Medium effects in electric form factors and transition strengths

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Electric transition strengths are calculated in a relativistic model. It is found that for electric transitions there is, in addition to the spin-orbit current, an extra medium-induced current coming from the Darwin term in the Hamiltonian. A medium-modified transition strength is defined and the effects of both medium-induced currents are studied. The application for the first 3^- excitation in ^{40}Ca yields unrealistic results. However, when vacuum polarization corrections are included the resulting $B(E3)$ value comes close to the nonrelativistic shell-model one.

I. INTRODUCTION

The role that the effective mass plays in relativistic mean field theories has been a subject of controversy ever since Miller[1] pointed out that the experimental spin-orbit splittings and magnetic moments of single-particle states in nuclei cannot be reproduced simultaneously by any single-particle Dirac Hamiltonian with local interactions. The primary origin of the discrepancy was initially ascribed to the enhancement of the convection current in the nuclear medium by a factor of M/M^* which roughly amounts to a factor of 2 enhancement in the current. Recently,[2] this difficulty was resolved by taking into consideration the polarization of the positive- and negative-energy core. The valence particle couples to $N\bar{N}$ excitations of the core nucleus thus reducing the convection current by a factor which approximately cancels the effect of M/M^*. Later on, however, it was pointed out[3] that the above argument on the cancellation was not enough to resolve the problems of the magnetic moments of finite nuclei. Nishizaki *et al.* argued that these arguments apply only to nuclear matter and, when one deals with finite nuclei, there is an additional term in the current stemming from the spin-orbit force (and related to it through the continuity equation) which affects magnetic moments even more than the effective mass does. Yet, more recently[4], we found that the contribution due to this term is only noticeable when dealing with hole-valence nuclei where the convection and Dirac magnetization currents cancel each other out. Because of this cancellation even a small medium-induced current (MIC) affects strongly the value of the magnetic moments. Furthermore, we found that the inclusion of the vacuum polarization, in the local density approximation, to quench the MIC contributions does not always improve the results.

All the arguments given above apply to the transverse magnetic operator in the limit of q going to zero. In Refs. 3 and 4 it was already noted that there is also a MIC contribution to the transverse electric form factor that may turn out to be important at low q values. Whereas in Ref. 3 the MIC contribution was purely of the magnetization type (i.e., depending on the spin), the one we found in Ref. 4 contained the same magnetization piece and, in addition, a term related, through the continuity equation, to the Darwin term of the Hamiltonian. This piece of the current does not contribute to magnetic transitions but it is present for the electric ones. The purpose of the present paper is to assess the importance of both MIC terms to transverse electric form factors. Besides, as electric transitions always involve at least one nuclear state that is not spherically symmetric, we consider it worthwhile to study the inclusion of the renormalization due to particle-vibration coupling as done previously for magnetic moments. In particular our calculations show that in the long wavelength limit (LWL) ($q\rightarrow 0$) transition amplitudes [$B(EJ)$ values] are strongly affected by the presence of the medium. If we include vacuum polarization corrections the effects of the medium are somewhat suppressed but the corrections are still sizeable.

In order to show clearly the origin of every term we shall work in a two-component model. The Hamiltonian for the nuclear sector in the mean-field theory through second order in v/c is given by[5]

$$H=\beta m_N+\left\{\beta\left[\frac{p^2}{2m_N}+U_s\right]+U_V\right\}+\left\{-\frac{1}{8m_N^3}\beta p^4-\frac{1}{4m_N^2}\beta(p^2U_s+U_sp^2)+\frac{1}{8m_N^2}[(\nabla^2U_V)-\beta(\nabla^2U_s)]-\frac{1}{4m_N^2}\boldsymbol{\sigma}\cdot[(\boldsymbol{\nabla}U_V)-\beta(\boldsymbol{\nabla}U_s)]\times\mathbf{p}\right\}, \quad (1)$$

where U_V and U_S are the vector and scalar mean fields. Notice that there are not m^*-like terms in the denominators. Third term within the second pair of curly brackets is the Darwin term. This Hamiltonian was obtained along with the other nine generators of the Poincaré group by (i) finding a realization of the algebra that satisfies the conmutation relations of the group, (ii) including in this algebra the constraint of charge conservation, that here translates into

$$[H,\rho]+[\mathbf{P},\mathbf{j}]=0\ , \tag{2}$$

and finally (iii) applying a unitary transformation to all the generators of the group (the Hamiltonain is one of them) which has the virtue of diagonalizing the Hamiltonian and at the same time maintaining the algebra of the group and preserving the gauge-invariant character of the one-body current (details about this transformation can be found in Ref. 6). The expression for the spatial part of the four-vector current which transforms covariantly under the generators of the group and conserves the charge is given by

$$\mathbf{j}=\sum_{a=1}^{A}\left\{e_N(q^2)\left[\frac{F(r)}{2m_N}\left[2\xi\frac{\nabla^{(a)}}{i}-\xi\mathbf{q}+i\mathbf{q}\times\boldsymbol{\sigma}^{(a)}\right]-\frac{i}{2m_N}\times\{[\nabla^{(a)}F(r)]+i\boldsymbol{\sigma}\times[\nabla^{(a)}F(r)]\}\right]+\frac{i\kappa_N(q^2)}{2m_N}\mathbf{q}\times\boldsymbol{\sigma}^{(a)}\right\}\ , \tag{3}$$

with

$$F(r)\equiv 2m_n[2m_N+U_S(r)-U_V(r)]^{-1}\ , \tag{4}$$

$$e_N(q^2)=\tfrac{1}{2}(1+\tau_3)eF_1(q^2)\ , \tag{5}$$

$$\kappa_N(q^2)=\tfrac{1}{2}(\mu_S+\tau_3\mu_V)eF_2(q^2)\ , \tag{6}$$

with e the proton charge and

$$\mu_s+\mu_V=2\times 1.793\ , \tag{7}$$

$$\mu_s-\mu_V=2\mu_n=2\times -1.913\ , \tag{8}$$

and where the normalization of $F_{1,2}$ is such that in the limit $q^2\to 0$

$$F_1(q^2)=F_2(q^2)=1\ . \tag{9}$$

In Eq. (3) the factor ξ takes into account the transformation of the gradient operator to the relative coordinate system. The first term in Eq. (3) is the convection piece, the second does not contribute to electric transitions; the third one is the Dirac magnetization current. The fourth term we shall call Darwin-induced current (DIC) and together with the fifth one [medium-induced magnetization (MIM) current in Ref. 4], which is related to the spin-orbit term, they give rise to the medium-induced currents. When dealing with magnetic form factors, and in particular magnetic moments, the Darwin current vanishes because of parity reasons, but the MIM current still contributes (see Refs. 3 and 4). Finally the last term is the anomalous magnetic moment contribution.

In the notation of Ref. 7 the effective current of Eq. (3) gives rise to the following transverse electric operator:

$$\hat{T}^{\mathrm{el}}_{JM}(\mathbf{r};\mathbf{q})=\frac{e_N}{2m_N}\left\{\frac{2(J+1)^{1/2}}{(2J+1)^{1/2}}F(r)j_{J-1}(qr)\mathbf{Y}_{J,J-1}\cdot\nabla-\frac{2(J)^{1/2}}{(2J+1)^{1/2}}F(r)j_{J+1}(qr)\mathbf{Y}_{J,J+1}\cdot\nabla+qF(r)j_J(qr)\mathbf{Y}_{J,J}\cdot\boldsymbol{\sigma}+\sqrt{J(J+1)}\frac{1}{qr}\frac{dF(r)}{dr}j_J(qr)\mathbf{Y}_{J,J}+\frac{1}{2J+1}\frac{dF(r)}{dr}[Jj_{J+1}(qr)-(J+1)j_{J-1}(qr)]\mathbf{Y}_J\cdot\boldsymbol{\sigma}\right\}+\frac{q\kappa_N}{2m_N}j_J(qr)\mathbf{Y}_{J,J}\cdot\boldsymbol{\sigma}\ . \tag{10}$$

In the preceding expression the two terms proportional to the derivative of the function $F(r)$ are due to the medium-induced currents referred to above and in Eq. (3). The first one comes from the DIC term and the second one is due to the MIM current.

We concentrate now on the $q\to 0$ limit of the matrix elements of this operator. In this limit (LWL) the matrix elements of the transverse electric operator depend on the divergence of the current in the form

$$T^{\mathrm{el}}_{JM}\overset{q\to 0}{\to}\frac{-i}{\sqrt{J(J+1)}}\int d^3r[(J+1)j_J(qr)-qrj_{J+1}(qr)]Y_{JM}(\hat{r})\nabla\cdot\mathbf{j}_{if}(\mathbf{r})\ . \tag{11}$$

We can make use of the continuity equation [Eq. (2)] to get rid of $\mathbf{j}_{if}(\mathbf{r})$ in terms of the full density. If we further keep only the first term in the expansion of the Bessel functions, then Eq. (11) becomes

$$T^{\mathrm{el}}_{JM}\overset{q\to 0}{\to}\left[\frac{J+1}{J}\right]^{1/2}\omega_{if}\frac{q^{J-1}}{(2J+1)!!}\int d^3r\, r^J Y_{JM}(\hat{r})[\rho_{if}(\mathbf{r})+\rho_{\mathrm{MID}}(\mathbf{r})]\ . \tag{12}$$

Here ω_{if} is the transition energy, $\rho_{if}(\mathbf{r})$ is the transition *charge* density, and finally $\rho_{\text{MID}}(\mathbf{r})$ is the "medium-induced charge density" which explicitly reads

$$\rho_{\text{MID}}(\mathbf{r})=\frac{e}{2m_N\omega_{if}}\frac{dF(r)}{dr}\left[-\frac{J}{r}\underline{\rho}_{if}(\mathbf{r})+i\nabla\underline{\rho}_{if}(\mathbf{r})\sigma\times\hat{r}\right], \tag{13}$$

where $\underline{\rho}_{if}(\mathbf{r})$ is the *nuclear transition* density. The origin of the two terms in $\rho_{\text{MID}}(\mathbf{r})$ can be traced, respectively, to the Darwin and spin-orbit terms in the Hamiltonian.

Now we notice that the transition charge density in Eq. (12) is slightly modified by the presence of the medium so, for all intents and purposes, it can take to be equal to the "free" transition charge density. Näively, one can argue that the medium introduces a modification to the standard definition [where only $\rho_{if}(\mathbf{r})$ is present] of the electric transition operator for photoabsorption via this medium-induced density. Thus, we may define a modified $B(EJ)$ as the square of the value of the integral in Eq. (12):

$$B(EJ)_{\text{free}}=\left|\int d^3r\, r^J Y_{JM}(\hat{r})\rho_{if}(\mathbf{r})\right|^2, \tag{14}$$

$$B(EJ)_{\text{mod}}=\left|\int d^3r\, r^J Y_{JM}(\hat{r})[\rho_{if}(\mathbf{r})+\rho_{\text{MID}}(\mathbf{r})]\right|^2. \tag{15}$$

In the following section we study the implications that this redefinition of the transition strength has for the nucleus ^{40}Ca. We also analyze the corrections that are introduced when the polarization of the vacuum is considered as was pointed out recently by Blunden and Horowitz.[8]

II. NUMERICAL RESULTS AND DISCUSSION

In the numerical example that follows we deal with transitions in ^{40}Ca. The relativistic single-particle basis wave functions are the eigenfunctions of the Hamiltonian of Eq. (1). The mean-field potentials are chosen to be of Woods-Saxon form and their strengths are adjusted so that (i) they reproduce (in the case of protons) the root mean square radius of the "core," (ii) they approximately reproduce the known experimental single-particle energies, and (iii) the description within the same formalism of the elastic-scattering electron data from the "core" is reasonable out to $q=4$ fm^{-1}. The value for the Woods-Saxon parameters that we obtain are reproduced in Table I.

As we are interested in electric (natural parity) transitions in ^{40}Ca we chose for the final state the first 3^- state at 3.75 MeV. For this wave function we take a linear combination of particle-hole pairs weighted by coefficients obtained from the random-phase approximations (RPA) calculation of Krewald and Speth[9] with a Landau-Migdal particle-hole interaction. In fact, for the results we present below, the collectivity of the wave function is not very relevant considering that all of them are shown normalized to the "free" transition value. This is in line with our interest in displaying only the effects introduced by the medium and not performing a sophisticated nuclear structure calculation. One must always bear in mind, however, that there may be slight modifications to these ratios when using different wave functions.

In Table II we present the results for the electric transition strengths as obtained from Eq. (12). The results are given as the increment fraction χ with respect to the "free" value, i.e., $B(EJ)_{\text{mod}}=(1+\chi)B(EJ)_{\text{free}}$. For the "free" case there is obviously no increment. When the medium modifications due to the relativistic treatment are introduced we notice the following.

(i) The first column (MIC) on the right hand side of Table II (Medium) corresponds to the case in which both medium-induced current terms, i.e., the Darwin and the spin-orbit current, are included. The resulting transition value is increased a hefty 159% with respect to the free value. The partial contribution of each of the medium-induced currents is displayed in the next two columns. Thus we notice that more than two-thirds of the increase of the $B(EJ)$ values comes from the MIM current while the rest is due to the Darwin term [in fact, there is small (1%) contribution coming from the interference between these two]. Thus, despite the fact that the Darwin term contribution is not as large as that of the MIM current term, it cannot be neglected.

(ii) The last column on the right hand side of Table II shows the same percentile increment as the first one but, this time, including the vacuum polarization. The way we included this corrections follows the approach developed in Ref. 2. The so-called "backflow" term is calculated in nuclear matter, in fact only for the isoscalar case, and then applied to ^{40}Ca by using a local density approximation. All in all, the backflow term is equivalent to a renormalization of the function $F(r)$ in Eq. (3) which becomes

$$F(r)\to F(r)\aleph(r), \tag{16}$$

where

$$\aleph(r)=\frac{1}{1+U_V(r)\eta(r)}, \tag{17}$$

TABLE I. Parameters of the interactions used in the calculation of the single-particle wave functions.

Nucleus	U_V (MeV)	U_S (MeV)	R (fm)	a (fm)
^{40}Ca proton	189.9	−254.23	4.50	0.53
neutron	189.9	−254.23	4.50	0.53

TABLE. II. Percentile increase of the transition strength [$B(EJ)$] with respect to the free value as described in the text. MIC: medium-induced currents. MIMC: medium-induced magnetization current. DIC: medium-induced Darwin current. RMIC: renormalized medium-induced current.

	Medium			
"Free"	MIC	MIMC	DIC	RMIC
0.00	1.59	1.16	0.42	0.22

$$\eta(r)=\frac{1}{\{[\frac{3}{2}\pi^2\rho(r)]^{2/3}+m^{*2}(r)\}^{1/2}}\ , \tag{18}$$

$$m^*(r)=m_N+U_S(r)\ , \tag{19}$$

and $\rho(r)$ is the nuclear density. The final result is a percentile increase of 22% with respect to the free value.

It can be concluded from the two observations above the electric transitions are dramatically affected by the presence of the strong relativistic potentials in the impulse approximation. Though the medium corrections that we have calculated in this paper appear somewhat large, asserting that they are definitely unrealistic is not as safe as in the case of magnetic moments. There, we have the Schmidt values and the experimental results to constrain the theory. Here there are model-independent (in some sense) sum rules and no experimental counterparts (except for the dipole). One might argue that in fact there is some model dependence to any sum rule but, on the other hand, in the long wavelength limit that we consider here only the dipole sum rule requires some model input (see Ref. 10 for a relativistic study of the dipole sum rule). Thus, granting the validity of the octupole sum rule for ^{40}Ca the increment we obtained for the transition strengths in this work, without including the vaccum polarization, does certainly overestimate the sum rule value. We are led to conclude, therefore, that for the determination of electric transition rates it is necessary to include the coupling of the spatial piece of the vector interaction to $N\bar{N}$ excitations. This is in line with the results arrived at by other authors (see Ref. 11 and references contained therein) who studied the nuclear response to the electromagnetic probe.

As a last remark we note that after the vacuum effects are included we are still left with a 22% increase in the transition strengths with respect to the simple impulse approximation. This increase is due to the presence of the medium which shows up in the current through the current conservation constraint. In fact, some recent calculations on quasielastic electron scattering in oxygen[12] have already employed medium-modified currents like the ones presented here. In these calculations, however, the radial dependences for the convection and spin-orbit medium currents are chosen *ad hoc*. In addition the Darwin current is treated at a very phenomenological level and without satisfying current conservation. It would be interesting to see whether a similar calculation but including the consistent current presented here would modify the results found in Ref. 12.

In summary, we have calculated electric transition strengths in a relativistic model. We find that in the LWL the corrections induced by the medium increase the nonrelativistic result by more than 100%. When the coupling to excitations of the vacuum is included these corrections are reduced to approximately 20%, for the particular case we studied. A similar calculation for transitions in other nuclei and the implications for sum rules is currently in progress.

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