Reconstruction of isospin and spin-isospin symmetries and double beta decay

J. Hirsch and F. Krmpotić

Departamento de Física, Facultad de Ciencias Exactas, Universidad Nacional de La Plata, 1900 La Plata, Argentina

(Received 23 December 1988)

It is shown that the nondynamical violation of the isospin and Wigner SU(4) symmetries, induced by the Hartree-Fock-BCS approximation, can be circumvented by using the quasiparticle random-phase approximation. The suppression mechanism for the $\beta\beta$-decay transition rates, proposed by Vogel and Zirnbauer, is found to be closely related to the restoration of SU(4) symmetry. It is suggested that the extreme sensitivity of the $\beta\beta$-decay amplitude on the proton-neutron coupling is a consequence of the explicit violation of the SU(4) symmetry and therefore an artifact of the model. A prescription is given for fixing this interaction strength within the quasiparticle random-phase approximation itself, which in this way acquires predicting power on both single and double β-decay lifetimes.

The inverse of the total lifetime for a double beta-decay ($\beta\beta$) process may be cast into the form[1]

$$T_{1/z}^{-1}=|\mathcal{M}_{2\nu}|^2\mathcal{G}_{2\nu}+|\eta|^2|\mathcal{M}_{0\nu}|^z\mathcal{G}_{0\nu}\ ,$$

where $\mathcal{M}_{2\nu}$ and $\mathcal{M}_{0\nu}$ are nuclear matrix elements for the two-neutrino mode ($\beta\beta_{2\nu}$) and the neutrinoless mode ($\beta\beta_{0\nu}$), respectively; the quantities $\mathcal{G}_{2\nu}$ and $\mathcal{G}_{0\nu}$ are the corresponding kinematical factors. The physically interesting quantity is the lepton violating parameter η, whose extraction from experimental data depends critically on the theoretical estimates of both $\mathcal{M}_{2\nu}$ and $\mathcal{M}_{0\nu}$.

Until recently, theoretical calculations of the $\beta\beta_{2\nu}$ decay rates were systematically larger than the corresponding experimental values; the discrepancy was particularly pronounced in the ^{128,130}Te isotopes. Vogel and Zirnbauer[2] (VZ) have recently made important progress in this regard. They have applied the quasiparticle random-phase approximation (QRPA) and shown that the ground-state correlations (GSC), induced by the proton-neutron (PN) residual interaction, play an essential role in reducing the $\beta\beta_{2\nu}$ decay probability. In observing this suppression mechanism a zero-range interaction was used and the consequences of the SU(4) symmetry on the $\mathcal{M}_{2\nu}$ amplitude were discussed in the context of a schematic model. Subsequent studies,[3,4] most of them performed with realistic effective interactions, lead essentially to the same conclusion: when evaluated within the QRPA, irrespective of the force employed, the predicted life times are very sensitive to GSC within the particle-particle (PP) channel. The same effect appears in the description of ordinary β^+-decay processes[3,4] and so far no satisfactory interpretation of this phenomenon has been put forward.

The aim of this paper is to point out that the extreme sensitivity of the Gamow-Teller (GT) $\beta\beta_{2\nu}$ amplitude, $\mathcal{M}_{2\nu}^{\rm GT}$, to model parameters is closely related to the self-consistence between the residual interaction and the average mean field, as well as to the supermultiplet structure in spin and isospin space. Moreover, it is suggested that physically meaningful nuclear parameters within the VZ model are those that lead to the maximal restoration of the Wigner SU(4) symmetry. As a frame of reference, the results for the Fermi (F) $\beta\beta_{2\nu}$ amplitude, $\mathcal{M}_{2\nu}^{\rm F}$, and the isospin invariance, will be used.

It is well known that, even when the isospin-breaking Coulomb force is not included in the Hamiltonian $\mathcal{H}$, the whole structure of isospin invariance may be demolished in an approximate treatment of the eigenstates of $\mathcal{H}$. In other words, the isospin-invariance breaking arises from the approximation that is introduced and not from the interaction. As a matter of fact, Engelbrecht and Lemmer[5] have pointed out that, in nuclei with ground-state isospin $T_0=(N-Z)/2>0$, the isospin invariance is *explicitly* broken by the Hartree-Fock (HF) field as well as by the Tamm-Dancoff approximation (TDA). They have also shown that the isospin-conserving description is recovered if the PN correlations, generated within the random-phase approximation (RPA), are included in the HF ground state in a self-consistent way. Lee[6] has explored this notion in more detail by introducing into the picture the Coulomb interaction, which leads to a *dynamical* breaking of isospin symmetry. His conclusion was that the isospin impurities in the ground state are also greatly reduced by the PN correlations.

While the RPA correlations for F transitions are closely related to the isospin symmetry, the quantitative features of correlations for GT transitions depend on the more detailed properties of $\mathcal{H}$. Owing to the strong spin-orbit interaction in the HF field, the SU(4) symmetry is badly broken in medium and heavy nuclei and the jj-coupling scheme is established. The PN correlations, which are responsible for building up the GT resonance, may be viewed, however, as a trend away from the jj coupling towards the LS coupling and the SU(4) symmetry. This is reflected by the experimental energy differences between the GT and F resonances[7]

$$E_{\rm res}^{\rm GT}-E_{\rm res}^{\rm F}=(26A^{-1/3}-37T_0A^{-1})\ \text{MeV}\ ,$$

which decreases as the mass number increases. The first and second terms arise, respectively, from the spin-orbit splitting and the PN correlations and have a tendency to cancel each other.

The $\beta\beta_{2\nu}$ amplitudes will be approximate as[8]

$$\mathcal{M}_{2\nu}(I)=\sum_{\alpha}\frac{\langle 0^+\|\mathcal{O}(I)\|I^+;\alpha\rangle\langle I^+;\alpha\|\mathcal{O}\|0^+\rangle}{E_\alpha^I-E_i+Q_{\beta\beta}/2+1}\ , \tag{1}$$

where $\mathcal{M}_{2\nu}(I=0)\equiv\mathcal{M}_{2\nu}^{\mathrm{F}}$ and $\mathcal{M}_{2\nu}(I=1)\equiv\mathcal{M}_{2\nu}^{\mathrm{GT}}$, $|0^+\rangle$ is the ground-state wave function of the initial nucleus, $\mathcal{O}(I)$ represent either the F operator t_+ $(I=0)$ or the GT operator σt_+ $(I=1)$, and the sum extends over a complete set of intermediate nuclear states $|I^+;\alpha>$; the notation for the energy denominator, given in units of m_e, is rather obvious.

The QRPA equations read

$$\begin{aligned}(\varepsilon_p+\varepsilon_n-\omega_\alpha^I)X_{pn}(I;\alpha)&=-\sum_{p'n'}[A(pn,p'n';I)X_{p'n'}(I;\alpha)+B(pn,p'n';I)Y_{p'n'}(I,\alpha)]\ ,\\(\varepsilon_p+\varepsilon_n+\omega_\alpha^I)Y_{pn}(I;\alpha)&=-\sum_{p'n'}[A(pn,p'n';I)Y_{p'n'}(I;\alpha)+B(pn,p'n';I)X_{p'n'}(I,\alpha)]\ ,\end{aligned} \tag{2}$$

with submatrices

$$A(pn,p'n';I)=(u_pv_nu_{p'}v_{n'}+v_pu_nv_{p'}u_{n'})F(pn,p'n';I)+(u_pu_nu_{p'}u_{n'}+v_pv_nv_{p'}v_{n'})G(pn,p'n';I)\ ,$$

$$B(pn,p'n';I)=(v_pu_nu_{p'}v_{n'}+u_pv_nv_{p'}u_{n'})F(pn,p'n';I)-(u_pu_nv_{p'}v_{n'}+v_pv_nu_{p'}u_{n'})G(pn,p',n';I)\ ,$$

where F and G are respectively the particle-hole (PH) and particle-particle (PP) matrix elements as defined in Ref. 9. The quasiparticle energies are expressed in standard form

$$\varepsilon_\tau=(e_\tau-\lambda_\tau)(u_\tau^2-v_\tau^2)+2\Delta_\tau u_\tau v_\tau\ , \tag{3}$$

where the subscript τ stands for p or n; e_τ and λ_τ are, respectively, the single-particle energies (SPE) and the chemical potentials, and

$$\Delta_\tau=-\tfrac{1}{2}\sum_{\tau'}\hat{j}_{\tau'}\hat{j}_\tau^{-1}u_{\tau'}v_{\tau'}G^{\mathrm{pair}}(\tau\tau,\tau'\tau';0)\ , \tag{4}$$

is the energy gap, with $\hat{j}_\tau\equiv(2j_\tau+1)^{1/2}$.

The energy difference that appears in (1) is related to the QRPA energy ω_α^I as

$$E_\alpha^I-E_i=\omega_\alpha^I+\lambda_p-\lambda_n\ , \tag{5}$$

and the transition matrix elements are expressed by means of the forward and backward going amplitudes X and Y in the form

$$\langle I^+;\alpha\|\mathcal{O}(I)\|0^+\rangle=\sum_{pn}\langle p\|\mathcal{O}(I)\|n\rangle[u_pv_nX_{pn}(I;\alpha)+v_pu_nY_{pn}(I;\alpha)]\ ,$$

$$\langle 0^+\|\mathcal{O}(I)\|I^+;\alpha\rangle=\sum_{pn}\langle p\|\mathcal{O}(I)\|n\rangle[v_pu_nX_{pn}(I;\alpha)+u_pv_nY_{pn}(I;\alpha)]\ .$$

Furthermore, the transition strengths

$$S_\pm(I)=\sum_\alpha s_\pm(I;\alpha)\ ,$$

with

$$s_+(I;\alpha)=|\langle I^+;\alpha\|\mathcal{O}(I)\|0^+\rangle|^2$$

and

$$s_-(I;\alpha)=|\langle 0^+\|\mathcal{O}(I)\|I^+;\alpha\rangle|^2\ ,$$

fulfill the sum rule

$$S_+(I)-S_-(I)=2T_0(2I+1)\ .$$

In the limit of exact isospin and Wigner SU(4) symmetries all of the $S_+(I)$ strength is concentrated in the resonant (collective) state $|I^+;\mathrm{res}\rangle$, there is no β^+ strength and the $\beta\beta_{z\nu}$ decay is forbidden, i.e.,

$$s_+(I;\mathrm{res})\equiv S_+(I),\quad S_-(I)\equiv 0,\quad \mathcal{M}_{2\nu}(I)\equiv 0\ . \tag{6}$$

While for the F transitions the relations (6) should be exact results when the Coulomb force is excluded, they only may be considered as first approximations for the GT processes. Within the BCS approximation and QTDA the above mentioned symmetries are always explicitly broken and therefore the conditions (6) are never fulfilled. Contrarily, within the QRPA these limits can be achieved if[5,6]

$$\begin{aligned}X_{pn}(I,\mathrm{res})&\sim u_pv_n\langle p\|\mathcal{O}\|n\rangle\ ,\\Y_{pn}(I,\mathrm{res})&\sim -v_pu_n\langle p\|\mathcal{O}\|n\rangle\ .\end{aligned} \tag{7}$$

After substituting (7) into (2) the following set of equations can be derived for the isobaric-analog state (IAS)

$$\begin{aligned}\varepsilon_p+\varepsilon_n-\omega_{\mathrm{res}}^{\mathrm{F}}&=-U_{j_p=j_n}^{\mathrm{F}}+\frac{u_n}{v_n}\Delta_n^{\mathrm{F}}+\frac{v_p}{u_p}\Delta_p^{\mathrm{F}}\ ,\\\varepsilon_p+\varepsilon_n+\omega_{\mathrm{res}}^{\mathrm{F}}&=U_{j_p=j_n}^{\mathrm{F}}+\frac{v_n}{u_n}\Delta_n^{\mathrm{F}}+\frac{u_p}{v_p}\Delta_p^{\mathrm{F}}\ ,\end{aligned} \tag{8}$$

where

$$U_{j_p=j_n}^{\mathrm{F}}=\sum_{j_{p'}=j_{n'}}\hat{j}_{p'}\hat{j}_p^{-1}(v_{n'}^2-v_{p'}^2)F(pn,p'n';0)\ ,$$

and

$$\Delta_\tau^{\mathrm{F}}=-\tfrac{1}{2}\sum_{\tau'}\hat{j}_{\tau'}\hat{j}_\tau^{-1}u_{\tau'}v_{\tau'}G(\tau\tau,\tau'\tau';O)\ .$$

From (4), (5), and (8) one immediately sees that

$$\Delta_\tau^{\mathrm{F}}=\Delta_\tau,\quad E_{\mathrm{res}}^{\mathrm{F}}-E_i=e_p-e_n+U_{j_p=j_n}^{\mathrm{F}} . \tag{9}$$

Thus, only within a self-consistent QRPA calculation the conditions (6) are fulfilled for the F transitions, or equivalently the explicit breaking of isospin symmetry induced by the QTDA is circumvented. In other words: (i) the same residual interaction should be used in solving the gap equations, both for protons and neutrons and for the PN particle-particle channel, i.e., $G^{\mathrm{pair}}=G$ and (ii) the symmetry energy contained in the proton SPE ($e_p=e_n+\Delta_C-U_{j_p=j_n}$, where Δ_C is the Coulomb displacement energy) should be equal to the symmetry energy $U_{j_p=j_n}^{\mathrm{F}}$.[10]

For the further discussion of $\beta\beta$ amplitudes we borrow now the δ interaction[7,9]

$$V=-C(v_sP_s+v_tP_t)\delta(r),\quad C\equiv 4\pi\ \mathrm{MeV\ fm}^3 ,$$

with different strength constants v_s and v_t for the PH, PP, and pairing channels and calculate the ^{128}Te nucleus. In order to determine the appropriate single-particle spectra and the interaction strengths v_s^{pair} we follow the prescription proposed by Conci *et al.*,[11] which consists in utilizing the experimental data together with a Wood-Saxon plus BCS calculation. This procedure yields $v_s^{\mathrm{pair}}=24$ and 31 for neutrons and protons, respectively. The coupling strengths in the PH channel are taken from Ref. 12, namely, $v_s^{\mathrm{PH}}=55$ and $v_t^{\mathrm{PH}}=92$ while v_s^{PP} and v_t^{PP} are treated as free parameters. Two different calculations are described.

Calculation I(CI): The experimental values of neutron SPE are used and the proton SPE are adjusted according to $e_p=e_n+\Delta_C-U_{j_p=j_n}^{\mathrm{F}}$; v_s^{pair} is fixed at the value of 28 for both neutrons and protons.

Calculation II (CII): The SPE as well as the strengths v_s^{pair} were taken from the experimental data as explained previously in the text.

The calculated values of S_- and $\mathcal{M}_{2\nu}$, within the QRPA,[13] are displayed in Fig. 1. The most relevant issue, which becomes self-evident at a first glance, is the great similarity between $\mathcal{M}_{2\nu}^{\mathrm{F}}$ and $\mathcal{M}_{2\nu}^{\mathrm{GT}}$. That is, both amplitudes are very sensitive to the GSC within the PP channel and pass through zero in the vicinity of the minima of S_-. In the following use will be made of this similarity in order to draw conclusions about the physical value of v_t^{PP}, after separating the explicit breaking of the SU(4) symmetry from the dynamical one.

As expected, when the condition of self-consistency (9) is satisfied (or equivalently when the isospin symmetry is strictly conserved) $\mathcal{M}_{2\nu}^{\mathrm{F}}\equiv 0$ and $S_-^{\mathrm{F}}\equiv 0$. In varying v_s^{PP} when v_s^{pair} is constant, a fictitious degree of freedom is introduced; thus, the variations of $\mathcal{M}_{2\nu}^{\mathrm{F}}$ and S_-^{F} within CI, exhibited in Fig. 1, just only put in evidence the lack of self-consistency between the residual interaction and the mean field. It is clear that even after introducing isospin impurities, i.e., when the condition (9) is not valid anymore, the value of v_s^{PP} should not be varied independently of the value of v_s^{pair}. Moreover, from CII it seems reasonable to state that the self-consistency between v_s^{PP} and v_s^{pair} is now achieved at the minimal value of S_-^{F}, which represents the amount of dynamical violation of isospin symmetry and for which $v_s^{\mathrm{PP}}\cong v_s^{\mathrm{pair}}$ and $\mathcal{M}_{2\nu}^{\mathrm{F}}\cong 0$.

The preceding results suggest that the physically sound value of v_t^{PP}, for a given value of v_s^{pair}, is that which minimizes S_-^{GT}. In other words, it might be expected that at this value of v_t^{PP} the explicit violation of SU(4) symmetry should be totally removed and that the minimum of S_-^{GT} should measure the extent to which the SU(4) is broken by the dynamics of $\mathcal{H}$. In this regard, one sees, from comparison of the lowest values of S_-^{GT} in CI and CII, that not only the spin-orbit splitting but also the isospin impurities play an important role. However, both minima of S_-^{GT} are located almost at the same value of v_t^{PP} ($\cong 37$) and the corresponding $\mathcal{M}_{2\nu}^{\mathrm{GT}}$ amplitudes are very nearly equal[14] ($\cong -0.03$). The location of the minimum of S_-^{GT} does depend on v_s^{pair} and it is moved to higher (lower) values of v_t^{PP} when v_s^{pair} is increased (decreased). The results for $\mathcal{M}_{2\nu}^{GT}$ obtained at the minimum

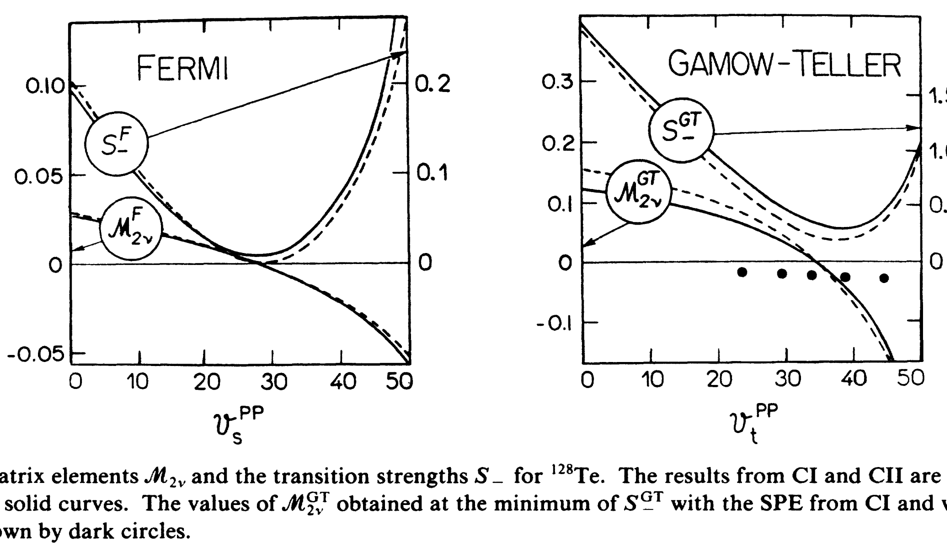

FIG. 1. The matrix elements $\mathcal{M}_{2\nu}$ and the transition strengths S_- for ^{128}Te. The results from CI and CII are indicated, respectively, by dashed and solid curves. The values of $\mathcal{M}_{2\nu}^{\mathrm{GT}}$ obtained at the minimum of S_-^{GT} with the SPE from CI and with $v_s^{\mathrm{pair}}=15$, 20, 25, 30, and 35 are shown by dark circles.

of S_{-}^{GT} with the SPE from CI and with $v_s^{pair} = 15$, 20, 25, 30, and 35 are shown in Fig. 1 by dark circles. Once the above-mentioned constraint between v_t^{PP} and v_s^{pair} is imposed, i.e., after circumvented the explicit violation of the SU(4) symmetry, the variation of $\mathcal{M}_{2\nu}^{GT}$ with respect to v_t^{PP} is of minor importance and $\mathcal{M}_{2\nu}^{GT}$ does not pass through zero anymore. This is consistent with the recent observation of Muto and Klapdor[4] in the sense that the vanishing of $\mathcal{M}_{2\nu}^{GT}$ is an artifact of the model (see also Ref. 15).

We summarize our viewpoints and conclusions as follows: (1) The extreme sensitivity of the $\mathcal{M}_{2\nu}$ amplitudes to the GSC within the PP channel, shown in Fig. 1, is artificially generated by the explicit violation of isospin and SU(4) symmetries. (2) The destructive interference between the forward and backward going terms in the β^+-GT amplitude, pointed out by VZ, is a consequence of the approximate validity of (7), or more precisely of the restoration of the SU(4) symmetry. (3) In order to get reliable theoretical results for the $\beta\beta$-decay lifetimes within the QRPA, the breaking of isospin and SU(4) symmetries induced by the HF-BCS approximation should be circumvented. (4) After fixing the PN coupling within the PP channel, as suggested here, it is not necessary to resort anymore to experimental data on β^+ strength in order to estimate this model parameter. Thus the VZ model is supplemented now with the predicting power on the lifetimes for both single and double β decays.

This work was supported by the Consejo Nacional de Investigaciones Científicas y Técnicas, Argentina. We would like to thank A. F. R. Toledo Piza for fruitful discussions.

[1]M. Doi, T. Kotani, and E. Takasugi, Prog. Theor. Phys. Suppl. **83**, 1 (1985).

[2]P. Vogel and M. R. Zirnbauer, Phys. Rev. Lett. **57**, 3148 (1986).

[3]J. Engel, P. Vogel, and M. R. Zirnbauer, Phys. Rev. C **37**, 731 (1988); O. Civitarese, A. Faessler, and T. Tomoda, Phys. Lett. B **194**, 11 (1987); J. Engel, P. Vogel, O. Civitarese, and M. R. Zirnbauer, *ibid.* **208**, 187 (1988).

[4]K. Muto and H. V. Klapdor, in *Neutrinos,* edited by H. V. Klapdor (Springer-Verlag, Berlin, 1988).

[5]C. A. Engelbrecht and R. H. Lemmer, Phys. Rev. Lett. **24**, 607 (1970).

[6]H. C. Lee, Phys. Rev. Lett. **27**, 200 (1971).

[7]K. Nakayama, A. Pio Galeão, and F. Krmpotić, Phys. Lett. **114B**, 217 (1982), and references therein.

[8]Usually two separate QRPA calculations are performed (one for the initial nucleus and one for the final nucleus) and some kind of average is carried out for the resulting matrix elements (Refs. 2-4). This effect is, however, only of minor importance regarding the objective pursued in this work.

[9]K. Ikeda, Prog. Theor. Phys. **31**, 434 (1964); J. I. Fujita and K. Ikeda, Nucl. Phys. **67**, 143 (1965).

[10]It is important to stress that, even when these conditions are satisfied, there is no reconstruction of isospin symmetry within the QTDA.

[11]C. Conci, V. Klemt, and J. Speth, Phys. Lett. **148B**, 405 (1984); C. Conci, Ph.D. thesis, University of Bonn, 1984.

[12]H. Castillo and F. Krmpotić, Nucl. Phys. **A469**, 637 (1987).

[13]Just for the sake of comparison, let us mention a few results for the QTDA. Within CI, $S_{-}^{F} = 0.55$ and $S_{-}^{GT} = 3.86$, and $\mathcal{M}_{2\nu}^{F}$ ($\mathcal{M}_{2\nu}^{GT}$) goes from 0.062 (0.26) to 0.075 (0.40) when v_s^{PP} (v_t^{PP}) is varied from 0 to 50.

[14]This theoretical estimate should be compared with the experimental value $|\mathcal{M}_{2\nu}| \cong 0.03$, obtained from $\mathcal{G}_{2\nu} = 8.54 \times 10^{24}\ \mathrm{yr}^{-1}$ (cf. Ref. 1) and the measured lifetime $T_{1/2}^{2\nu} = (1.4 \pm 0.4) \times 10^{24}$ yr (cf. Ref. 4).

[15]C. R. Ching, T. H. Ho, and X. R. Wu, Phys. Rev. C **40**, 304 (1989).