since IAV and HM are both considered to be of leading order, UT may also be of leading order, as is seen from Eq. (3.14). Equation (4.3) shows that the UT formula represents the DWBA cross section via the interaction $(\hat{U}_{xA}+U_{bA}-U_{aA})$. Therefore it is natural to consider the UT formula as a part of the leading order.

Such ambiguity about the order of processes stems from manipulations in which a Green's function is eliminated by rewriting interactions in the numerator as a sum of the inverse of the Green's function and the rest, as is done in the derivation of Eq. (2.23) from Eq. (2.20). Remember that the latter contains one Green's function and one interaction whereas the former does not. This ambiguity about the number of perturbative steps is inherent for rearrangement processes between different clusters. A typical example of this kind is seen in the simultaneous plus sequential two-nucleon transfer formalism in which the two step process in the prior-post representation can be reexpressed by the sum of the one- and two-step processes in the post-post representation.[17]

D. UT vs other formalisms

Even after a lengthy dispute,[4,9–15] the UT group still criticizes all the other formalisms on the argument that they erroneously include processes other than inclusive breakup. In their latest paper,[13] they present two kinds of arguments for the relation

$$\sigma_b^{\text{IAV}}\approx\sigma_b^{\text{HM}}\approx\sigma^{\text{rec}}>>\sigma_b \ , \tag{4.5}$$

where σ_b, σ_b^{IAV}, and σ_b^{HM} are angle and energy integrated inclusive breakup cross sections, of the form

$$\sigma_b=\int\frac{d^2\sigma^{\text{inc}}}{d\Omega_b dE_b}d\Omega_b dE_b \ , \tag{4.6}$$

for the exact, IAV, and HM formalisms, respectively.

In the first argument they rewrite the angle-energy integrals of Eq. (4.6) in terms of the operator

$$\hat{\Lambda}_b\equiv\int dE_b d\Omega_b\rho(E_b)\chi_b^{(-)*}(\mathbf{k}_b,\mathbf{r}_b')\chi_b^{(-)}(\mathbf{k}_b,\mathbf{r}_b) \ , \tag{4.7}$$

and claim that if U_{bA} is real then $\langle\hat{\Lambda}_b\rangle$ is unity, due to completeness. For completeness the integral over E_b must run up to infinity; however, the E_b variable in the cross section is restricted by energy conservation [i.e., $E_b\leq E-\min(E_c)$]. Therefore completeness is not applicable in Eq. (4.7).

Their second argument has already been described in Ref. 11 and rebutted by Ref. 12. The point of this argu-

ment is the following. The replacement $(E-H+i\epsilon)^{-1}\rightarrow(e-K_b-K_x-U_{bA}-V_{xA}+i\epsilon)^{-1}$ in the total reaction cross section (2.30) may not change the value very much and may be considered as a good approximation for $\sigma^{\rm rec}$. Therefore the relation $\sigma_b^{\rm IAV}\approx\sigma_b^{\rm HM}\approx\sigma^{\rm rec}$ is obtained. We think that not only is this statement without any theoretical foundation but it also contradicts their original derivation of the UT formula based upon the replacement $(E-H+i\epsilon)^{-1}\rightarrow(E-K_b-K_x-U_{bA}-U_{xA}+i\epsilon)^{-1}$ to extract the breakup process.

E. A common problem

A common shortcoming of all the theories discussed in this paper is that the particle x is assumed to be either structureless or a cluster in a particular state (usually the ground state). The difficulty of relaxing this restriction prevents easy generalization of the optical reduction (2.11) in a practical calculation. This may be a reason for the discrepancy between $\sigma^{\rm IAV}(\sigma^{\rm HM})$ and the experimental data of (α,p) in the MUT calculation.[13] A reasonable fit of $\sigma^{\rm UT}$ to the data of (α,p) does not necessarily mean UT theory is superior to others, because it only includes the EB and EBF processes.

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