# Forward scattering of a proton from a $J=1$ nucleus: Selection rules for flipping the spin of the target by two units

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We show that in the plane-wave impulse approximation an incident proton cannot, in the case of elastic forward ($q=0$) scattering, change the polarization state $(J,M)$ of a $J=1$ target nucleus from $(1,+1)$ to $(1,-1)$, where the direction of polarization is normal to the scattering plane. We then show that this double spin-flip process can take place when the effects of double scattering are taken into account. As a corollary, the spin-quadrupole moment of the deuteron must be zero.

## I. SCATTERING OF A PROTON FROM A SPIN-ONE TARGET IN THE PLANE-WAVE IMPULSE APPROXIMATION

We consider the elastic scattering of protons, which of course have spin $\frac{1}{2}$, from spin-one ($J=1$) targets like deuteron or $^6$Li. We will consider the situation where the target nucleus is polarized normal to the scattering plane formed by the incoming and outgoing proton momenta $\mathbf{k}_i$ and $\mathbf{k}_f$. We then consider the amplitude for the process in which the proton flips the spin direction of the target from $M_i=+1$ to $M_f=-1$.

We use the plane-wave impulse approximation (PWIA) for the scattering amplitude

$$F_{fi}(\mathbf{q})=\langle \psi_f\chi_f | \sum_i^A \exp(i\mathbf{q}\cdot\mathbf{r}_i)f_i|\chi_i\psi_i\rangle , \tag{1}$$

where $\psi$ is the wave function of the target and $\chi$ is the spin state of the proton. The nucleon-nucleon scattering amplitude $f_i$, in the form given by Wallace[1] and expressed in the Breit frame, is

$$\begin{aligned} f_i(k,\mathbf{q})&=2ik_f[A+B\boldsymbol{\sigma}_0\cdot\boldsymbol{\sigma}_i+iq(C_1\boldsymbol{\sigma}_0+C_2\boldsymbol{\sigma}_i)\cdot\hat{\mathbf{N}}+D(\boldsymbol{\sigma}_0\cdot\mathbf{q})(\boldsymbol{\sigma}_i\cdot\mathbf{q})+E(\boldsymbol{\sigma}_0\cdot\hat{\mathbf{K}})(\boldsymbol{\sigma}_i\cdot\hat{\mathbf{K}})]\\ &=2ik_f[A+B\boldsymbol{\sigma}_0\cdot\boldsymbol{\sigma}_i+iq(C_1\sigma_{0z}+C_2\sigma_{iz})+q^2D\sigma_{0x}\sigma_{ix}+E\sigma_{0y}\sigma_{iy}]\\ &=\langle\psi_f|\sum_i^A\exp(i\mathbf{q}\cdot\mathbf{r}_i)(\alpha+\beta\sigma_{i+1}+\gamma\sigma_{i0}+\delta\sigma_{i-1})|\psi_i\rangle , \end{aligned} \tag{2}$$

where $\hat{\mathbf{x}}=\mathbf{q}/q$ with $\mathbf{q}$ the three momentum transfer, $\hat{\mathbf{y}}=\hat{\mathbf{K}}=(\mathbf{k}_i+\mathbf{k}_f)/|\mathbf{k}_i+\mathbf{k}_f|$, thus the polarization of the target is along $\hat{\mathbf{z}}=\hat{\mathbf{N}}=\hat{\mathbf{q}}\times\hat{\mathbf{K}}$ direction. Parameters $A,..,E$ are functions of $k=k_i=k_f$ and $q$. Clearly, if the spin direction ($2m_0=+1/-1$ for up/down) of the incident proton remains the same before and after the scattering, then $\alpha=2ik_f(A+2im_0qC_1)$, $\beta=0$, $\gamma=2ik_f(2m_0B+iqC_2)$, and $\delta=0$; if it changes then $\alpha=0$ and $\gamma=0$.

By making a multipole expansion

$$\exp(i\mathbf{q}\cdot\mathbf{r}_i)=4\pi\sum_{lm} i^l j_l(qr_i)Y_{lm}^*(\Omega_q)Y_{lm}(\Omega_{r_i}) , \tag{3}$$

we can obtain the following expression for the scattering amplitude:

$$\begin{aligned} F_{M_fM_i}(\mathbf{q})=\frac{4\pi}{\sqrt{2J_f+1}}\sum_{lm} i^lY_{lm}^*(\Omega_q)\Big[&\alpha(J_ilM_im|J_fM_f)A_l\\ &+\sum_\lambda[\beta(l1m1|\lambda m+1)(J_i\lambda M_im+1|J_fM_f)+\gamma(l1m0|\lambda m)(J_i\lambda M_im|J_fM_f)\\ &+\delta(l1m-1|\lambda m-1)(J_i\lambda M_im-1|J_fM_f)]B_{l\lambda}\Big] , \end{aligned} \tag{4}$$

 

where we have defined the reduced matrix elements

$$A_l=\langle \psi^{J_f}\| \sum_i j_l(qr_i)Y_l(i)\|\psi^{J_i}\rangle ,$$

$$B_{l\lambda}=\langle \psi^{J_f}\| \sum_i j_l(qr_i)[Y_l(i)\sigma(i)]^{\lambda}\|\psi^{J_i}\rangle .$$

For the problem at hand, we have $J_i=M_i=J_f=-M_f=1$ and $\Omega_q=(\theta_q,\phi_q)=(\pi/2,0)$. Noting that $(J\lambda J\mu|J-J)=0$ unless $\mu=-\lambda=-2J$ and $B_{l\lambda}=0$ for odd $l$ (parity), we find

$$\begin{aligned}F_{-11}(\mathbf{q})&=\sqrt{\pi}(\gamma B_{22}-\sqrt{3/2}\alpha A_2)\\&=2i\sqrt{\pi}k_f[(2m_0B+iqC_2)B_{22}\\&\quad-\sqrt{3/2}(A+2im_0qC_1)A_2] .\end{aligned} \tag{5}$$

One sees that, in this case, the proton spin-flip scattering does not contribute because only $\alpha$ and $\gamma$ parameters enter the expression of the scattering amplitude.

The scattering amplitude obviously vanishes for forward scattering ($q=0$) because of the presence of $j_2(qr)$ in $A_2$ and $B_{22}$. In fact, as we will show in the next section, $B_{22}$ is zero for all $q$. So in the PWIA, only the $A$ and $C_1$ terms contribute to the scattering amplitude.

## II. VANISHING OF DIAGONAL MATRIX ELEMENT $B_{22}$

The reduced matrix element $B_{22}$ for any transition can be written as

$$B_{22}(J_i\to J_f)=\langle \psi^{J_f}\| \sum_i j_2(qr_i)[Y_2(i)\sigma(i)]^{\lambda=2}\|\psi^{J_i}\rangle . \tag{6}$$

This term is responsible, for example, for inelastic scattering from a $J=0^+$ ground state of an even-even nucleus to the giant spin-quadrupole state which in $LS$ coupling would be a particle hole state with quantum numbers $L=2$, $S=1$, and $J=2$.

But here, for elastic scattering from a $J=1$ target, we have only the diagonal reduced matrix element

$$B_{22}(J,J)=\langle \psi^{J=1}\| \sum_i j_2(qr_i)[Y_2(i)\sigma(i)]^{\lambda=2}\|\psi^{J=1}\rangle , \tag{7}$$

We claim that this matrix element is zero.

The Wigner-Eckart theorem gives

$$B_{22}=\frac{\sqrt{2J+1}}{(J2M0|JM)}\langle \psi_M^J| \sum_i j_2(qr_i)O_0(i)|\psi_M^J\rangle , \tag{8}$$

where $O_0=[Y_2\sigma]_0^{\lambda=2}$, i.e., the $z$ component. By using the following properties:

$$(\sigma_{-1})^\dagger=-\sigma_{+1} ,$$

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$$Y_{l,m}^*=(-1)^mY_{l,-m} ,$$

we note that

$$O_0=\frac{1}{\sqrt{2}}(Y_{2,1}\sigma_{-1}-Y_{2,-1}\sigma_{+1})$$

is anti-Hermitian:

$$O_0^\dagger=-O_0 . \tag{9}$$

We now take time reversal into the picture, a topic extensively discussed in the context of proton-deuteron scattering by Seyler.[2] From the general properties of time reversal, see, for example, the work of Frauenfelder and Henley,[3] we have the identity (their Eq. 4.190)

$$\langle \psi_T O_T \phi_T\rangle=\langle \phi O^+\psi\rangle , \tag{10}$$

where $O_T=TOT^{-1}$ and where $T\psi(t)=\psi_T(-t)=U_TK\psi(-t)$ with $K$ the complex-conjugation operator and $U_T=-i\sigma_y=\binom{0\ -1}{1\ \ 0}$. This is true always and does not require time-reversal invariance to hold.

We can then show by explicit calculation

$$TO_0T^{-1}=O_0 . \tag{11}$$

If $\psi=\phi=\phi_M^J$, we find, from Eqs. (9)–(11),

$$\langle \phi_{-M}^J O_0\phi_{-M}^J\rangle=-\langle \phi_M^J O_0\phi_M^J\rangle .$$

However since $O_0$ is a quadrupole operator,

$$\langle \phi_{-M}^J O_0\phi_{-M}^J\rangle=\langle \phi_M^J O_0\phi_M^J\rangle .$$

We thus have the result

$$\langle \phi_M^J O_0\phi_M^J\rangle=-\langle \phi_M^J O_0\phi_M^J\rangle , \tag{12}$$

and the matrix element vanishes. As a corollary, we see that the spin-quadrupole moment of the deuteron is zero. A clear exposition of how time-reversal invariance affects the matrix elements of a tensor operator is given in the recent text by Sakurai.[4]

Summarizing this section we have found, in the single scattering approximation, that when a proton changes the state of a $J=1$ object from $M=1$ to $-1$ (with the $z$ axis normal to the scattering plane) the proton itself cannot flip its spin. From time-reversal invariance considerations, the amplitude given by Eq. (5) simplifies to

$$\begin{aligned}F_{-11}(\mathbf{q})=&\ 2i\sqrt{\pi}k_f(-\sqrt{3/2})[A(q)+2im_0qC_1(q)]\\&\times\langle J=1\| \sum_i j_2(qr_i)Y_2(i)\|J=1\rangle .\end{aligned} \tag{13}$$

For the case of the deuteron the last factor $[A_2(q)]$ will be nonzero because of the presence of $D$-state admixtures in the deuteron wave function. More generally, the $J=1$ system must possess *orbital* angular momentum in order for the amplitude $F_{-11}(\mathbf{q})$ to be nonzero. For single scattering we have an orbital flip—not a spin flip.

## III. BEYOND THE PWIA—EFFECTS OF DOUBLE SCATTERING

Although the forward scattering amplitude for flipping a $J=1$ object from a state $M=+1$ to a state $M=-1$ with a spin $\frac{1}{2}$ nucleon as a projectile vanishes in PWIA, we do not expect this to be an exact result. For example, in the second-order Born approximation, we would expect that the first interaction can change $M$ to $+1$ to 0 and the second interaction from 0 to $-1$.

We will here consider proton-deuteron scattering in the eikonal approximation, where it is known, from the work of Franco and Glauber,[15] and of Harrington,[6] that the second-order scattering terms are important. Allowing for spin-dependent terms in the nucleon-nucleon scattering amplitude, Franco and Glauber[5] have shown that an approximate expression for the proton-deuteron elastic scattering amplitude is

$$F_{fi}(q)=\langle\psi_f\chi_f|M_n(q)\exp(\tfrac{1}{2}i\mathbf{q}\cdot\mathbf{r})+M_p(q)\exp(-\tfrac{1}{2}i\mathbf{q}\cdot\mathbf{r})|\psi_i\chi_i\rangle + \frac{i}{2\pi k}\langle\psi_f\chi_f|\int\exp(i\mathbf{q}'\cdot\mathbf{r})\tfrac{1}{2}\{M_n(\mathbf{q}'+\tfrac{1}{2}\mathbf{q}),M_p(-\mathbf{q}'+\tfrac{1}{2}\mathbf{q})\}d^2q'|\psi_i\chi_i\rangle \,, \tag{14}$$

where $\mathbf{r}$ is the relative coordinate of the two nucleons in the deuteron and the symbol { } represents the anticommutator. We can also write the above equation as

$$F_{fi}(q)=F_{fi}^{(1)}(q)+F_{fi}^{(2)}(q) \tag{15}$$

corresponding to single scattering (first two terms) and double scattering. Only $F_{fi}^{(2)}(q)$ will contribute to the double spin-flip process $M=+1\rightarrow M=-1$.

The above expression is approximate because spin-dependent eikonals do not commute. One neglects spin contributions where the two nucleons in the deuteron strongly overlap. An explicit expression for the spin structure of the *p*-*d* scattering amplitude has been given by Alberti, Bertocchi, and Gregorio.[7]

In this work, in order to set the *scale* for the double scattering contribution to the double spin-flip process, we will be content to evaluate the amplitude for the forward direction ($q=0$) with an *S*-wave deuteron wave function

$$\psi=\Phi(r)\xi_M^{S=1} \,. \tag{16}$$

Clearly, $\xi_{+1}=\uparrow_p\uparrow_n$ and $\xi_{-1}=\downarrow_p\downarrow_n$.

Recall that our coordinate system is such that the polarization of the deuteron is the $z$ direction and, in the case of forward scattering, the incident direction is along the $y$ axis. We not only take $q$ to be zero but also assume that $q'$ is sufficiently small so that $\mathbf{k}_i$, $\mathbf{k}_f$, and $\mathbf{k}'$ (the momentum carried by the incident proton between two collisions) all lie approximately along $y$ axis.

The vector $\mathbf{q}'$ is then in the $x$-$z$ plane making an angle $\phi$ with the $x$ axis

$$\mathbf{q}'=\mathbf{k}_i-\mathbf{k}'=\mathbf{k}_f-\mathbf{k}'=q'(\cos\phi\hat{\mathbf{x}}+\sin\phi\hat{\mathbf{z}}) \,,$$

$$\mathbf{K}'=(\mathbf{k}_i+\mathbf{k}')/2=(\mathbf{k}'+\mathbf{k}_f)/2=K'\hat{\mathbf{y}} \,.$$

Hence

$$\hat{\mathbf{N}}'=\hat{\mathbf{q}}'\times\hat{\mathbf{K}}'=-\sin\phi\hat{\mathbf{x}}+\cos\phi\hat{\mathbf{z}} \,.$$

We then are able to work out the amplitude

$$F_{-1,+1}(q=0)=\frac{i}{2\pi k}\int S(\mathbf{q})\langle\xi_{-1}\chi_f|\tfrac{1}{2}\{M_n(\mathbf{q}),M_p(-\mathbf{q})\}|\xi_{+1}\chi_i\rangle d^2q$$
$$=4ik\int_0^\infty S(q)[(B_nE_p+B_pE_n+E_nE_p)-q^2(B_nD_p+C_{2n}C_{2p}+B_pD_n)/2-q^4D_nD_p/2]q\,dq \,. \tag{17}$$

We emphasized that this expression is valid only when we approximate the deuteron wave function by the pure $S$ wave. In the above, $S(\mathbf{q})$ is the form factor of the deuteron in the $S$-wave approximation

$$S(\mathbf{q})=\int\exp(i\mathbf{q}\cdot\mathbf{r})|\phi(r)|^2d^3r \,,$$
$$=4\pi\int\frac{\sin(qr)}{qr}|\phi(r)|^2r^2dr \,. \tag{18}$$

With a Hulthen wave function

$$\Phi(r)=N(e^{-\alpha r}-e^{-\beta r})/r$$

with $\alpha=0.232\ \mathrm{fm}^{-1}$, $\beta=1.202\ \mathrm{fm}^{-1}$, the deuteron form factor is then

$$S(q)=\frac{2\alpha\beta(\alpha+\beta)}{q(\alpha-\beta)^2}\left[\tan^{-1}\left[\frac{q}{2\alpha}\right]-2\tan^{-1}\left[\frac{q}{\alpha+\beta}\right]+\tan^{-1}\left[\frac{q}{2\beta}\right]\right] \,. \tag{19}$$

An approximate $q$ dependence of the nucleon-nucleon scattering amplitude is given by Wallace,[1]

$$A(q)=A_0\exp(-\eta_Aq^2)$$

with similar forms for $B$, $C_1$, $C_2$, $D$, and $E$. The parameters $A_0$ and $\eta_A$, etc., are taken from Table I in Ref. 8 for an 800-MeV proton beam. With the above assumptions and with the above parameters, we find

$$F_{-1,+1}(q=0)=(22.11+i7.20)\times10^{-3}\ (\mathrm{fm})\ ,$$

or

$$\frac{d\sigma}{d\Omega}(q=0,\text{double spin flip})=5.4\times10^{-4}\ (\mathrm{fm}^2)\ .$$

This is much smaller (by approximately four orders of magnitude) than the unpolarized cross section for which one averages over initial states and sums over final ones. This is not surprising because the single scattering amplitude is zero for the double spin-flip process and we are left with *pure* double scattering. It should be remembered that in the case of unpolarized scattering near the forward direction *pure* double scattering, proportional to $|F^{(2)}|^2$, is very small: the double scattering makes itself felt mainly through its interference with single scattering.

We can use a more elaborate wave function[5]

$$\Phi(r)=N(e^{-ar}-e^{-dr})(1-e^{-cr})(1-e^{-gr})/r\ ,$$

with $a=0.232\ \mathrm{fm}^{-1}$, $d=1.90\ \mathrm{fm}^{-1}$, $c=1.59\ \mathrm{fm}^{-1}$, and $g=2.5\ \mathrm{fm}^{-1}$. We then obtain the following result,

$$M_{-1,+1}(q=0)=(15.67+i2.92)\times10^{-3}\ (\mathrm{fm})$$

or

$$\frac{d\sigma}{d\Omega}(q=0,\text{spin flip})=2.5\times10^{-4}(\mathrm{fm}^2)\ .$$

This is only half of that with the Hulthen wave function.

We thus have the interesting result that there is a significant dependence of the double scattering on the details of the deuteron wave function. It will be of particular interest to further study the effects of the short-range behavior of the deuteron wave function on the double scattering process.

It would obviously be interesting to test this result experimentally by attempting to measure the double spin-flip reaction from a polarized spin-one target. In this regard the extensive work with polarized deuteron targets by George Igo and collaborators should be mentioned.[9] We realize that forward scattering experiments are difficult. Perhaps the most practical experiment is to use a polarized deuteron beam, such as they have in SATURNE, and scatter from a hydrogen target.

Not too much work has been done on proton-$^6$Li scattering at high energies. However, at low energies, theoretical work by Thompson[10] indicates that coupled channel effects (i.e., the virtual excitation of low-lying levels) are extremely important.

We hope that this work will stimulate and help to clarify experimental work on the scattering of nucleons from or by polarized deuterons.

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