Interacting boson fermion model description for the levels of ⁷¹Ge₃₉ populated in the beta decay of 65.30-h ⁷¹As

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We have studied the level properties of the N = 39 nucleus ⁷¹Ge by gamma-ray spectroscopy following the beta decay of ⁷¹As, for which we measure a half-life of 65.30±0.07 h. When we calculated the level structure of ⁷¹Ge within the framework of the interacting boson fermion model, we found good agreement with the experimentally determined level properties. Some evidence was found for the occurrence of levels built on the coexisting ⁷⁰Ge-core intruder state.

I. INTRODUCTION

The level structure of transitional nuclei has been difficult to account for within the framework of early nuclear models. The advent of quadrupole collective and interacting boson models has met with success in the description of widely varying classes of even-even nuclei. This has included even-even nuclei in which coexisting sets of excitations have been shown to occur.^{1,2} The interacting boson model (IBM) and its equivalent quadrupole phonon model (TQM) have been extended to account for odd mass nuclei. This extension is referred to as the interacting boson fermion model (IBFM). Here, we use the capability of this type of model to account for the low-energy level structure of the N=39 nucleus ⁷¹Ge. We then point to possible evidence for the coupling of the valence nucleons to the known cross subshell "intruder" band that was taken as a nonmodel state in the calculation.

The experimental information on the level properties of $^{71}\text{Ge}_{39}$ and the beta decay of ^{71}As was reviewed by Kearns and Mo³ and more recently by Bhat and Alburger.⁴

II. EXPERIMENTAL PROCEDURES

Sources of 61-h 71 As were made by subjecting targets of enriched 68 Zn to the alpha beam of the Lawrence

Berkeley Laboratory 88-in. cyclotron. Following irradiation, the targets were chemical-purified using standard radiochemical separation procedures at the Nuclear Chemistry Division of the Lawrence Livermore National Laboratory (LLNL).

The sources were measured using several Ge(Li) spectrometers ranging in characteristics from thin planar (xray) detectors to large volume (gamma-x) detectors. Energy calibration was performed by simultaneously measuring the sources with known multi-gamma-ray standards.⁵ A megachannel spectrometer⁶ was used to accumulate and analyze the coincidence spectra, while the code GAMANAL (Ref. 7) was used to analyze the singles spectra.

The half-life of ⁷¹As was measured in a separate set of experiments. Elsewhere⁸ we have reported the techniques we use to perform precision measurements of half-lives.

III. EXPERIMENTAL RESULTS

Our half-life value came from two separate measurements, one of which had 210 separate measurement points and another than had 183 separate measurement points. These yielded a value of 2.721 ± 0.003 d (65.30 ±0.07 h).

The energies and relative intensities of the gamma rays assigned to the decay of 71 As are given in Table I. In ad-

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Energy	Intensity ^a	Assignment		Coincident gamma rays
(keV)	(relative)	From	То	(and notes)
23 438(15)	0.226(13)	198	175	
64.69(-)	≤0.05	589	525	footnote b
174.954(5)	1000(3) ^c	174	g.s.	
195.22(15)	0.10(5)	1026	831	footnote d
247.351(5)	2.37(4)	747	499	279,348,392,465,499,571,659, (685),759,(852),882,(919),1044
264.21(15)	0.10(5)	1095	831	footnote d
279.379(7)	2.27(7)	1026	747	175,247,(331),(361),499,572,595, 774
287.319(35)	0.26(3)	1095	808	
306.217(25)	0.30(4)	831	525	
308.240(35)	0.14(2)	808	499	footnote d
	0.10(2)	1139	831	footnote d
311.15(15)	0.05(2)	1406	1095	footnote d
315.450(85)	0.03(1)			
324.922(62)	0.21(5)	499	175	
326.785(15)	37.0(3)	525	198	(185),306,308,386,570,616,680, 1033,(1073),1104,1267,1255,1276
331.479(44)	0.25(4)	1139	808	
331.4(2)	0.02(1)	831	499	footnote d
348.270(45)	0.58(9)	1095	747	247,572,747
350.163(6)	4.59(5)	525	175	175,(596)
373.837(12)	1.04(3)	1205	831	306,(572),(635)
375.700(85)	0.09(3)			
380.075(57)	0.10(2)	1406	1026	
387.305(42)	0.15(2)	1095	808	
391.383(18)	7.36(6)	589	198	174,(204),448,572,614,(951),1040, 1202,1348
392.156(54)	0.42(3)	1139	747	247,499,598,747
410.416(83)	0.10(5)			
414.386(95)	0.04(1)	589	175	
431.281(23)	0.25(1)	1139	708	
445.069(171)	0.02(1)	1743	1298	
448.519(98)	0.04(2)	1038	589	
457.718(117)	0.04(1)	1205	747	
465.228(1)	1.14(2)	1212	747	175,247,499,573,747
470.597(162)	0.02(1)			footnote e
499.876(10)	44.2(2)	499	g.s.	247,308,331,386,526,595,639, 680,705,712,798,906,1006,1058, 1098,1129,1292,1280,1437
504.281(47)	2.1(6)	1212	708	175,708
526 642(3)	10.5(1)	1026	499	175 324 499 924
533.2(2)	0.23(2)	708	174	175,521,755,521
533.6(2)	0.04(2)	1629	1095	920.1095, footnote d
551.5(1)	0.05(2)	1298	747	, 1 0,10,0, 1000000 u
570.42(21)	0.27(2)	1095	525	
572.255(15)	3.21(5)	747	175	175,218,348,392,457,465,551, 631,659,759,851,881,(996),1044
590.5(1)	0.3(1)	1298	708	
595.6(1)	1.0(1)	1095	499	175,499, footnote d
614.265(50)	0.17(3)	1139	525	
615.365(2)	6.42(4)	1205	589	
622.705(35)	0.16(1)	1212	589	
631.52(15)	2.4(4)	1378	747	footnote d
633.440(25)	0.58(7)	808	175	175,218,595,(749)
639.477(14)	0.59(3)	1139	499	
659.428(19)	0.83(3)	1406	747	
674.332(85)	0.07(1)	1506	831	

TABLE I. Energies, intensities, and coincidence relationships for the gamma rays in the decay of 65.30-h⁷¹As to the levels of ⁷¹Ge.

Energy (keV)	Intensity ^a Assignment (relative) From To		Coincident gamma rays	
	(Telative)	FIOIII	10	(and notes)
680.035(9)	1.18(5)	1205	525	326,350
696.575(12)	0.11(2)	1792	1095	
698.440(20)	0.24(1)	1406	708	
702.5(3)	702.5(3) 0.4(2)		747	footnote d
705.1(3)	0.03(2)	1205	499	footnote d
708.195(5)	3.27(7)	708	g.s.	
711.6(3)	0.08(2)	886	174	
712.598(5)	3.96(7)	1212	499	
727.531(22)	0.22(1)	1558	831	
741.6(3)	0.015(8)			
747.28(1)	1.98(5)	747	g.s.	249,348,(457),465,485,551,634 659,702,851,882,998,1044
754.4(3)	0.018(9)	1780	1026	
759.110(28)	0.18(2)	1506	747	
765.886(65)	0.07(1)	1792	1026	
788.915(47)	0.11(2)	1378	588	
798.0(2)	0.2(1)	1629	831	footnote d
798.4(2)	0.28(2)	1506	708	footnote d
808.271(33)	0.36(3)	808	g.s.	
828.0(1)	0.04(2)	1026	198	
831.294(10)	1.05(2)	831	g.s.	195,264,374,(647),727
839.3(3)	0.014(6)	1038	198	footnote e
851.3(2)	0.5(2)	1598	747	footnote d
				174,247,258,572,747
851.63(7)	2.2(2)	1026	174	footnote d
881.893(25)	0.37(3)	1629	747	247,572,747
886.983(97)	0.05(1)	886	g.s.	
890.0(2)	0.03(1)	1598	708	
906.696(11)	0.54(2)	1406	499	
920.553(7)	3.7(1)	1095	175	footnote d 175,533,708
921.(2)	0.2(1)	1096	175	footnote d
935.175(14)	0.34(4)	1743	808	175.308.499.633.808
964.479(9)	0.83(2)	1139	174	· · · · · · · · · · · · · · · · · · ·
983.666(50)	0.06(2)	1792	808	
996.064(60)	0.12(1)	1743	747	
1006.466(17)	0.53(2)	1506	499	
1009.9(3)	0.04(3)			footnote e
1026.512(17)	3.8(1)	1026	g.s.	
1030.200(80)	0.13(1)	1205	174	
1033.542(17)	2.38(3)	1558	525	175.326.350
1037.530(15)	2.46(3)	1212	174	174
1039.343(61)	0.11(1)	1629	589	
1044.845(19)	0.23(2)	1792	747	(247).(572).(747)
1050.6(2)	0.015(6)			
1055.9(2)	0.008(5)			
1058.817(16)	0.30(1)	1558	499	(217).499
1073.4(2)	0.023(5)	1598	525	(=,
1083.864(34)	0.14(1)	1792	708	
1090.490(10)	0.19(2)		,00	footpote e
1095.490(10)	49.8(7)	1095	g.s.	312
1098.64(7)	1.49(4)	1598	499	512
104.162(26)	0.14(1)	1629	525	
106.93(8)	0.06(1)	102/		
123.74(8)	0.036(6)	1298	175	
129.366(54)	0.07(1)	1629	499	
139.461(19)	9.75(5)	1139	8.5	
191.18(9)	0.06(2)	1780	589	
1000 0((5)	0.125(0)	1,00	500	

TABLE I. (Continued).

Energy	Energy Intensity ^a Assignment		ment	Coincident gamma rays
(keV)	(relative)	From	То	(and notes)
1211.350(78)	0.30(2)	1801	589	
1212.496(23)	3.39(6)	1212	g.s.	
1218.160(74)	0.030(6)	1743	525	
1231.692(15)	0.84(2)	1406	174	
1243.56(8)	0.021(4)	1743	499	
1247.0(1)	0.009(3)	1422	175	
1255.759(45)	0.050(5)	1780	525	
1267.008(20)	0.28(1)	1792	525	
1276.04(45)	0.09(3)	1801	525	footnote d
1280.914(65)	0.020(8)	1780	499	
1292.13(15)	0.016(5)	1792	499	
1297.507(47)	0.24(2)			
1298.729(15)	2.32(5)	1298	g.s.	
1307.980(34)	0.115(6)	1506	198	
1312.0(2)	0.011(3)			footnote e
1321(1)	0.006(4)			footnote e
1331.526(23)	0.96(3)	1506	174	
1347.7(5)	0.005(3)	1937	589	
1360.442(128)	0.104(4)	1558	198	
1379.0(5)	0.011(5)	1378	g.s.	
1383.864(35)	0.042(4)	1558	174	
1406.5(1)	0.026(6)	1406	g.s.	
1423.579(25)	0.34(1)	1598	174	
1454.26(15)	0.009(3)	1629	174	
1533.6(1)	0.007(3)			footnote e
1568.4(2)	0.02(1)	1743	175	
1582.334(65)	0.036(3)	1780	198	
1587.9(3)	0.004(2)			footnote e
1598.505(25)	0.43(2)	1598	g.s.	
1602.74(14)	0.031(4)	1801	198	
1605.749(21)	0.17(1)	1780	174	
1617.121(29)	0.247(6)	1792	174	
1629.154(15)	0.30(1)	1629	g.s.	
1722.1(5)	0.002(1)			footnote e
1743.399(35)	0.37(1)	1743	g.s.	
1762.494(58)	0.095(3)	1937	174	
1785.3(4)	0.003(2)			
1792.0(4)	0.005(3)	1792	g.s.	
1800.4(8)	0.003(1)			
1937.414(44)	0.137(3)	1937	g.s.	
1965.029(70)	0.014(3)	1965	g.s.	

TABLE I. (Continued).

^aThe γ -ray intensities are relative to te 174.954-keV intensity being taken as 1000 units. For conversion to absolute intensities, a factor of 0.082(3) should be used (see text) (Ref. 4).

^bA total intensity of ≤ 0.05 was determined from the 615 gate.

^cFiducial. Error representative of the statistics and peak shape quality of fit. An error of 2% should be added in quadrature when calculating the absolute intensities. This 2% represents the error in the knowledge in the overall shape of the efficiency curve.

^dIntensity derived from coincidence gate.

^eAssignment to the decay of ⁷¹As should be taken as tentative.

dition, the table lists the results of the coincidence experiments and the placement of the gamma rays in the proposed level scheme which is shown in Fig. 1. Since the parent ⁷¹As has a ground-state spin of $\frac{5}{2}^{-}$, we can assume that there is negligible beta feeding to the $\frac{1}{2}^{-}$ ground state of ⁷¹Ge. Using this fact and taking the multipolarity of the 175-keV transition to the ground state as *E*2 (and hence a total conversion coefficient of 0.093) leads to a conversion factor of 0.082(3) for conversion from the relative intensities in Table I to absolute intensities.⁴ The $\log f_t$ and $\log f_1 t$ values, which are given in Table II, were calculated for each level using the tables of Gove and Martin,⁹ a 2013±4 keV $Q_{\rm EC}$ value,⁴ and a half-life of 65.30 h. The $Q_{\rm EC}$ was taken from the A=71 evaluation in Nuclear Data Tables by Bhat and Alburger.⁴ The spin and parity values of the ground state (g.s.), and of the

175- and 198-keV levels, have been established as $\frac{1}{2}^{-}$, $\frac{5}{2}^{-}$, and $\frac{9}{2}^{+}$, respectively.¹⁰⁻¹² Below we give the spin parity values that we deduced using our data and previously published transfer reaction and in-beam results.

The 499.88-keV level: The ⁷²Ge(p,d)⁷¹Ge transfer reaction^{11,13} exhibits a strong L=1 transition implying a $\frac{1}{2}^{-}$ or $\frac{3}{2}^{-}$ assignment. This state is also strongly populated in the ⁷¹Ga(p,n)⁷¹Ge reaction.¹⁴ Our observation of the 1006-keV transition from the known $\frac{7}{2}^{-}$ level supports the assignment of $\frac{3}{2}^{-}$ and eliminates the $\frac{1}{2}^{-}$ possibility.

the assignment of $\frac{3}{2}^{-}$ and eliminates the $\frac{1}{2}^{-}$ possibility. *The 525.12-keV level:* The (p,n) reaction study of Malan *et al.*¹⁴ gives an L=2 distribution and our observation of a transition to the known $\frac{9}{2}^{+}$ level at 198 keV limits the assignment to $\frac{5}{2}^{+}$.

The 589.81-keV level: The angular distribution of the 391-keV transition observed in $(p,n-\gamma)$ studies¹² leads to a J-value range of $\frac{3}{2}$ to $\frac{11}{2}$. However, the branching ratios

of depopulating transitions as well as transitions from higher energy levels limit the value to $\frac{7}{2}^+$.

The 708.20-keV level: The (p,d) reaction shows an L=1 distribution^{11,13} limiting the assignment to $\frac{1}{2}^{-}$ or $\frac{3}{2}^{-}$. Polarization measurements¹⁵ and (p,n- γ) intensity data lead to an assignment of $\frac{3}{2}^{-}$.

The 747.23-keV level: The angular distribution of the g.s. transition observed in the $(p,n-\gamma)$ study¹² uniquely assigns the spin as $\frac{5}{2}$. An assignment of $\frac{5}{2}^{-}$ then results from the L=3 distribution found in the (p,d) studies.¹¹

The 808.20-keV level: The gamma-ray angular distribution from the $(p,n-\gamma)$ studies supports assignments of $\frac{1}{2}$ and $\frac{3}{2}$, while their intensity data has been used to limit the value to $\frac{1}{2}^{-}$. In our study the level is populated, within experimental error, solely by transitions from higher lying levels, which results in a limit of log ft > 11for any possible beta population (as should be expected



FIG. 1. Decay scheme for ⁷¹As: levels populated in ⁷¹Ge (a) up to 900 keV; (b) 1100 to 1200 keV; (c) 1201 to 1400 keV; (d) 1410 to 1509 keV; (e) 1510 to 1699 keV; and (f) 1700 to 2000 keV.





FIG. 1. (Continued).

from a $\frac{5}{2}^{-}$ parent).

The 831.29-keV level: A (p,d) study¹³ has shown an L=1 distribution and (p,n- γ) studies further limit its value to only $\frac{3}{2}^{-}$. Also, our observation of a transition to the known $\frac{5}{2}^{+}$ level excludes a $\frac{1}{2}^{-}$ assignment, as does the level's population from the known $\frac{7}{2}^{-}$ level at 1506 keV.

The 1026.51-keV level: The $(p,n-\gamma)$ studies¹² uniquely show that the J value must be $\frac{5}{2}$ and their data strongly support negative parity. Both the deexcitation and the population of the level from higher energy levels in our data are also consistent with a $\frac{5}{2}$ assignment.

The 1038.33-keV level: The $(p,n-\gamma)$ study¹² gives a $\frac{9}{2}^+$ assignment to this level. Our $\log f_1 t$ value and deexcitation transition pattern support this value.

The 1095.50-keV level: The polarized (d,p) studies of Bieszk *et al.*¹⁵ assign this level as $\frac{3}{2}^{-}$, and our data support such an assignment.

The 1096.1-keV level: The $(p,n-\gamma)$ studies established a level at 1096.1 keV based on the coincidence placement of 921.2- and 596.2-keV transitions. A value of $\frac{\gamma}{2}^{-}$ has been assigned from their data and the fact that the deexcitation populates the known $\frac{3}{2}^{-}$ level at 500 keV. Analysis of our data shows a doublet nature of the 920keV transition and leads to a log ft value of 8.8 for its population.

The 1139.33-keV level: This level was observed in one of the (p,d) studies,¹³ and the angular distribution results of the (p,n- γ) study¹² lead to a $\frac{3}{2}^{-}$ value, which is consistent with our gamma-ray branching results.

The 1205.16-keV level: The polarized (d,p) studies as-

sign a value of $\frac{5}{2}^+$ for this level and our observed deexciting transitions support this assignment.

The 1212.47-keV level: The $(p,n-\gamma)$ results uniquely select a spin of $\frac{5}{2}$ for this level. Also, they give a preference for negative parity based on the angular distribution of the transition to the g.s. Our branching ratios agree with this assignment.

The 1298.72-keV level: Spins of $\frac{1}{2}$ or $\frac{3}{2}$ are possible from the angular distribution data in the (p,n- γ) study. Our observation of the level's beta population from the ⁷¹As $\frac{5}{2}^{-}$ parent eliminates the $\frac{1}{2}$ value. The gamma-ray branching suggests negative parity.

The 1378.75-keV level: This level, which is weakly populated in beta decay, has been determined to have an L=2 distribution in the (d,p) studies.¹⁶ Our observation of a g.s. transition thus limits the assignment to $\frac{3}{2}^+$.

The 1406.69-keV level: The (p,d) studies^{11,13} observe an L=3 distribution. Our observation of a transition to the g.s. thus limits the assignment to a $\frac{5}{2}$ value.

The 1422.0-keV level: In-beam studies assign this level as possessing a $\frac{9}{2}^{-}$ value. Our observation of this level in the beta decay of $\frac{5}{2}^{-71}$ As presumably results from its population by gamma-ray transitions from higher energy levels.

The 1449.8-keV level: Our coincidence data places a 702-keV gamma ray as the sole observed deexciting transition of this level. We tentatively take this to be the same level as observed in (d,p) studies.

The 1506.48-keV level: This level is observed with an L=3 distribution in the (p,d) reaction^{11,13} as well as the (p,n) study.¹⁴ Our observation of a 1308-keV transition

to the 198-keV $\frac{9}{2}^+$ level limits the assignment to $\frac{7}{2}^-$.

The 1558.69-keV level: This level is observed to have an L=2 distribution in the polarized (d,p) study and was given a $\frac{5}{2}^+$ assignment.¹⁵ Our observation of a transition to the 198-keV $\frac{9}{2}^+$ level supports this choice.

The 1598.51-keV level: This level has been observed to have an L=1 distribution in (p,d) studies^{11,13,16} and is observed in the (d,p) and (p,n) studies as well. Our observation of its direct population in the beta decay of ⁷¹As, which has a $\frac{5}{2}^{-}$ g.s., leads to a $\frac{3}{2}^{-}$ assignment.

The 1629.15-keV level: This level has been observed in (p,n) studies¹⁴ and is limited to a value of $\frac{3}{2}^+$ or $\frac{5}{2}^-$. Our data are consistent with the $\frac{3}{2}^+$ assignment.

The 1743.24-keV level: This level is populated by an L=1 distribution in the (p,d) studies, and its direct population in the ⁷¹As beta decay limits the assignment to $\frac{3}{2}^{-1}$.

TABLE II. Population of ⁷¹Ge levels in the beta decay of 65.30 h $\frac{5}{2}$ - ⁷¹As.

Level		EC+beta		
(keV)	J^{π}	(percent)	logft	$\log f_1 t$
0	$\frac{1}{2}$ -			
175	$\frac{2}{5}$ -	84.0	5.9	
198	$\frac{2}{9} +$	0.5		9.02
499	$\frac{2}{3}$ -	1.9	7.2	
525	$\frac{2}{5}$ +	3.0	7.0	
589	$\frac{2}{7}$ +			
708	$\frac{2}{3}$ -			
747	$\frac{5}{2}$ -	0.5	7.6	
808	$\frac{1}{2}$ -	≤0.02	≥11	
831	$\frac{2}{3}$ -			
886	$\frac{3}{2}$ -	0.011	9.1	
1026	$\frac{5}{2}$ -	1.53	6.9	
1038	$\frac{2}{9} +$	0.0044		9.96
1095	$\frac{2}{3}$ -	4.6	6.3	
1096	$\frac{2}{7}$ -	0.02	8.7	
1139	$\frac{3}{2}$ -	1.0	6.9	
1192	$\frac{11}{2}$ +			
1205	$\frac{5}{2}$ +	0.81	7.0	
1212	$\frac{5}{2}$ -	1.08	6.8	
1298	$\frac{\tilde{3}}{2}$ -	0.22	7.4	
1378	$\frac{3}{2}$ +	0.01	8.7	
1406	$\frac{5}{2}$ -	0.23	7.3	
1421	$\frac{\bar{9}}{2}$ -			
1449	-	0.03	8.1	
1506	$\frac{7}{2}$ -	0.17	7.2	
1558	$\frac{5}{2}$ +	0.25	7.0	
1598	$\frac{3}{2}$ -	0.19	7.0	
1629	$\frac{3}{2}^{+}$	0.01	8.2	
1743	$\frac{3}{2}$ -	0.08	7.0	
1780	$\frac{7}{2}$ -	0.03	7.3	
1792	$\frac{5}{2}$ -	0.011	7.7	
1801	7 ⊥	0.03	7.2	
1937	$\frac{3}{2}$	0.02	6.4	
1965	<u>3</u> -	0.001	7.2	

The 1780.70-keV level: Our observation of a transition to the known $\frac{9}{2}^+$ 198-keV level and an L=3 distribution in the (p,d) study leads to a $\frac{7}{2}^-$ assignment.

The 1792.08-keV level: Although observed in (p,d) and (d,p) studies, no angular distribution is reported for this level. Our branching ratio data suggest a value of $\frac{5}{2}^{-}$.

The 1937.45-keV level: The log ft transition to the g.s. and transitions to the 174- and 594-keV levels suggest a $\frac{3}{2}^+$ value. However, no definite value can be given.

The 1965.03-keV level: This level has an L=1 distribution in the (p,d) studies and is populated directly in the ⁷¹As beta decay. Thus the assignment is limited to $\frac{3}{2}^{-}$.

IV. IBFM CALCULATION OF THE LEVEL STRUCTURE OF ⁷¹Ge₃₉

We have calculated the level structure of ${}^{71}\text{Ge}_{39}$ by coupling a neutron quasiparticle in the N=28 to 50 shell to the ${}^{70}\text{Ge}_{38}$ even-even core. We have described the core nucleus within the IBM, ${}^{17-19}$ and the even-odd ${}^{71}\text{Ge}$ within the IBFM. ${}^{20-22}$

The parametrization for the core nucleus ⁷⁰Ge has been adjusted to the positions of the low-lying levels. However, we have left out from the fit the experimental 0^+ level at 1216 keV and its associated 2^+ band member at 1708 keV (Ref. 23) because it has been suggested that these can be thought of as arising from the occurrence of a cross subshell intruder band built on the excitation of a pair of



FIG. 2. Comparison of the experimentally known levels of ⁷⁰Ge with our IBM core calculation. The experimental data was taken from Nuclear Data Sheets (Ref. 23). [Nota bene: Elsewhere (Ref. 25) we have shown that the intruder band and the g.s.-band excitations mix. This results in (1) allowing the intruder state to deexcite to the g.s. band excitations [as, in the case here, exhibited by the B(E2) ratio shown in this figure]; and (2) altering the energy of the mixed states: thus to zero order $2^+(2)$ states originally predicted at 1840 keV can be expected to be at a higher energy in the mixed system.]

 $1g_{9/2}$ protons and $1g_{9/2}$ neutrons; hence, we will refer to it as an intruder band similar to those found in this region¹ (see caption of Fig. 2). The core parameters used were (*h* in units of MeV) $h_1 = 1.036$, $h_2 = -0.17$, $h_3 = 0.03$, $h_{40} = 0.2$, $h_{42} = -0.4$, and $h_{44} = -0.08$, and the boson number was taken as N=7, which is half the number of valence shell particles. The parameters h_i are defined in Ref. 19. The relation to the parameters in Ref. 17 is

$$h_1 = \varepsilon_d - \varepsilon_s ,$$

$$h_2 = \widetilde{v}_0(2)^{-1/2} ,$$

$$h_3 = \widetilde{v}_2 ,$$

$$h_{4L} = \frac{1}{2}(2L+1)^{1/2}C_L$$

Figure 2 shows a comparison of our core description to the experimentally known levels.

It should be noted that N=39 is exactly the midshell and therefore a description in terms of 10 neutron holes in the N=28-50 shell should be equally applicable. In the latter case the core nucleus would be ⁷²Ge. However, the low-lying model states of both possible core nuclei (⁷⁰Ge and ⁷²Ge) are similar and without much influence on the present results. On the other hand, the experimental 0⁺(2) and 2⁺(2) levels exhibit the characteristics of the states of an intruder band and are not model states. In particular, the position of the 0⁺(2) level is low, close to the 2⁺(1), and strongly changes from isotope to isotope, being the lowest in ⁷²Ge, while the B(E2) value for the 2⁺(2) to 0⁺(2) transition is much larger than for the 2⁺(2) to 0⁺(1) transition.

In the calculation of the negative parity states, we include the negative parity neutron quasiparticle states $p_{1/2}$, $f_{5/2}$, and $p_{3/2}$ in the N=28 to 50 valence shell. We take the quasiparticle energies as $(e_{f_{5/2}})-(e_{p_{1/2}})=0.63$ MeV and $(e_{p_{3/2}})-(e_{p_{1/2}})=0.95$ MeV, and the occupation probabilities as $v^2(p_{1/2})=0.327$, $v^2(f_{5/2})=0.949$, and $v^2(p_{3/2})=0.941$, which is in accordance with the Kisslinger and Sorensen BCS parametrization.²⁴ The fermion-boson interaction strengths employed in the calculation are $A_0=0.08$ MeV, $\Gamma_0=0.05$ MeV, $\Lambda_0=2.2$ MeV, and $\chi=-\frac{1}{2}(7)^{1/2}$. These four parameters are defined as in Ref. 22, which coincides with the standard IBFM definitions for A_0 in Ref. 28 and Ref. 20 for the remainder.

For the calculation of positive parity states, we have included the $g_{9/2}$ valence shell quasiparticle state. In addition, we included the $d_{5/2}$ configuration from the shell above. Because of the large non-spin-flip matrix element $\langle d_{5/2} || Y_2 || g_{9/2} \rangle$, this configuration has a noticeable effect on the positive parity states arising from the $g_{9/2}$ configuration. In particular, the $J = (j-2) = \frac{5}{2}^+$ state is lowered in energy by a considerable amount.²² In the calculation we employ $v^2(g_{9/2}) = 0.09$ and $v^2(d_{5/2}) = 0.01$, with the $d_{5/2}$ state lying 3 MeV above the $g_{9/2}$ quasiparticle state. The boson-fermion interaction strengths are $A_0 = 0.1$ MeV, $\Gamma_0 = 0.38$ MeV, $\Lambda_0 = 5.0$ MeV, and $\chi = -\frac{1}{2}(7)^{1/2}$. The higher values for these parameters than for the negative parity states can be understood as a consequence of only one valence-shell quasiparticle configuration of unique parity.²² In principle, we might expect that Γ_0 and Λ_0 are equal for odd- and even-parity states. However, this difference may be caused by two factors: First, as pointed out in Ref. 26, in deriving expressions for the interaction strengths, all radial integrals have been taken equal. Second, according to Ref. 27, the renormalization effects on Γ_0 and Λ_0 can be also caused by the exclusion of the positive-parity single-particle levels other than $g_{9/2}$ and $d_{5/2}$.

We first compare our experimental and calculational



FIG. 3. Energy spectrum of ⁷¹Ge calculated using the IBFM compared with experimentally known levels: (a) comparison of negative-parity levels, and (b) positive parity levels.

results. In Fig. 3 the level spectra of 71 Ge that we have calculated are compared with the experimentally known levels, which were taken from the literature^{3,4,10-16} and from our results. We then use the occurrence of additional levels beyond those expected to suggest that the additional levels arise from the coupling of the valence nucleons to the core-intruder configuration that was not included in the model core.

Using the wave functions obtained by diagonalization of the Hamiltonian, we have calculated the reduced transition probabilities for the E2 and M1 transitions among the low-energy states employing the standard form of E2 and M1 operators given in Refs. 20 and 22. In the calculation of the E2 properties, we take the standard parameter values: $e^{\text{s.p.}} = 0.5$, $\chi = -\frac{1}{2}(7)^{1/2}$, and adjust the boson charge to the experimental data, $e^{\text{vib}} = 0.8$. In the calculation of the M1 properties we use the standard values of the gyromagnetic ratios, $g_R = Z/A$, $g_1 = 0$, and $g_s = 0.7g_s$ (free). Here the effective charges and gyromagnetic ratios are defined according to Ref. 22. The boson gyromagnetic ratio g_R is taken in accordance with the standard procedure in the IBM.^{26,28} This g_R value is consistent with the observed magnetic moment of the $2^+(1)$ state in ⁷⁰Ge (Ref. 29). For $g_R = Z / A = 0.46$ there is μ {2⁺(1)}=0.92 μ_N ; on the other hand, using the g_R value corresponding to the valence space only, the resulting magnetic moment μ {2⁺(1)}=0.57 μ _N would be too small. The calculated static moments and transitions are presented in Tables III and IV, where they are compared with the experimental data. The agreement between theory and experiment is good for both positive and negative parity states.

As can be seen in Fig. 3(a), there is a good agreement between the calculated level spectra and the experimentally determined negative parity levels up to approximately 800 keV. However, in the measurable range from 800 to ~ 1500 keV, a larger than predicted number of levels occur. As similar calculations for nuclei in this region²² have provided a good description of levels up to ~2000 keV, it is reasonable to suspect an origin different from a breakdown of the model. However, as we pointed out earlier, we have explicitly excluded the intruder band in the effective core that we have used in the calculations. Thus we might expect this to give rise to the additional levels.

Elsewhere,¹ we have shown that the neutron-proton interaction between particles that populate the 1g orbital can produce coexisting structures in this region. As shown in Fig. 4, we can visulize the process of promotion and mutual polarization as giving rise to the coreintruder state. When this occurs in the ⁷¹Ge core, the valence neutrons can be expected to populate the $p_{3/2}$ and $f_{5/2}$ orbits, and thus we can expect additional lowenergy $\frac{3}{2}^{-}$ and $\frac{5}{2}^{-}$ levels. As seen in Fig. 3, this is what we observe. Further, if the odd neutron is promoted to the $1g_{9/2}$ orbit, we might expect a $\frac{9}{2}^+$ level. In the experimental spectra, a $\frac{9}{2}^+$ level is observed to occur at approximately the expected energy. However, at present there is insufficient evidence to differentiate this from the level that arises out of the $1g_{9/2}$ odd neutron coupled to the g.s. core excitation.

As there has been little evidence of valence nucleons in odd mass nuclei acting as spectators coupled to core-intruder-state excitations, further work such as inbeam gamma-ray/conversion electron and lifetime measurements would greatly help clarify the nature of the "excess" levels in 71 Ge.

V. CONCLUSION

We have investigated the levels of ⁷¹Ge both experimentally and theoretically. The calculation of the low energy positive and negative parity levels of ⁷¹Ge within the IBFM reproduces rather well the experimental low

Level	Q (e b)		μ (μ	$\frac{\mu (\mu_N)}{\mu}$	
(keV)	Theor.	Expt.	Theor.	Expt.	
	N	egative parity levels			
$\frac{1}{2}(1)$			0.45	0.55	
$\frac{5}{2}(1)$	0.11		0.99	1.02	
$\frac{3}{2}(1)$	0.01		0.45		
$\frac{3}{2}(2)$	0.10		-1.13		
$\frac{5}{2}(2)$	0.02		0.52		
$\frac{7}{2}(1)$	-0.01		1.33		
	Р	ositive parity levels			
$\frac{9}{2}(1)$	-0.31	±0.34	-1.22	-1.04	
$\frac{5}{2}(1)$	-0.06		-1.72		
$\frac{7}{2}(1)$	-0.15		-1.09		
$\frac{3}{2}(1)$	0.14		-0.69		
$\frac{13}{2}(1)$	-0.43		-0.31		
$\frac{1}{2}(1)$	-0.30		-0.38		

TABLE III. Static moments of ⁷¹Ge.

Transition		Intensity		Mix	Mixing ratio	
From	То	Theor.	Expt.	Theor.	Expt. ^a	
		Negativ	e parity levels			
$\frac{3}{2}(1)$	$\frac{5}{2}(1)$	7	0.5	3.44		
-	$\frac{1}{2}(1)$	100	100	-0.03	M1 + E2	
$\frac{3}{2}(2)$	$\frac{3}{2}(1)$	0.3		0.07		
	$\frac{5}{2}(1)$	0.01	7	0.41		
	$\frac{1}{2}(1)$	100	100	0.01		
$\frac{5}{2}(2)$	$\frac{3}{2}(2)$	0.07		-0.05		
	$\frac{3}{2}(1)$	91	74	0.06	M 1	
	$\frac{5}{2}(1)$	100	100	5.59	3.33	
	$\frac{1}{2}(1)$	13	62			
$\frac{1}{2}(2)$	$\frac{5}{2}(1)$	0.00				
-	$\frac{3}{2}(2)$	2		0.09		
	$\frac{3}{2}(1)$	359	41	-0.04		
	$\frac{5}{2}(1)$	100	100			
	$\frac{1}{2}(1)$	7	62	0		
$\frac{5}{2}(3)$	$\frac{3}{2}(3)$	0.1		0.02		
	$\frac{1}{2}(2)$	0.02				
	$\frac{5}{2}(2)$	53	22	0.04		
	$\frac{3}{2}(2)$	5		-0.38		
	$\frac{3}{2}(1)$	100	100	-0.26		
	$\frac{5}{2}(1)$	59	25	-1.43		
	$\frac{1}{2}(1)$	75	36			
		Positiv	e parity levels			
$\frac{7}{2}(1)$	$\frac{5}{2}(1)$	1.2	2.7	0.02		
	$\frac{9}{2}(1)$	100	100	-0.22	2.9	
$\frac{5}{2}(2)$	$\frac{7}{2}(1)$	100	100	-0.14		
	$\frac{5}{2}(1)$	7	16.2	1.05		
	$\frac{9}{2}(1)$	2				
$\frac{1}{2}(1)$	$\frac{5}{2}(2)$	0.01				
	$\frac{5}{2}(1)$	100	100			

TABLE IV. Branching and mixing ratios for ⁷¹Ge.

^aTaken from Ref. 3.



FIG. 4. Mechanism suggested for the occurrence of the coexisting band in the 70 Ge core (for a complete discussion of the excitation mechanism and its consequences, see Ref. 1).

energy levels and their electromagnetic properties. A crucial assumption in our calculations was that the nonspin-flip interaction between the $g_{9/2}$ and $d_{5/2}$ states has a strong influence on the nature of some of the low energy positive parity states. Also, in describing the effective boson core we excluded the known intruder 0⁺ level at 1206 keV and its associated 2⁺ level. We find evidence for the occurrence of levels in excess of those expected from the calculation which has been found to accurately predict the level spectra up to about 2 MeV in neighboring nuclei that do not possess core-intruder-state (nonmodel) excitations. We associated their occurrence with the coupling of the valence nucleons to the intruder state known to occur in the ⁷⁰Ge core.

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