Constraints on the self-consistent relativistic Fermi-sea particle formalism in the quantum hadrodynamical model

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The self-consistent relativistic Fermi-sea particle formalism in the quantum hadrodynamics is discussed in terms of properties of conserving approximations. The relativistic Dirac-Hartree-Fock approximation (σ, ω, ρ, and π), for example, is examined from the point of view of the Hugenholtz–Van-Hove theorem and thermodynamics as a check for internal consistency. These two conditions sufficient for constructing conserving approximations are strict constraints on modification and simplification of the Fermi-sea particle approximations and should be used in order to define consistent quasiparticle approximations. It is shown that the pion vertex and retardation which enters through exchange corrections prevent conserving properties to be maintained in the Dirac-Hartree-Fock approximation, and this suggests a careful analysis of self-consistency in the relativistic formalism when correlation effects are considered. The relativistic Fermi-sea particle approximations can be defined uniquely when both constraints are incorporated into self-consistency of the approximations.

I. INTRODUCTION

The Schrödinger equation is used for the starting point to understand nonrelativistic quantum many-body systems, and based on this formalism, nuclear matter equations of state have been discussed in order to derive them microscopically. The consistency of approximations and assumptions within this formulation can be checked, for example, in terms of properties of conserving approximations.[1] However, we know that most of proposed approximations are nonconserving, and they violate the known properties of conserving approximations: the Hugenholtz–Van-Hove (HV) theorem[2] and certain fundamental thermodynamic relations. The nonrelativistic Brueckner-Hartree-Fock approximation has been rediscussed, and the preceding properties are used as constraints in order to check the computational consistency of the approximation and formulate a general and convenient method for deriving correct quasiparticle approximation.[3]

The extension of the preceding formalism to study high-energy and high-density phenomena (neutron stars,[4] supernova, and heavy-ion collisions) must be investigated in a relativistic field theory. We consider here the quasiparticle approach in the approximations of a renormalizable relativistic quantum field theory, quantum hadrodynamics (QHD),[5] which is based on mesons and baryons and maintains covariance, gauge invariance, and causality. In a renormalizable relativistic field theory, the fundamental problem is how to define approximations systematically including the quantum vacuum, which is technically complicated and formidable in practical calculations and still not known.[6,7] However, there are approximations motivated by nonrelativistic results that may correspond to the nonrelativistic limit of the QHD approximations, such as the mean-field theory (MFT),[8] Dirac-Hartree-Fock (DHF),[9] and relativistic Brueckner-Bethe-Goldstone (RBBG).[10] They are defined by neglecting the modifications of the Dirac sea particles (negative-energy particles), but self-consistently defined using real (valence) nucleons inside the Fermi sea. Therefore, the approximations should reproduce the ground-state properties of nonrelativistic calculations with certain limits and give appropriate relativistic corrections. We also discuss an effective vacuum fluctuation correction to the DHF approximation.

Internal consistency of the QHD approximations as conserving approximations has not been discussed except the MFT and the relativistic Hartree approximation (RHA) (MFT plus vacuum corrections); both approximations satisfy the HV theorem and thermodynamics exactly. In addition to the calculational and phenomenological consistency, the MFT reproduces empirical data reasonably well.[11] However, the systematic higher-order corrections to the MFT are difficult to achieve since retardation interaction causes the violation of the HV theorem and thermodynamics. The problem of the retardation effects is discussed explicitly in the DHF (σ,ω) approximation.[12] When retardation interaction is ignored (the static limit), the DHF(σ,ω) satisfies the HV theorem and thermodynamic relations exactly. This shows that self-consistency should be examined carefully when correlation effects are included.

We will discuss the DHF approximation by including σ, ω, ρ, and π and show whether or not the model restores thermodynamic consistency (the HV theorem and thermodynamics). This is necessary when thermodynamic quantities such as compressibility and symmetry energy are calculated, and Landau Fermi-liquid theory can be applied exactly if an approximation maintains thermodynamic consistency. The static limit of σ, ω, and ρ sectors maintains thermodynamic consistency, but the momentum-dependent pion-vertex violates the requirement because the momentum-dependent vertex now carries off the energy-momentum of the system. To remedy this problem, one might follow a procedure analogous to nonrelativistic calculation, that is, since the momentum-dependent pion-vertex generates a contact interaction, we should remove the contact interaction terms which lead to strong short-range repulsion. However, this causes serious inconsistency in a thermodynamic relation although the improved approximation can maintain the HV theorem. Therefore, the theorem is not the sufficient condition for constructing conserving approximations. This will be discussed in Sec. III.

Thermodynamic consistency of the QHD approximations can be checked unambiguously by calculating energy density, pressure and single-particle energy spectrum. We can calculate dynamical pressure rigorously for the homogeneous infinite nuclear matter, and the pressure must vanish at the saturation density of nuclear matter. This is also one of the constraints for nuclear matter approximations in order to define the correct saturation density, however, the condition is not carefully considered in nuclear matter approximations. We should notice that if our approximation breaks thermodynamic consistency seriously, it does not make sense to determine compressibility, symmetry energy, and also the equation of state. Conserving approximations can maintain all the constraints already mentioned. The sufficient condition in order to construct conserving approximations will be discussed specifically using the DHF approximation. We emphasize that one should examine thermodynamic consistency to extract consistent calculations from the relativistic approximations for nuclear matter; and together with successful construction of thermodynamic consistency, the relativistic Fermi-sea particle approximations can be defined uniquely.

II. THE DIRAC-HARTREE-FOCK APPROXIMATION

Dyson's equation is used to derive the full relativistic HF equation systematically to sum to all orders self-consistent direct and exchange diagrams to the baryon Green function.[5] We can write the full Green function in terms of the particle-antiparticle propagator, $G_F(k)$, and the hole propagator inside the Fermi sea, $G_D(k)$,

$$G(k)=G_F(k)+G_D(k) \ , \tag{2.1}$$

$$G_F(k)=[\gamma^\mu k^*_\mu + M^*(k)]\frac{1}{k^{*2}_\mu - M^{*2}(k)+i\epsilon} \ , \tag{2.2}$$

$$G_D(k)= [\gamma^\mu k^*_\mu + M^*(k)]\frac{i\pi}{E^*(k)} \times\delta(k^0-E(k))\theta(k_F-|\mathbf{k}|) \ . \tag{2.3}$$

The dynamical variables $k^*(k)$ and $M^*(k)$ are defined by the proper self-energy $\Sigma(k)$, which is decomposed generally in the infinite matter system as

$$\Sigma(k)=\Sigma^s(k)-\gamma^0\Sigma^0(k)+\gamma\cdot\mathbf{k}\Sigma^v(k) \ .$$

Using $\Sigma^v(k)$ and $\Sigma^s(k)$, we have $\mathbf{k}^*(k)=\mathbf{k}[1+\Sigma^v(k)]$ and $M^*(k)=M+\Sigma^s(k)$, where M is the nucleon mass $M=939$ MeV. It is assumed that the baryon Green function has simple poles with unit residue, and that at finite baryon density the quasiparticle levels are filled up to $|\mathbf{k}|=k_F$. The relativistic Fermi-sea particle approach is defined by taking contributions from real nucleons in the Fermi sea, and $G_D(k)$ is employed in the calculation of self-energies, energy density and pressure. The self-consistent single-particle energy spectrum, $E(k)$, which is the solution to the transcendental equation is given by self-energies as

$$\begin{aligned} E(k)&=[E^*(k)-\Sigma^0(k)]_{k_0=E(k)} \\ &=\{\mathbf{k}^2[1+\Sigma^v(|\mathbf{k}|,E(k))]^2 \\ &\quad +[M+\Sigma^s(|\mathbf{k}|,E(k))]^2\}^{1/2}-\Sigma^0(|\mathbf{k}|,E(\mathbf{k})) \ , \end{aligned} \tag{2.4}$$

and self-energies depend on $E(k)$, $|\mathbf{k}|$, and k_F.

We can calculate the Fermi energy according to the HV theorem,

$$\varepsilon+\frac{d}{d\rho_B}\left[\frac{\varepsilon}{\rho_B}\right]=E_{\mathrm{HV}}(k_F)\rho_B \ , \tag{2.5}$$

where ε is the energy density, ρ_B the baryon density, and $E_{\mathrm{HV}}(k_F)$ the Fermi energy. The second term can be interpreted as pressure of the system and it will vanish at nuclear matter saturation point, where we will have the relation,

$$\frac{\varepsilon}{\rho_B}=E_{\mathrm{HV}}(k_F) \ , \tag{2.6}$$

which shows the equality between the Fermi energy and the energy per particle. In the MFT, RHA, and the static limit of the DHF (σ,ω) approximation, $E(k_F)$ given by the Eq. (2.4) and $E_{\mathrm{HV}}(k_F)$ given by the Eq. (2.5) are equal at every density, and the approximations are physically well defined. The analytical proof of thermodynamic consistency of the static DHF (σ,ω) approximation is discussed,[12] and so we will discuss self-energies of π- and ρ-meson sectors in order to check thermodynamic consistency.

There is an ambiguity in the HF approximation to the pseudoscalar (PS) model of pion. Due to the large effective PS-πN coupling, scalar self-energies calculated with the coupling are extremely large, and the ground-state configuration at normal saturation density is "Fermi-shell" state (see Chap. 8 of Ref. 5). The HF approximation to the PS model of pion is inadequate for a description of the ground state of nuclear matter. Therefore instead of the PS model of pion, we use the pseudovector (PV) model of pion, which is transformed from the PS model of pion by a nonlinear chiral transformation and renormalizable in spite of the resulting PV-πN coupling.[13] The large coupling constants in the PS model are now replaced by small effective coupling constants in the PV model, and one may treat nonlinear interactions in the PV model Lagrangian in a perturbative fashion. Note that the PS-HF approximation maintains the HV theorem and thermodynamics in the static limit, $E(k)-E(q)=0$, and so we should check the properties in the PV-HF approximation.

We use the PV-πN model of pion and rewrite the self-energies for ρ and π mesons for the analysis of thermodynamic consistency:

$$\Sigma^s(k)=\frac{g_\rho^2(\zeta-1)}{2(2\pi)^3}\int^{k_F}d^3q\frac{M^*(q)}{E^*(q)}D_\rho^0(k,q)+\frac{\zeta-1}{2(2\pi)^3}\left[\frac{g_\pi}{2M}\right]^2\int^{k_F}d^3q\frac{M^*(q)}{E^*(q)}\{[E(k)-E(q)]^2-(\mathbf{k}-\mathbf{q})^2\}D_\pi^0(k,q)\ , \tag{2.7}$$

$$\Sigma^0(k)=\frac{g_\rho^2(\zeta-4)}{4m_\rho^2(2\pi)^3}\int^{k_F}d^3q+\frac{g_\rho^2(\zeta-1)}{4(2\pi)^3}\int^{k_F}d^3qD_\rho^0(k,q)$$
$$+\frac{(\zeta-1)}{(2\pi)^3}\left[\frac{g_\pi}{2M}\right]^2\int^{k_F}\frac{d^3q}{(2\pi)^3}D_\pi^0(k,q)$$
$$\times\left\{\frac{\{[E(k)-E(q)]^2+(k-q)^2\}}{2}+\frac{[E(k)-E(q)][qq^*(q)-\mathbf{k}\cdot\mathbf{q}^*(q)]}{E^*(q)}\right\}, \tag{2.8}$$

$$\Sigma^v(k)=\frac{g_\rho^2(\zeta-1)}{4k(2\pi)^3}\int^{k_F}d^3q\frac{q^*(q)\cos\theta}{E^*(q)}D_\rho^0(k,q)$$
$$+\frac{(\zeta-1)}{(2\pi)^3}\left[\frac{g_\pi}{2M}\right]^2\int^{k_F}d^3q\left[E(k)-E(q)+\frac{qq^*(q)}{E^*(q)}\right]D_\pi^0(k,q)$$
$$-\frac{\zeta-1}{2(2\pi)^3k}\left[\frac{g_\pi}{2M}\right]^2\int^{k_F}d^3q\cos\theta D_\pi^0(k,q)$$
$$\times\left[\frac{q^*(q)}{E^*(q)}\{k^2+q^2+[E(k)-E(q)]^2\}+2q[E(k)-E(q)]\right], \tag{2.9}$$

where $q=|\mathbf{q}|$ and $q^*(q)=|\mathbf{q}[1+\Sigma^v(q)]|$, and $D_i^0(k,q)^{-1}=(k-q)_\lambda^2-m_i^2+i\epsilon$. All self-energies are evaluated on shell at the self-consistent single-particle energies $q^0=E(q)$ given by the transcendental equation (2.4). ζ is the spin-isospin degeneracy factor, $\zeta=2$ (neutron matter), $\zeta=4$ (nuclear matter). The self-energies $\Sigma^s(k)$, $\Sigma^0(k)$, and $\Sigma^v(k)$ together with dynamical variables, $k^*(k)$, $M^*(k)$, and $E(k)$ constitute a system of coupled, nonlinear integral equations for determining the DHF approximation.

The energy density and pressure of the system are calculated from the expectation value of the energy-momentum tensor $\hat{T}^{\mu\nu}$. The energy density is given by $\varepsilon=\langle\hat{T}^{00}\rangle$, and the hydrodynamic pressure is $\mathcal{P}_{\text{hy}}=\frac{1}{3}\langle\hat{T}^{ii}\rangle$, where i is summed ($i=1,2,3$),[14] and off-diagonal terms are identically zero because of the rotational invariance of the system. The explicit expressions for the energy density and pressure are

$$\varepsilon=-\frac{g_\rho^2(4-\zeta)}{16m_\rho^2}\rho_B^2+\frac{\zeta}{(2\pi)^3}\int^{k_F}d^3k\,E(k)$$
$$+\frac{\zeta(\zeta-1)}{2(2\pi)^6}\int^{k_F}\frac{d^3k}{E^*(k)}\int^{k_F}\frac{d^3q}{E^*(q)}\left[\frac{g_\rho^2}{2}D_\rho^0(k,q)\{\tfrac{1}{2}-[E(k)-E(q)]^2D_\rho^0(k,q)\}[k^{*\mu}q_\mu^*-2M^*(k)M^*(q)]\right.$$
$$\left.+\left[\frac{g_\pi}{2M}\right]^2D_\pi^0(k,q)\{\tfrac{1}{2}-[E(k)-E(q)]^2D_\pi^0(k,q)\}F(k,q)\right], \tag{2.10}$$

and

$$\mathcal{P}_{\text{hy}}=\frac{g_\rho^2(4-\zeta)}{16m_\rho^2}\rho_B^2+\frac{1}{3}\frac{\zeta(\zeta-1)}{(2\pi)^3}\int^{k_F}d^3k\frac{\mathbf{k}\cdot\mathbf{k}^*(k)}{E^*(k)}$$
$$-\frac{\zeta(\zeta-1)}{2(2\pi)^6}\int^{k_F}\frac{d^3k}{E^*(k)}\int^{k_F}\frac{d^3q}{E^*(q)}\left[\frac{g_\rho^2}{2}D_\rho^0(k,q)[\tfrac{1}{2}+\tfrac{1}{3}(\mathbf{k}-\mathbf{q})^2D_\rho^0(k,q)][k^{*\mu}q_\mu^*-2M^*(k)M^*(q)]\right.$$
$$\left.+\left[\frac{g_\pi}{2M}\right]^2D_\pi^0(k,q)[\tfrac{1}{2}+\tfrac{1}{3}(\mathbf{k}-\mathbf{q})^2D_\pi^0(k,q)]F(k,q)\right], \tag{2.11}$$

where

$$F(k,q)=2k_\mu^*(k-q)^\mu q_\nu^*(k-q)^\nu-(k-q)_\lambda^2[k^{*\mu}q_\mu^*+M^*(k)M^*(q)]\ .$$

We can calculate the single-particle energy (2.4) using self-energies just given and check a thermodynamic relation using the energy density ε and the pressure $\mathcal{P}_{\text{hy}}$. However, the thermodynamic quantities of the system such as energy density and pressure are not simply given by the energy-momentum tensor when dynamical quantities $[M^*(k), k^*(k)$, and $E(k)]$ are determined, but are self-consistently related to the dynamical quantities through the Eq. (2.5); this is the important consequence of the HV theorem.

III. THERMODYNAMIC CONSISTENCY

We will show whether the DHF approximation including ρ and π is a conserving approximation or not. The energy density and pressure are related by the first law of thermodynamics:

$$\mathcal{P}_T=\rho_B^2\frac{\partial}{\partial\rho_B}\left[\frac{\varepsilon}{\rho_B}\right]=-\varepsilon+\mu\rho_B \ , \tag{3.1}$$

where μ is the chemical potential of the system. In infinite nuclear matter, we have the binding energy curve (total-energy/particle versus baryon density) which saturates at $k_F=1.30$ fm^{-1}; $\varepsilon/\rho_B-M=-15.75$ MeV. The remarkable conclusion of the HV theorem is that in any interacting many-body system which has a saturating binding energy curve, the chemical potential must be equal to the Fermi energy $E(k_F)$ at saturation (pressure $=0$): $\mu=\varepsilon/\rho_B=E(k_F)$. This is the constraint that the energy density and single-particle energy must satisfy.

From the energy density and the pressure given by (2.10) and (2.11) and the dynamical variables which are defined by self-energies, we can check the HV theorem of the DHF approximation analytically. First, we will prove that the static limit of the DHF approximation with the PV pion does not maintain the HV theorem, (3.1) with $\mu=E(k_F)$ at every density. The static limit means that we will ignore the energy transfers in self-energies, energy density and pressure: $E(k)-E(q)=0$, and we will show that the violation of the HV theorem comes solely from the momentum-dependent pion vertex. We can calculate the chemical potential, μ, of the DHF approximation,

$$\begin{aligned}\varepsilon+\mathcal{P}_{\rm hy}=&\frac{1}{3}\frac{\zeta}{(2\pi)^3}\int^{k_F}d^3k\frac{\mathbf{k}\cdot\mathbf{k}^*(k)}{E^*(k)}+\frac{\zeta}{(2\pi)^3}\int^{k_F}d^3k\,E(k)\\&-\frac{1}{2}\frac{\zeta(\zeta-1)}{(2\pi)^6}\int^{k_F}\frac{d^3k}{E^*(k)}\int^{k_F}\frac{d^3q}{E^*(q)}\left[\frac{g_\rho^2}{2}D_\rho^0(\mathbf{k},\mathbf{q})^2[k^{*\mu}q_\mu^*-2M^*(k)M^*(q)]\tfrac{1}{3}(\mathbf{k}-\mathbf{q})^2\right.\\&\left.+\left[\frac{g_\pi}{2M}\right]^2D_\pi^0(\mathbf{k},\mathbf{q})^2F(k,q)\tfrac{1}{3}(\mathbf{k}-\mathbf{q})^2\right] .\end{aligned}\tag{3.2}$$

The second term on the right-hand side of (3.2), which is the sum of nucleon single-particle energy spectrum, is rewritten by performing partial integration as

$$\frac{\zeta}{(2\pi)^3}\int^{k_F}d^3k\,E(k)=\rho_BE(k_F)-\frac{1}{3}\frac{\zeta}{(2\pi)^3}\int_0^{k_F}d^3k\,k\frac{\partial E(k)}{\partial k} . \tag{3.3}$$

The derivative of the baryon single-particle energy $E(k)$ with respect to the momentum k in the second term is calculated from the self-consistent transcendental equation (2.4) for $E(k)$ and the self-consistent equations for $M^*(k)$, $k^*(k)$, and $\Sigma^0(k)$. We obtain

$$\begin{aligned}\frac{\zeta}{(2\pi)^3}\int_0^{k_F}d^3k\,k\frac{\partial E(k)}{\partial k}=&\frac{\zeta}{(2\pi)^3}\int^{k_F}d^3k\frac{\mathbf{k}\mathbf{k}^*}{E^*(k)}\\&-\frac{\zeta(\zeta-1)}{2(2\pi)^6}\int^{k_F}\frac{d^3k}{E^*(k)}\int^{k_F}\frac{d^3q}{E^*(q)}\\&\times\left[\frac{g_\rho^2}{2}(\mathbf{k}-\mathbf{q})^2[k^{*\mu}q_\mu^*-2M^*(k)M^*(q)]D_\rho^0(\mathbf{k},\mathbf{q})^2\right.\\&\left.+\left[\frac{g_\pi}{2M}\right]^2F(\mathbf{k},\mathbf{q})[D_\pi^0(\mathbf{k},\mathbf{q})+(\mathbf{k}-\mathbf{q})^2D_\pi^0(\mathbf{k},\mathbf{q})^2]\right] .\end{aligned}\tag{3.4}$$

Therefore, by substituting the final result (3.4) into (3.3), we can show

$$\varepsilon+\mathcal{P}_{\rm hy}=\rho_BE(k_F)+\tfrac{1}{3}\frac{\zeta(\zeta-1)}{2(2\pi)^6}\int^{k_F}\frac{d^3k}{E^*(k)}\int^{k_F}\frac{d^3q}{E^*(q)}\left[\frac{g_\pi}{2M}\right]^2F(\mathbf{k},\mathbf{q})D_\pi^0(\mathbf{k},\mathbf{q}) . \tag{3.5}$$

Neither $\mathcal{P}_{\rm hy}$ nor the second term on the right-hand side vanishes at nuclear matter saturation, therefore the HV theorem is not maintained, and also the correct nuclear matter saturation density is not well defined. Although the PS-HF approximation maintains thermodynamic consistency in the static limit and hence defines the correct nuclear matter saturation density, the PV-HF approximation does not. It is because the PV-HF approximation was defined in a perturbative fashion in which higher-order nonlinear terms in the model Lagrangian were neglected. Therefore, the PS-HF approximation is not completely transformed to the PV-HF approximation, and this is the reason why the static limit of the PV-HF approximation does not maintain thermodynamic consistency.

In Fig. 1, we compared the Fermi energy $E(k_F)$ and $E_{\rm HV}(k_F)=(\varepsilon+\mathcal{P}_{\rm hy})/\rho_B$ defined by the DHF (σ,ω,π,ρ) approximation; they are different about 9.5 MeV at saturation and the difference between $E_{\rm HV}(k_F)$ and $E(k_F)$ become larger for the high-density region. In Fig. 2, we compared hydrodynamic pressure, $\mathcal{P}_{\rm hy}=\frac{1}{3}\langle T^{ii}\rangle$, thermodynamic pressure $\mathcal{P}_T=\rho_B^2\partial(\varepsilon/\rho_B)/\partial\rho_B$, and pressure consistent with the requirement of the HV theorem, $\mathcal{P}_{\rm HV}=E(k_F)\rho_B-\varepsilon$. Coupling constants are adjusted to reproduce the saturation property, $k_F=1.30\ {\rm fm}^{-1}$, $\varepsilon/\rho_B-M=-15.75$ MeV, and the symmetry energy $a_4=35.0$ MeV. The coupling constants and thermodynamic properties for several approximations are listed in the Table I. At nuclear matter saturation point, the pressure of the system vanishes because of the balance of attractive and repulsive forces, and we can use this condition to check internal consistency. $\mathcal{P}_T$ vanishes at saturation since the pressure is directly defined from the curvature of ε/ρ_B-M, but $\mathcal{P}_{\rm hy}$ and $\mathcal{P}_{\rm HV}$ becomes zero at different densities, which produce different saturation densities. Therefore, saturation properties of nuclear matter should be defined carefully when retardation interactions are included.

The static DHF approximation can be modified in order to maintain the HV theorem: $(\varepsilon+\mathcal{P}_{\rm hy})/\rho_B=E_{\rm HV}(k_F)=E(k_F)$. Following the common procedure in the nonrelativistic calculations, one can subtract terms that lead to contact interactions (delta-function interactions) from self-energies. The requirement of the HV theorem defines the subtraction terms uniquely. They are given by

$$\Delta\Sigma^s(k)=\Sigma^s(k)+\left[\frac{g_\pi}{2M}\right]^2\frac{\zeta-1}{2(2\pi)^3}\int^{k_F}d^3q\frac{M^*(q)}{E^*(q)}\ln[D_\pi^0(k,q)]\ , \tag{3.6}$$

$$\Delta\Sigma^0(k)=\Sigma^0(k)-\left[\frac{g_\pi}{2M}\right]^2\frac{\zeta-1}{2(2\pi)^3}\int^{k_F}d^3q\,\ln[D_\pi^0(k,q)]\ , \tag{3.7}$$

$$\Delta\Sigma^v(k)=\Sigma^v(k)+\left[\frac{g_\pi}{2M}\right]^2\frac{\zeta-1}{2(2\pi)^3}\frac{1}{k}\int^{k_F}d^3q\,\cos\theta\ln[D_\pi^0(k,q)]\ , \tag{3.8}$$

where $\Sigma^v(k)$ is not exactly subtracted, but it is sufficient for the comparison between $E_{\rm HV}(k_F)$ and $E(k_F)$ up to the relatively high-density region ($k_F\sim3.0\ {\rm fm}^{-1}$).

We can compare $(\varepsilon+\mathcal{P}_{\rm hy})/\rho_B=E_{\rm HV}(k_F)$ and the self-consistent single-particle energy $E(k)$ at the Fermi surface with self-energies modified as Eqs. (3.6)–(3.8). The difference between $E_{\rm HV}(k_F)$ and $E(k_F)$ is less than 1% up to $k_F=3.0\ {\rm fm}^{-1}$. However, we found large difference between $\mathcal{P}_{\rm HV}$ and $\mathcal{P}_T$. The modified approximation can maintain the HV theorem, but violate thermodynamics seriously; we have $E_{\rm HV}(k_F)=E(k_F)$ and $\mathcal{P}_{\rm hy}=\mathcal{P}_{\rm HV}\neq\mathcal{P}_T$. Since the requirement of the HV theorem defines uniquely how to modify self-energies, it is not possible to resolve the preceding problem. Therefore, the procedure given by the subtraction without considering the self-consistent structure of an approximation is not physically reliable and well defined. The HV theorem is a constraint to which dynamical calculations must obey, but the theorem does not necessarily assure correct thermodynamic relations. This example demonstrates the important conclusion that the sufficient condition for consistent quasiparticle approximations is thermodynamic consistency.

The problem of the pion correction comes from the momentum-dependent pion vertex. In order to show this, we set the momentum dependence of the vertex constant, $\mathbf{k}-\mathbf{q}\rightarrow\mathbf{c}$ (*constant*), together with $E(k)-E(q)=0$. In this case, thermodynamic consistency of the approximation is recovered exactly: $\mathcal{P}_{\rm hy}=\mathcal{P}_{\rm HV}=\mathcal{P}_T$; $E_{\rm HV}(k_F)=E(k_F)$; a conserving approximation defined by $G_D(k)$ can be found within certain limits. The constant

TABLE I. Fermi-liquid properties of nuclear matter. Results at saturation are given for the MFT of QHD-I and QHD-II (Ref. 18), and the static Dirac-Hartree-Fock (SDHF) approximation (σ, ω, and ρ). The coupling constants g_s^2 and g_v^2 are obtained by fitting the binding energy and saturation density of equilibrium nuclear matter ($\varepsilon/\rho_B-M=-15.75$ MeV, $k_F=1.30\ {\rm fm}^{-1}$). Note that g_ρ is fixed by reproducing the value of the symmetry energy 35.0 MeV (Refs. 10 and 16) in the SDHF_2. K is the compression modulus and a_4 the symmetry energy. F_0, F_1, and F_0' are the values of dimensionless Landau particles at saturation. The hadron masses used in all calculations are $m_s=550$ MeV, $m_v=783$ MeV, and $m_\rho=770$ MeV.

	g_s^2	g_v^2	g_ρ^2	F_0	F_1	F_0'	K (MeV)	a_4 (MeV)
MFT-I	122.43	190.06		0.559	−1.15		540	19.3
RHA	78.17	102.58		0.679	−0.62		450	15.0
MFT-II	122.43	190.06	36.79	0.559	−1.15	0.459	540	28.1
SDHF_1	109.07	149.79		0.891	−0.97	0.938	570	33.8
SDHF_2	107.68	148.44	6.1	0.856	−0.98	1.01	580	35.0

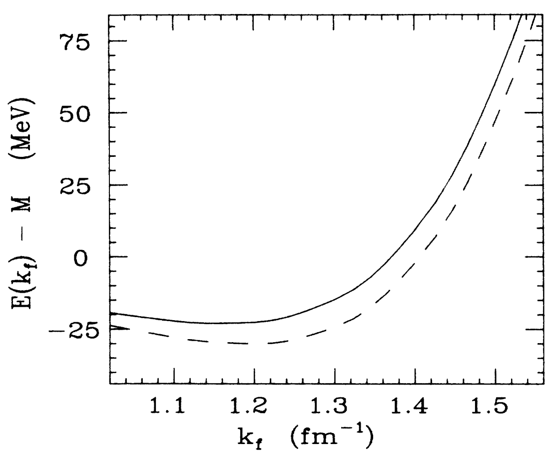

FIG. 1. The DHF (σ,ω,π,ρ) self-consistent single-particle energy at Fermi surface $E(k_F)$ (solid line) and the Fermi energy $E_{\rm HV}(k_F)$ (dashed line). M is the nucleon mass: $M=939$ MeV.

pion-vertex strength is now $c^2(g_\pi/2M)^2$, and the curvature of the pion energy density versus k_F is similar about normal density, for example $c=|\mathbf{c}|\sim 100$ MeV, to the result given by the momentum-dependent vertex calculation, but the equation of state becomes stiffer, resulting in the large compressibility. The momentum-dependent vertex and retardation have a significant effect on the nuclear matter calculation

We can include phenomenological vertex factors, such as

$$\exp(-a^2q^2) \text{ and } \frac{1}{1-q^2/\lambda^2}, \tag{3.9}$$

where a and λ are a range and a cutoff, respectively. However, it is obvious that these phenomenological factors work to reduce exchange corrections from relatively large momentum q, and so the DHF approximation becomes close to the MFT result. Moreover, we can include the vacuum fluctuation correction effectively. Let us assume that the self-energy arising from the modified Dirac sea at finite density can be calculated in the mean-field approximation as it is discussed in the RBBG approximation.[10] Note that the procedure given in the RBBG is valid with neglecting retardation correction when the energy density is calculated, and so we consider the static DHF approximation when we include the effective vacuum fluctuation correction.

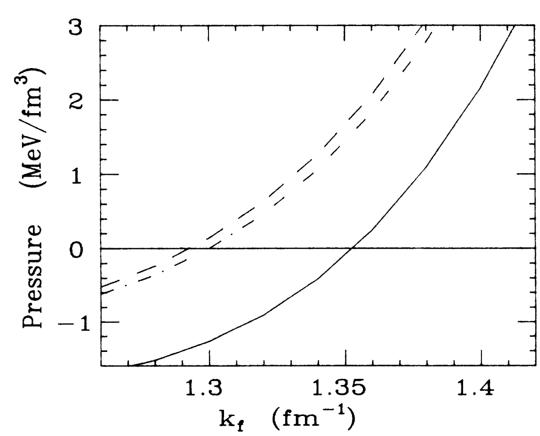

FIG. 2. Pressures calculated by the DHF approximation are shown. $\mathcal{P}_{\rm hy}=\frac{1}{3}\langle T^{ii}\rangle$ (solid line), $\mathcal{P}_{\rm HV}=E(k_F)\rho_B-\varepsilon$ (dashed line) and $\mathcal{P}_T=\rho_B^2(\partial/\partial\rho_B)(\varepsilon/\rho_B)$ (dot-dashed line). Note three different saturation densities at $\mathcal{P}=0$.

The vacuum correction to the self-energy comes only from a scalar term, and the renormalization procedure discussed in Refs. 5 and 15 gives the following correction to the scalar self-energy:

$$\Sigma^s_{\rm vac}=i\frac{g_s^2}{m_s^2}\int\frac{d^4q}{(2\pi)^4}{\rm Tr}[G_F(q)]+{\rm CTC}$$

$$=\frac{g_s^2}{m_s^2\pi^2}[M^{*3}\ln(M^*/M)-M^2(M^*-M) -\tfrac{5}{2}M(M^*-M)^2-\tfrac{11}{6}(M^*-M)^3], \tag{3.10}$$

where CTC denotes "counterterm contributions" arising from renormalization procedure. The energy density is calculated as

$$\varepsilon=\langle\hat{T}\rangle+\tfrac{1}{2}\langle\hat{V}\rangle+\Delta\varepsilon_1+\Delta\varepsilon_2, \tag{3.11}$$

where the *kinetic* term $\langle\hat{T}\rangle$ and the *potential* term $\langle\hat{V}\rangle$ are, respectively,

$$\langle\hat{T}\rangle=\frac{\zeta}{(2\pi)^3}\int^{k_F}d^3k\frac{M^*(k)M+k^*(k)k}{E^*(k)}, \tag{3.12}$$

$$\langle\hat{V}\rangle=\frac{\zeta}{(2\pi)^3}\int^{k_F}d^3k\left[\frac{M^*(k)}{E^*(k)}\Sigma^s(k)+\frac{k^*(k)}{E^*(k)}k\Sigma^v(k) -\Sigma^0(k)\right]. \tag{3.13}$$

The vacuum fluctuation corrections to energy density, $\Delta\varepsilon_1$ and $\Delta\varepsilon_2$, are given by

$$\Delta\varepsilon_1=-\frac{1}{4\pi^2}[M^{*4}\ln(M^*/M)+M^3(M-M^*) -\tfrac{7}{2}M^2(M-M^*)^2 +\tfrac{13}{3}M(M-M^*)^3-\tfrac{25}{12}(M-M^*)^4], \tag{3.14}$$

and

$$\Delta\varepsilon_2=\frac{m_s^2}{2g_s^2}[\Sigma^s_{\rm vac}]^2. \tag{3.15}$$

Using self-energies given by (σ,ω,ρ,π) and Eqs. (3.10)–(3.15), the vacuum correction is self-consistently included in the DHF approximation.

The effective vacuum fluctuation correction gives a repulsive contribution to the DHF approximation. The correction softens the equation of state in the normal nuclear matter range; consequently, it gives the smaller value of compressibility than the one found in the DHF calculation. However, we have $E(k_F)\neq E_{\rm HV}(k_F)$ and $\mathcal{P}_T\neq\mathcal{P}_{\rm HV}$, and thermodynamic consistency is violated seriously in this calculation; correct or physically reasonable self-consistency of QHD approximations should be carefully redefined when vacuum fluctuation corrections are included.

We understand that the sufficient condition for a conserving approximation is $\mathcal{P}_{\rm hy}=\mathcal{P}_{\rm HV}=\mathcal{P}_T$. Equality of pressures is sufficient condition for the HV theorem: $E_{\rm HV}(k_F)=E(k_F)$, but theorem is not sufficient for equality of pressures and consistency between dynamical and thermodynamic calculations as it is discussed in this section. Ambiguity of an approximation arises when the approximation maintains the HV theorem, but fails thermodynamic consistency, and in this case, we have to reconsider assumptions and self-consistent structure of the approximation. If an approximation maintains the HV theorem, we only need to check the condition $\mathcal{P}_{\rm HV}=\mathcal{P}_T$, since $E_{\rm HV}(k_F)=E(k_F)$ assures $\mathcal{P}_{\rm hy}=\mathcal{P}_{\rm HV}$. This is useful since $\mathcal{P}_{\rm HV}$ is readily calculated when the coupled nonlinear equation for $E(k)$ is solved. Consistency condition is rewritten in terms of energy-momentum tensor as

$$\tfrac{1}{3}\langle \hat{T}^{ii}\rangle=\rho_B^2\frac{\partial(\langle \hat{T}^{00}\rangle/\rho_B)}{\partial\rho_B}\ , \tag{3.16}$$

with

$$\frac{\partial\langle \hat{T}^{00}\rangle}{\partial\rho_B}=E(k_F)\ , \tag{3.17}$$

for the quasiparticle energy spectrum. Therefore, the diagonal components of energy-momentum tensor of ideal perfect fluid is related to each other through the Eq. (3.16), and together with (3.17), they define new self-consistency for relativistic approximations. The HV theorem and thermodynamics are strict criteria in order to choose the exact Green function and self-consistency to an approximation.

IV. THERMODYNAMIC PROPERTIES

Since the static DHF approximation is a conserving approximation, the thermodynamic properties, such as

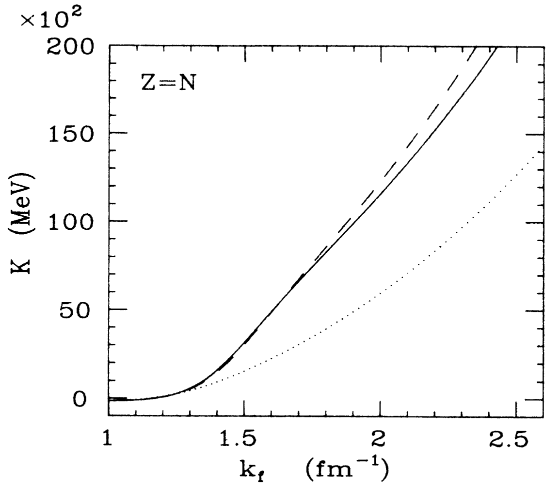

FIG. 4. Compression moduli for symmetric nuclear matter ($Z=N$), vs the Fermi momentum; the SDHF_2 (solid line), MFT-I (dashed line), and RHA (dotted line).

compression modulus, K, symmetry energy, a_4, and Landau parameters (F_0,F_1,F_0') are given exactly by following expressions:

$$K=9\rho_B\frac{\partial^2\varepsilon}{\partial^2\rho_B}=9\rho_B\frac{\partial\mu}{\partial\rho_B}=9\rho_B(1+F_0)/N_F\ , \tag{4.1}$$

where μ is the chemical potential, $\mu=E(k_F)$, and N_F is the density of states at the Fermi surface:

$$N_F=\frac{\zeta k_F^2}{2\pi^2}[\partial k/\partial E(k)]_{k=k_F}\ .$$

The density of states is related to the Landau parameter F_1 as

$$\left(\frac{\partial k}{\partial E}\right)_{k=k_F}=\frac{\mu}{k^*(k_F)}(1+\tfrac{1}{3}F_1)\ . \tag{4.2}$$

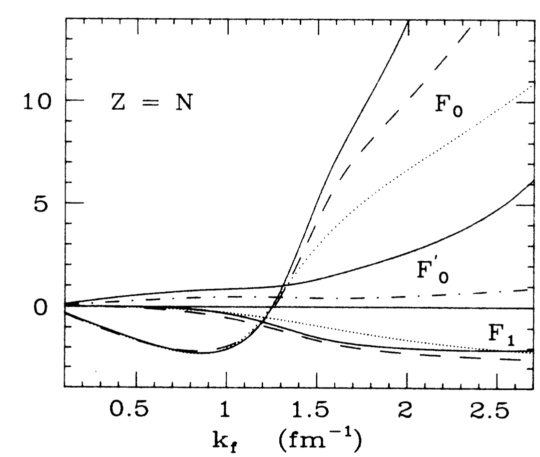

FIG. 3. Dimensionless relativistic Landau parameters for symmetric nuclear matter ($Z=N$). The solid lines are the results of the SDHF_2 approximation (σ,ω,ρ); the dashed and dotted lines are the MFT-I and RHA, respectively. The dot-dashed line of F_0' is calculated from MFT-II (Ref. 18).

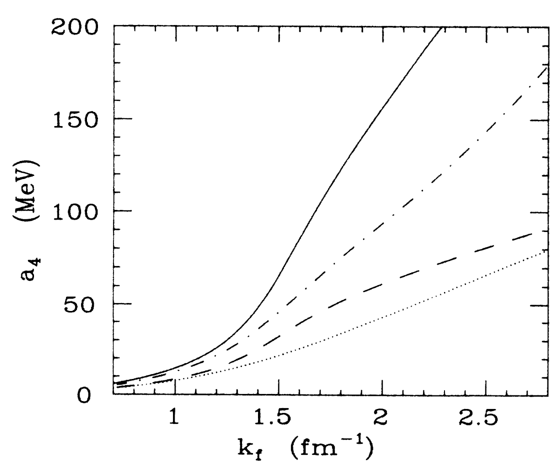

FIG. 5. Symmetry energies. The SDHF_2 (solid line), MFT-I (dashed line), RHA (dotted line), and MFT-II (dot-dashed line) are shown.

The symmetry energy is also related to the (isospin-dependent) Landau parameter F_0' as

$$a_4=\frac{1}{2}\rho_B\left[\left[\frac{\partial^2\varepsilon}{\partial^2\rho_3}\right]_{\rho_B}\right]_{\rho_3=0}=\frac{1}{6}\frac{k_F}{[\partial k/\partial E(k)]_{k=k_F}}(1+F_0') \ . \quad (4.3)$$

The higher-order Landau parameters can be calculated by coupled nonlinear integral equations as discussed in the Ref. 12. Landau parameters, compression modulus and symmetry energy are calculated in the static DHF (σ,ω,ρ), and compared with the results of the MFT, RHA, and MFT-II approximations.

In the SDHF, ρ-meson contribution has a small effect on the binding energy of the σ-ω results, but the symmetry energy is changed considerably; and so the coupling constant g_ρ is decreased further in order to reproduce the symmetry energy which is taken here to be $a_4=35$ MeV.[16] If Fermi liquid properties of the MFT and RHA are compared in Figs. 3–5, the Dirac sea particle correction to the MFT is not so large ($\sim 20\%$) at saturation as expected, and so the MFT and RHA may be good nonperturbative starting points at normal density. The particle-antiparticle correlation effect is important for high-density region, while the correlation effect between real nucleons in the Fermi sea may be more important at saturation, which implies that the Fermi-sea particle approach should be useful to understand dynamics about normal density. The consistent retardation correction including pion needs to be discussed, since the binding energy curve at saturation is sensitive to the change caused by retardation, and subsequently it changes compressibility and symmetry energy substantially.

V. CONCLUSION

The concept of quasiparticle in the strongly interacting nuclear many-body system should be defined consistently by the relation between Fermi energy, density of particles and total energy of the system according to the HV theorem. However, as it has been discussed in this paper, the theorem is not the sufficient condition for constructing a conserving approximation, and one should examine certain thermodynamic relations in order to check internal consistency of an approximation. One can modify approximations in terms of the HV theorem uniquely, however, there may appear serious inconsistency in a thermodynamic relation and consequently, in the definition of the density of nuclear matter at saturation. The problem raises a question whether assumptions and calculational scheme to an approximation are physically compatible or not. On the other hand, thermodynamic consistency, $\mathcal{P}_{\rm hy}=\mathcal{P}_{\rm HV}=\mathcal{P}_T$ and $E(k_F)=E_{\rm HV}(k_F)$ is the correct consistency condition that is maintained in conserving approximations. This conclusion is compatible with the one of the Refs. 1, and it will give us the criteria for modifications to the approximation. A consistent quasiparticle approximation can be defined uniquely when one achieves correct self-consistency and the self-consistent Green function to the approximation.

The specific form of the full Green function $G(k)$ can be written down using Dirac equation and Dyson's equation as the solution to the self-consistent QHD approximations. The same self-consistency is employed with assumptions explained in Sec. II when $G(k)$ is replaced by $G_D(k)$, and this is the reason why several assumptions must be checked; we used the HV theorem and thermodynamics in order to examine internal consistency of approximations. Retardation interaction, which is absent in the nonrelativistic HF approximation, breaks the requirement of the HV theorem in the DHF approximation. The single-particle energy spectrum at the Fermi surface defined in the DHF approximation is not equal to the average energy per particle at saturation $[E(k_F)\neq\varepsilon/\rho_B]$; consequently, it defines different nuclear matter saturation densities, which are shown by calculating pressures, $\mathcal{P}_{\rm hy}$, $\mathcal{P}_{\rm HV}$, and $\mathcal{P}_T$. The properties of nuclear matter saturation ($k_F=1.30$ fm^{-1}, $\varepsilon/\rho_B-M=-15.75$ MeV) are reproduced by fitting the binding energy curve, ε/ρ_B-M, with the effective coupling constants g_s and g_v, but we should note that we have an additional constraint at nuclear matter saturation: pressure of nuclear matter must vanish at saturation. The correct self-consistent calculation should be constructed to maintain required constraints when modifications and vacuum fluctuation corrections are introduced and retardation interaction plays an important role in an approximation.

The DHF approximation including pion breaks the HV theorem even in the static limit, $E(k)-E(q)=0$, and we have found that inconsistency also comes from the momentum-dependent pion vertex. The HV theorem can be recovered by subtracting terms which lead to strong short-range correlations between nucleons. The subtractions are uniquely defined by the requirement of the HV theorem. However, we have also found serious inconsistency in a thermodynamic relation, and this shows that the subtractions are not physically reliable and affect the self-consistent solution to the approximation. Therefore, the HV theorem and thermodynamics clarify the validity of modifications to an approximation and where inconsistency of an approximation arises. The similar problem in the nonrelativistic calculation including pion should be carefully examined for the check of internal consistency and the application to Landau Fermi-liquid theory. A static DHF approximation can be defined in order to restore thermodynamic consistency by setting pion-vertex energy-momentum transfer constant; we obtained $\mathcal{P}_{\rm hy}=\mathcal{P}_{\rm HV}=\mathcal{P}_T$ and $E(k_F)=E_{\rm HV}(k_F)$. In this case, $G_D(k)$ is the exact self-consistent solution to the approximation. A conserving approximation can be defined using $G_D(k)$ when appropriate self-consistency is built in an approximation; self-consistency and the Green function of the Fermi particle approximations should be improved in order that it maintains thermodynamic consistency.

The Fermi-sea particle approximations are nonconserving except the MFT and RHA approximations, but it is possible that we can obtain conserving approximations from other QHD approximations when appropriate self-consistency and certain modifications are employed. Simplifications and modifications of complicated approximations are necessary because of practical calculations and qualitative understanding of a physical system. To construct a conserving approximation in RBBG, for example, certain simplification could be suggested,[17] however, the validity of modifications to the approximation should be examined in terms of the HV theorem and thermodynamics, and these criteria will help to construct and find consistent approximations. The physical importance and usefulness of thermodynamic consistency in practical calculations are emphasized through specific examples. We can build self-consistent relativistic approximation uniquely when the HV theorem and thermodynamics are incorporated into the $G_D(k)$ formalism. Thermodynamically consistent or conserving approximations can preserve all physically required constraints, and the general solution to the relativistic Fermi-sea particle approximations is an open question. It is important in the Fermi-sea particle formalism to understand how to construct the systematic self-consistent quasiparticle approximations which maintain properties of conserving approximations.

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