Rotational g factors of ^{158}Er at low spins

A. Ansari
Institute of Physics, Bhubaneswar 751 005, India

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Using a pairing-plus-quadrupole-model interaction Hamiltonian in a variation-after-exact-angular-momentum-projection approach we have calculated g factors of ^{158}Er for the low-spin yrast states with $I=2-8$. We find that g factors decrease very slowly with the increase of spin ($g_8=0.86g_2$) even though shape parameters (β,Δ_n,Δ_p) change appreciably as a function of angular momentum. This is in contrast to a rapid fall predicted in the cranked Hartree-Fock-Bogoliubov approach which happens because of a too early ($I<8$) strong rotation alignment of neutron $i_{13/2}$ orbitals.

The cranked Hartree-Fock-Bogoliubov (CHFB) method[1-3] has provided a very simple and elegant approach to study the effect of collective rotation on the single-particle motion. That is, the effect of interplay between collective and single-particle degrees of freedom is naturally present in the CHFB approach. However, the CHFB method is well known for its two shortcomings, namely, the wave function is not an eigenfunction of the particle number and angular momentum operators. These are conserved only on an average. To shed some light on these weaknesses, we have investigated recently at Giessen[4,5] the effect of spin and particle number projection of CHFB wave functions on the variation of g factors with spin for a rare-earth backbender ^{158}Dy. We found that the CHFB method, as is usually practiced, is quite adequate for a qualitative study of high-spin properties. Figure 9 of Ref. 4 and Fig. 8 of Ref. 5, for example, illustrate this point. In Ref. 5 it is established (see the discussion there on page 486) that a correction of the spin projected energy for errors in the value of the average particle number with respect to the good angular momentum wave function is essential, and this, to a great extent, can be taken into account by following a simple recipe of Allart *et al.*[6] Furthermore, we have also shown in Ref. 5 (page 495) that the effect of particle number projection on the spin projected CHFB wave function is not significant as far as the variation of g factors with spin is concerned.

^{158}Er is one of the most interesting and well-studied backbending nuclei. Besides the yrast energy spectrum up to $I=46$, where the band is terminated,[7,8] its $B(E2)$ transition rates have also been measured[9] up to $I=22$, though uncertainties are rather large. From the $B(E2)$ rates transition quadrupole moments and thereby change in collectivity with spin is deduced. The g factors of some $N=90$ isotones like ^{156}Dy (Ref. 10) and ^{152}Sm (Refs. 11 and 12) are now measured for a few excited states with $I\leq 10$. In ^{152}Sm g factors for $I=2-10$ are almost a constant, and in ^{154}Sm it shows only a small reduction with spin. It should, therefore, be interesting to see how g factors of ^{158}Er vary with spin. In the usual CHFB approach we have already[13] studied the changes of g factors with spin up to $I=42$ where it starts decreasing fast at $I=4$ itself. We known that strong rotation alignment effects are inherent in the CHFB approach. So, it may be worthwhile to see how the corresponding values turn out in the axially symmetric variation after spin projection (VAP) approach. Then by comparison with the experimental data, as and when these are available, one can learn about the pairing and single-particle structure of the yrast levels of ^{158}Er at low spins. In view of this we present here axial VAP results for g factors of ^{158}Er up to $I=8$.

First we will briefly present the formalism and then our results will be presented and discussed. Finally we present a brief conclusion.

As already mentioned, we take the pairing-plus-quadrupole-model Hamiltonian of Baranger and Kumar (BK) (Ref. 14)

$$H=\sum_{\alpha}\epsilon_\alpha c_\alpha^\dagger c_\alpha-\tfrac{1}{2}\chi\sum_{\alpha,\beta,\gamma,\delta}\langle\alpha|Q_{2\mu}|\gamma\rangle\langle\beta|(-1)^\mu Q_{2-\mu}|\delta\rangle c_\alpha^\dagger c_\beta^\dagger c_\delta c_\gamma-\tfrac{1}{4}G\sum_{\alpha,\gamma}c_\alpha^\dagger c_{\bar\alpha}^\dagger c_{\bar\gamma}c_\gamma\ , \qquad (1)$$

where the quadrupole moment operator

$$Q_{2\mu}=r^2Y_{2\mu}(\hat r)\ . \qquad (2)$$

The quadrupole moment interaction strength χ (the same for proton-proton, neutron-proton, and neutron-neutron interactions) and the pairing interaction strengths G for protons and neutrons used here are [see Eq. (4) of Ref. 4].

$$\chi=70/A^{1.4},\quad G_p=26/A,\quad G_n=21/A \qquad (3)$$

(all in MeV). ϵ_α are the spherical single-particle energies. The model space consists of $N=4,5$ major shells for protons and $N=5,6$ major shells for neutrons with an innert core of 40 protons and 70 neutrons. Solving the HFB equations[14] with Hamiltonian (1) we obtain the axially symmetric intrinsic many-body wave function. Then using the standard angular momentum projection technique[15,16] we calculate the total energy for a given spin, I, for a fixed set of shape parameters

$$E_I(\beta,\Delta_p,\Delta_n)=\frac{\int_0^{\pi/2}d\theta\,\sin\theta\, d^I_{00}(\theta)H(\theta)}{\int_0^{\pi/2}d\theta\,\sin\theta\, d^I_{00}(\theta)N(\theta)}\ , \quad (4)$$

where

$$\begin{Bmatrix} H(\theta) \\ N(\theta)\end{Bmatrix}=\langle \mathrm{HFB}|\begin{Bmatrix} H \\ 1\end{Bmatrix}e^{-i\theta J_y}|\mathrm{HFB}\rangle\ . \quad (5)$$

In Eq. (4) $d(\theta)$ is a rotation matrix, θ denoting the angle of rotation about the y axis. Energy depends upon shape parameters: the quadrupole deformation parameter, β, and proton and neutron pairing gaps, Δ_p and Δ_n, respectively. As already discussed, we have not performed a particle number projection of the HFB wave function, but instead have corrected[5,6] the energy for the error in the average particle number

$$E_I^c=E_I-\lambda_p(N_p-N_p^0)-\lambda_n(N_n-N_n^0)\ , \quad (6)$$

where $N^0_{p,n}$ is the exact number and $N_{p,n}$ is the expectation value of the number operator with respect to the spin projected state Ψ_I, namely,

$$N_p=\langle \Psi_I|\hat{N}_p|\Psi_I\rangle\ . \quad (7)$$

Then the spin projected energy E_I^c is minimized with respect to the shape parameters.

After the energy minimization is carried out, the corresponding wave function is used to calculate the magnetic moment

$$\mu_I=\langle \Psi_{I,M=I}|\mu^1_0|\Psi_{I,M=I}\rangle$$
$$=\begin{bmatrix} I & 1 & I \\ I & 0 & I\end{bmatrix}\sum_\nu \begin{bmatrix} I & 1 & I \\ -\nu & \nu & 0\end{bmatrix}\int_0^{\pi/2}d\theta\,\sin\theta\, d^I_{-\nu 0}(\theta)\sum_{\tau_3}\langle \mathrm{HFB}|\mu_\nu^{1(\tau_3)}e^{-i\theta J_y}|\mathrm{HFB}\rangle\Big/\int_0^{\pi/2}d\theta\,\sin\theta\, d^I_{00}(\theta)N(\theta)\ , \quad (8)$$

where τ_3 is the isospin projection quantum number and the brackets denote a Clebsch-Gordan coefficient. For an even-even nucleus, as is the case here, the values of I are even and the $\nu=0$ term in (8) will not survive. Also, it can easily be seen that contributions from the $\nu=1$ and $\nu=-1$ terms would be equal. Thus Eq. (8) is finally simplified to

$$\mu_I=\frac{I}{\sqrt{I(I+1)}}\,\frac{\int_0^{\pi/2}d\theta\,\sin\theta\, d^I_{-10}(\theta)\sum_{\tau_3}\langle \mathrm{HFB}|\mu_+^{(\tau_3)}e^{-i\theta J_y}|\mathrm{HFB}\rangle}{\int_0^{\pi/2}d\theta\,\sin\theta\, d^I_{00}(\theta)N(\theta)} \quad (9)$$

with the usual definition of step-up operators

$$\mu_+=g_l\sum_i l_+(i)+g_s\sum_i s_+(i)\ , \quad (10)$$

where g_l and g_s are the free nucleon orbital and spin g factors; for protons $g_l=1$ and $g_s=5.586$, whereas for neutrons $g_l=0$ and $g_s=-3.836$. However, spin gyromagnetic ratios are assumed to be attenuated[17] by a factor 0.6. Then the rotational g factors are calculated from

$$\mu_I=Ig_I\ . \quad (11)$$

As we know, the best aspect of the CHFB theory is that it includes the possibility of alignment of s.p. orbitals along the collective rotation axis. But at the same time, producing alignment in ^{158}Er at very low spins[13] like $I\approx 4$ is unphysical. On the other hand, in the axial VAP approach the rotation alignment mechanism is not explicitly present. Only the process of angular momentum projection and minimization of the projected energy with respect to the shape parameters, particularly Δ_n, produce some changes in structure as a function of spin. The axial VAP approach is, therefore, limited in application and is suitable only to study the structure of states well below the band crossing region where the effect of mixing with the s band is negligible. For $I\lesssim 8$ in ^{158}Er this should be the case.

Following the method already outlined, we have computed the g factors of ^{158}Er in the axial VAP approach for the low-spin yrast states with $I=2$–8. We should point out that the pairing interaction strengths Gp and G_n in Eq. (3) are slightly reduced as compared to G_p 27/A and $G_n=22/A$ taken by BK.[14] This is done to ensure that in this calculation the values of Δ_p and Δ_n at $I=0$ (ground state) more or less agree with their self-consistent intrinsic values employing the standard BK Hamiltonian[14] (see also Ref. 4).

In Table I the values of the shape parameters, γ-ray excitation energies, and g factors are presented as a function of spin. Variation in the values of β showing stretching (for the low spins considered) is qualitatively similar to CHFB results as well as the experimental trend.[9] The values of the shape parameters at $I=0$ are closer to the ground-state intrinsic values of $\beta=0.243$, $\Delta_p=1.217$, and $\Delta_n=0.890$ computed with the Hamiltonian parameters of Ref. 14. As expected, the decrease of Δ_p with the increase in spin is rather slow $[\Delta_p(I=8)=0.72\ \Delta_p(I=0)]$ whereas Δ_n decreases at a faster rate $[\Delta_n(I=8)=0.58\Delta_n(I=0)]$. The γ-ray excitation energies, after somewhat arbitrary correction[18] (a constant multiplying factor, independent of spin) for the core polarization such that $E_2^{\mathrm{theo}}\approx E_2^{\mathrm{expt}}$, look in reasonable agreement with the experimental numbers.[19] For the valence particles considered, the total binding energy for a given angular

TABLE I. Quadrupole deformation parameter, β and proton and neutron pairing gap, Δ_p and Δ_n, respectively, as a function of spin for the yrast states of ^{158}Er. Also excitation energies and g factors are listed. For the computed numbers $E_\gamma(I)=0.7\times(E_I-E_{I-2})$. The only known experimental value (Ref. 20) of the g factor is $g_2\approx0.37$.

I	β	Δ_p (MeV)	Δ_n (MeV)	$E_\gamma(I)$ (MeV)	$E_{\gamma(I)}^{\rm expt}$ (MeV)	g_I
0	0.245	1.195	0.920			
2	0.255	1.125	0.853	0.196	0.192	0.400
4	0.265	1.025	0.755	0.376	0.335	0.391
6	0.275	0.940	0.651	0.462	0.443	0.375
8	0.287	0.860	0.530	0.494	0.523	0.346

momentum state is about 300 MeV, whereas the excitation energies are only of the order of a few hundred keV that are computed as differences of those large numbers. Therefore, the calculations have to be very accurate and these are very sensitive to even slight modifications in the Hamiltonian parameters.

Experimental data on g factors of ^{158}Er are still not available, except that quoted recently by Sugawara-Tanabe and Tanabe[20] at $I=2$ and 16: $g_2\approx0.37$ and $g_{16}\approx0.0$. We get a somewhat larger value of $g_2=0.40$. This is understandable because, as evident from the calculated excitation energies, our effective moment of inertia is smaller. Furthermore, with the increase of spin the g factors are decreasing, but very slowly ($g_8\approx0.86g_2$), particularly in view of the cranking results[13,21,22] where even $g_4/g_2\approx0.4$ (using the same Hamiltonian). Here we should point out that recent CHFB calculations of Sugawara-Tanabe and Tanabe[20] which also include the quadrupole pairing term (besides the usual monopole pairing) in the Hamiltonian predict a similar slow decreasing trend. It may be emphasized that the inclusion of the quadrupole pairing term hinders the rapid rotation alignment of single-particle orbitals in the CHFB approach. On the other hand, in our VAP approach these extra degrees of freedom are not required.

In view of these results on g factors we would also like to add that in calculations of Bengtsson and Aberg[23] g factors of ^{160}Yb, an isotone of ^{158}Er, turn out to be almost a constant ($\approx$0.4) for $I\leq10$. Also the experimental data for some other deformed rare-earth nuclei like ^{160}Dy, ^{170,174}Yb (Ref. 24), and ^{156,160}Gd (Ref. 25) show only a small reduction of g factor with the increase of spin for $I\leq10$.

The g factors of ^{158}Er in the low-spin region, $I=2-8$, have been calculated in the axial (asymmetry parameter, $\gamma=0$) VAP approach. Of course, it is not the most general variation, but is rather restricted to the variation of the macroscopic shape parameters β, Δ_p, and Δ_n. We find that g factors are decreasing very slowly with the increase of spin. This appears quite reasonable in view of the rotational nature of the energy spectrum (soft rotor, $E_4/E_2=2.74$) and known data[10,17,26] for many nuclei at low spins.

Thus, it again emerges, like in Ref. 5, that the structure of deformed rare-earth nuclei in the low-spin region should be studied in the axial VAP approach rather than the cranking one, particularly if the Hamiltonian in the latter approach contains only the monopole pairing interaction. Also in the VAP approach the wave functions are exact eigenfunctions of the angular momentum operator.

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